STAFF MEMO

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A model of credit risk in the corporate sector based on bankruptcy prediction*

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Abstract: We propose a method for assessing the risk of losses on bank lending to the non-financial corporate sector based on bankruptcy probability modelling. We estimate bankruptcy models for different industries and attach a risk weight to each firm's debt in a given year. The risk weight is equal to the probability of bankruptcy. By summing all risk-weighted debt in an industry, we obtain an estimate of the share of debt in bankruptcy accounts in a given year. A key feature of our model is the inclusion of economic indicators at the industry level, observed in real time, as explanatory variables together with standard financial accounting variables and real-time credit rating information. We find that historically, during 2000–2014, there is good correspondence between our estimated measure of risk-weighted debt and actual debt in bankruptcy accounts. Moreover, bank losses according to bank statistics and debt in bankruptcy accounts display a similar pattern over time in most industries.

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1 Introduction

A large proportion of banks' lending is to the corporate sector. Losses on corporate loans have historically exceeded losses on household loans by a wide margin both during banking crises and in periods without major solvency crises. This applies both to Norway during the banking crisis of 1988-1993 and internationally, e.g. during the 2008 financial crisis (see Kragh-Sørensen and Solheim, 2014). The purpose of this paper is to describe a framework for assessing the risk of losses on bank lending to the non-financial corporate sector. Banks' losses on loans to corporations are strongly related to bankruptcy. This link has been demonstrated both empirically, e.g. in Norges Bank's SEBRA model (see Bernhardsen, 2001 Bernhardsen and Larsen, 2007), and in numerous theoretical papers, such as the seminal contribution by Merton (1974).

In this paper, we use a reduced-form framework to *i)* assess the degree of banks' credit risk by estimating bankruptcy models for different industries and *ii)* attach a risk weight to each firm's debt – the risk weight being equal to the probability of bankruptcy during a given year conditional on the information at the time of prediction. By summing all risk-weighted debt in an industry, we obtain an estimate of the share of debt in bankruptcy accounts in a given year.

Our data set covers the whole population of Norwegian limited liability enterprises during the period 1999–2016Q3. The basis for the bankruptcy probability prediction (the information set) consists of key financial variables from companies' financial accounts and real-time information about credit rating from a credit rating agency. In addition, we use economic indicators relevant for the industry under study as proxies for business conditions. The typical prediction problem during a given year will be to predict the probability of bankruptcy either during the same year (nowcasting) or in a later year (forecasting) given real-time information about key financial indicators, credit ratings and economic indicators. The novelty of our approach compared to the previous literature is that we combine real-time economic indicators, real-time rating information and accounts data to produce nowcasting and forecasting predictions.

The rest of this paper is organised as follows. In Section 2, we discuss bankruptcy modelling in the existing literature and present definitions and operationalizations of main concepts, including the key concept of risk-weighted debt. In Section 3, we present the data and our sample. In Section 4, we introduce our econometric model of bankruptcy prediction and present estimates of parameters, marginal effects and risk-weighted debt. We also consider the out-of-sample performance of our preferred model and compare it to several alternative specifications. Further specification issues are discussed in an Appendix. Finally, Section 5 concludes.

2 Bankruptcy models

Before the pioneering work of Beaver (1966) and Altman (1968), financial institutions' analyses of credit risk on corporate loans were largely subjective judgments ("expert" opinions) based on a few key variables such as leverage, collateral and earnings. Modern credit score systems combine information on several accounting variables to obtain a credit risk score, or default probability. Such credit score systems are typically based on either probability models (linear, probit or logit) or discriminant analysis. An example of the former is Norges Bank's SEBRA model (see Bernhardsen, 2001) and of the latter the influential ZETA discriminant model (Altman et al., 1977). Discriminant models are generally criticised for not adapting to fast-moving changes in borrowers' conditions and for being linear, so that a given increment in an explanatory variable will have the same effect regardless of the level of the variable. Moreover, they do not have any theoretical foundation, merely lumping together a large number of financial indicators into a "black box" statistical model.

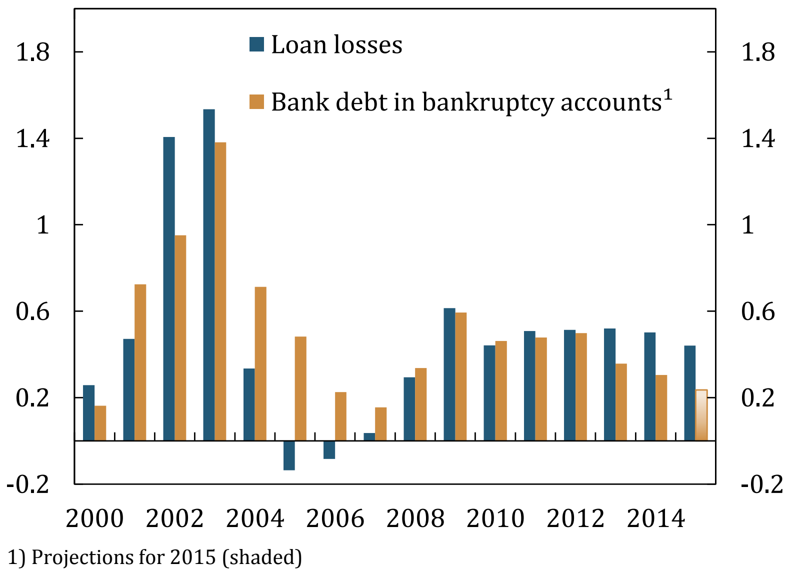
A class of bankruptcy models with a theoretical foundation is the risk-of-ruin model (Wilcox, 1973 Scott, 1981) and the option pricing model of Merton (1974). In the risk-of-ruin model, a firm will go bankrupt if the value of its assets falls below that of its debt obligations, whereas in the Merton model a firm's probability of bankruptcy depends on asset value relative to outside debt, and the asset value volatility. Golombek and Raknerud (2015) estimate a continuous–discrete choice model derived from stochastic dynamic programming that determines both exit and investment simultaneously. In their model, the exit decision is a trade-off between the value of installed capital if production is continued and the value of installed capital if the firm exits. A critical discussion of the merits of structural versus reduced-form bankruptcy models is found in Wu et al. (2010). Their main conclusion is that there is little to gain in terms of improved predictive ability by invoking option-theory models of bankruptcy compared to reduced-form models (e.g. logit or probit). The latter typically include accounting information, market data and comprehensive firm characteristics as explanatory variables.

2.1 Operationalizations

Banks' loan losses There are no precise measures of a bank's actual loan losses in any given period. Before realising a loss, a default on the loan must occur, i.e., the debtor has failed to make interest or principal payments for 30 or 90 days (i.e., the loan is non-performing) or the bank must expect to realise a loss based on the information concerning the loan. The bank must then write the debt down and finally net the write-downs against any reversals. The write-downs also include collective impairment losses on loans to problem sectors where banks expect losses to occur without knowing which customers will generate the losses. In this paper, we use banks' annual net write-downs as a proxy for loan losses.

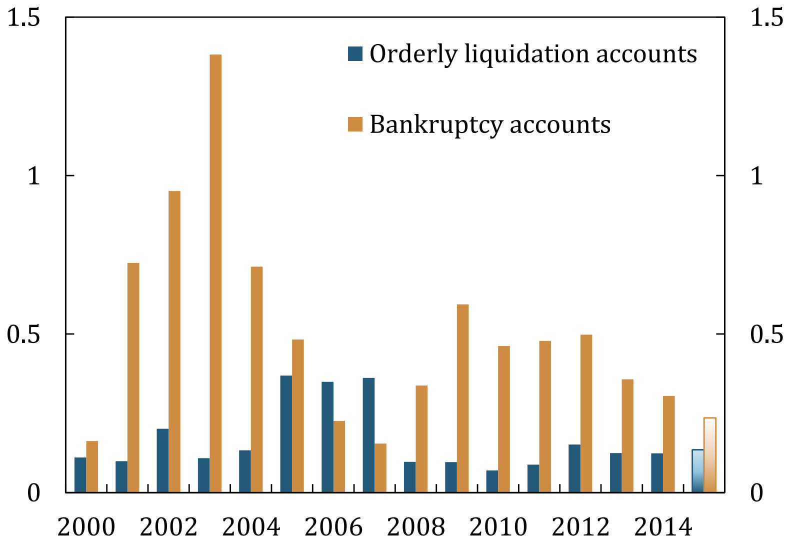
Bankruptcy There is obviously a strong relationship between debt in bankruptcy accounts and loan losses in the corporate sector. A firm's failure to serve loan obligations is perhaps the most common cause of bankruptcy (according to bankruptcy legislation, an insolvent debtor must begin bankruptcy proceedings). However, only a subset of banks' loan losses is due to bankrupt firms. Banks recognise substantial impairment losses also on loans to firms that have not gone bankrupt, e.g. related to debt restructuring. Debt restructuring may occur if the debtor believes that the firm will become profitable in the future conditional on debt reduction, payment deference, etc. On the other hand, when collateral is pledged for a loan, the bank will not normally lose the entire sum of the loan when the enterprise goes bankrupt. Despite discrepancies, the debt of bankrupt enterprises has historically shown a strong correlation with banks' loan losses, as shown in Figure 1. Debt in bankruptcy accounts behaves much more smoothly, but has the same cyclical pattern as loan losses. There are visible timeliness problems related to using write-downs as a measure of losses: the negative "losses" in 2005–2006 reflect reversals of the relatively high write-downs in 2002–2003.

Figure 1: Banks' loan losses as a share of total loans to the corporate sector and bank debt in bankruptcy accounts as a share of total bank debt. Percent. 2000–2015



Our data show that orderly liquidations (liquidations without bankruptcy, excluding mergers and demergers, i.e., when the activity continues under a new firm identifier) are typically associated with very little debt. Thus little is gained by including debt in orderly liquidation accounts in a measure of potential loan losses. Figure 2 shows debt in both bankruptcy accounts and orderly liquidation accounts (mergers and demergers excluded). First, debt in bankruptcy accounts is about three times larger than debt in orderly liquidation accounts during 2000–2015. Second, the latter shows very little variation over the business cycle, indicating that the orderly liquidations are not responding to economic conditions but are motivated by other factors. It is reasonable to assume that only a limited number of orderly liquidations lead to losses on bank loans and, hence, that bankruptcy is a much stronger signal of loan losses than liquidations. This is our reason for only focusing on bankruptcies in this paper.

Figure 2: Bank debt in bankruptcy accounts and orderly liquidation accounts as a share of total bank debt. Percent. 2000–2015[1)]



1) Projections for 2015 (shaded)

One problem with bankruptcy is that its actual timing is somewhat arbitrary, both because bankruptcy proceedings take time and because of a delay in the registration of bankruptcies in the statistics. There is typically a lag of one or two years, or sometimes even longer, between the date of the last approved financial accounts and the official date of bankruptcy. To address this issue, we define our bankruptcy indicator variable, B_{it}, as follows: $B_{it} = 1$ if $t-1$ is the last year the firm is registered as being active at the end of the year and the firm is declared bankrupt within year $t+1$ (i.e., in t or $t+1$).[1] Otherwise, $B_{it} = 0$. The year of bankruptcy is thus assumed to be the year when the firm's activity comes to a stop. Registered activity either means that the firm filed (approved) financial

[1]The firm is also defined as bankrupt if the liquidation of the firm was registered as compulsory. These firms are shown to have some of the same properties as bankrupt firms.

Table 1: Timing of bankruptcy registration

Year of last registered financial accounts (t)	No. of bankruptcies registered within t+2 years	Share of bankruptcies registered within t+2 years
1999	1118	84.4
2000	1431	86.3
2001	1650	89.8
2002	1526	91.2
2003	1177	86.2
2004	959	86.7
2005	733	79.7
2006	931	79.6
2007	1714	88.0
2008	1391	85.4
2009	1296	87.1
2010	1086	84.1
2011	1039	86.2
2012	1158	87.4
2013	1016	86.7

accounts for the year $t-1$ or that there was a new credit rating of the firm at the end of year $t-1$. In the case of missing financial accounts for a firm that was deemed active, these were imputed using the financial accounts from the previous year. Figure 1 indicates that our timing of the bankruptcy, as defined by B_{it}, coincides well with the timing of banks' write-downs on loans to bankrupt firms (see also Figure 6).

Table 1 shows that our definition picks up about 85 percent of the bankruptcies in our sample (about 30 percent of bankruptcies are registered in the first year after the last approved accounts). The requirement could be relaxed to maximum three years' lag instead of two. However, this comes at a cost. First, the estimation sample would be reduced by one year. Second, and more importantly, a three-year lag is a long time between the last available information and the registered bankruptcy, which could weaken the predictive ability of our model.

Risk-weighted debt The share of bank debt in bankruptcy accounts for industry S in year t is:

$$L_t^S = \frac{\sum_{i \in S} B_{it} \; D_{i,t-1}}{\sum_{i \in S} D_{i,t-1}}$$

where $D_{i,t-1}$ is the amount of bank debt at the end of year $t-1$ of firm i (this information is available from the financial accounts of year $t-1$) and S is the set of firms in the industry. Clearly, given our definition of B_{it}, L_t^S cannot be completely determined until the end of year $t+1$. Hence it must be assessed based on predictions from observed data.

Our estimate of the share of debt in bankruptcy accounts in t for a given industry S is assumed to have the following mathematical form:

$$\mathrm{RW}_t^S(\theta^S) = \frac{\sum_{i \in S} p_{it}(\theta^S) D_{i,t-1}}{\sum_{i \in S} D_{i,t-1}}$$

where $p_{it}(\theta^S)$ is the probability of bankruptcy during t given explanatory variables $X_{i,t-1}$ with unknown parameter vector θ^S (the dependence of p_{it} on $X_{i,t-1}$ is suppressed in the notation for simplicity). The probability $p_{it}(\theta^S)$ is obtained from an econometric model (see below), with the parameter vector θ^S being estimated using data for $X_{i,t-1}$ and B_{it}. We will henceforth denote $\mathrm{RW}_t^S(\theta^S)$ as the risk-weighted debt.[2] To assess the fit of our model to actual bank losses, we compare the estimated risk-weighted debt to both the share of bank debt in bankruptcy accounts, L_t^S, and reported losses in bank statistics.

3 Data and sample

Data on non-financial firms Norges Bank uses a database with balance sheets, income statements and firm-specific information for all Norwegian-registered firms that submit their financial accounts to the Brønnøysund Register Centre. The data is delivered by Bisnode, which also delivers its own credit rating and information on bankruptcies and liquidation dates. At present, the data covers the financial accounts for the fiscal years 1999–2015, and bankruptcy registration and rating data for 1999–2016Q3. Our dataset covers a period without any major solvency crises in Norway, hence the average bankruptcy frequency is relatively low in most industries. However, the period covers two downturns where loan losses and bankruptcies increased.

[2]Note that $\mathrm{RW}_t^S(\theta^S)$ is the debt-weighted average bankruptcy probability in year t among firms that are active at the end of $t-1$ (with weights equal to $D_{i,t-1}/\sum_{i \in S} D_{i,t-1}$)

We only include non-consolidated financial accounts in our analyses and exclude all financial firms. Moreover, our sample includes only limited liability enterprises, covering about 90 percent of the total bank debt of all non-financial firms. Observations of firms for which no industry code has been registered are excluded from the dataset. The accounts data include information about debt to credit institutions (here referred to as "bank debt") at the end of the accounting year for each firm. Since we are interested in banks' credit risk associated with loans to the corporate sector, we exclude observations of firms without bank debt.

Table 2 shows the number of firms in our sample for the fiscal year 2014, together with the share of bank debt in each industry and each industry's average annual bankruptcy frequency for the period 2000–2014. 35 percent of bank debt is held by firms in Commercial real estate and the industry has a very low bankruptcy frequency during the period. This does not mean that lending to commercial real estate is always less risky than lending to other industries. During the banking crisis of 1988–1993, a large share of Norwegian banks' loan losses came from commercial real estate. Property-related corporate lending also appear to have been the main cause of bank losses in several other crises (see Kragh-Sørensen and Solheim, 2014).

The group Other industries is a heterogeneous residual group, including firms in shipping, oil and gas exploration, support activities for oil and gas exploration and electricity and water supply. While Norwegian banks experienced significant losses to the shipping industry after the financial crisis, this was not accompanied by a comparable increase in bankruptcies or bankruptcy debt in our firm sample. There are also some very large publicly owned companies in this group, especially in electricity and water supply (e.g., Statkraft) and in oil exploration (e.g., Statoil). These companies have a very low risk of bankruptcy and they tend to dominate risk-weighted debt in the industry. Because of these idiosyncrasies, we exclude Other industries from our analyses below.

The Bisnode credit rating system consists of five categories: AAA, AA, A, B and

Table 2: Descriptive statistics

Industry	No. of firms (2014)	Share of bank debt (2014)	Annual bankruptcy frequency (2000-2014)
Fishing and fish farming	993	2.9	1.8
Manufacturing, mining and quarrying	4,747	6.9	2.3
Retail trade, hotels and restaurants	15,026	5.6	3.2
Construction	12,202	8.2	2.1
Commercial real estate	23,678	35.3	0.4
Services[1)]	14,735	14.7	1.8
Other industries[2)]	1,844	26.5	1.4

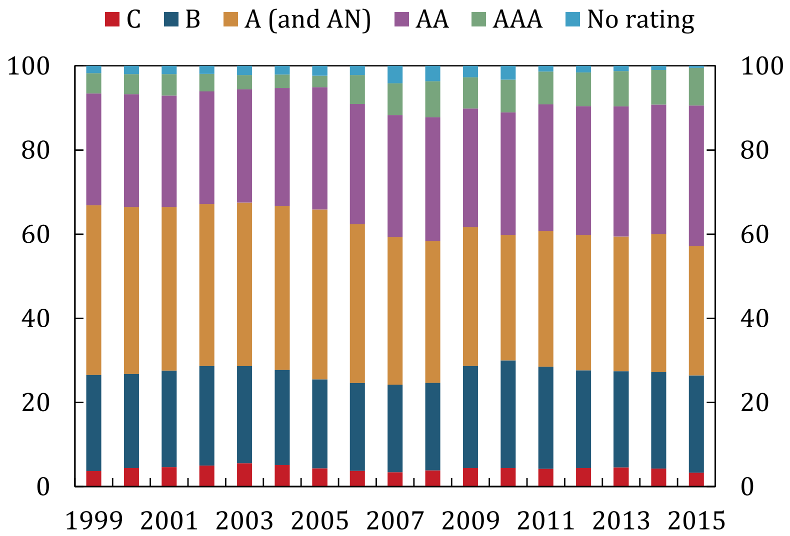
1) Includes all service activities, information and communication and transportation and storage excluding international shipping.
2) Oil and gas exploration, support activities for oil and gas exploration, international shipping, electricity and water supply and renovation activities, agriculture and forestry.

C.[3] Each firm's rating is determined based on assessments from four areas: basic facts, ownership information, financial figures and payment history. Figure 3 shows the share of firms in each rating category per year: about 85–90 percent of the firms have one of the mid ratings (AA, A or B), 5–10 percent have the highest rating (AAA) and 3–5 percent the lowest rating (C). There are visible, albeit modest, changes in the share of firms in each rating category along the business cycle – most visible for the rating categories B and C during the financial crisis. Regarding mobility between the rating categories, we find that the share of firms whose rating is the same in two consecutive years (conditional on survival) lies roughly between 55 and 70 percent for the four lowest categories, but is only just above 40 percent for AAA. The probability that an AAA-ranked firm is downgraded to AA at the end of the next year is 45 percent.

Data on banks' loans and loan losses Banks report their loans and loan losses by industry directly to Finanstilsynet (Financial Supervisory Authority of Norway) once a year. Due to data availability, only Norwegian banks' reported loans and loan losses are used in our analyses. Loans and loan losses of branches of foreign banks operating in Norway are thus excluded. We use annual data for loans and loan losses per industry

[3]Non-rated firms are in a separate category. Firms rated AN (”new firm”) are lumped together with firms in category A because they perform similarly with regard to bankruptcy risk.

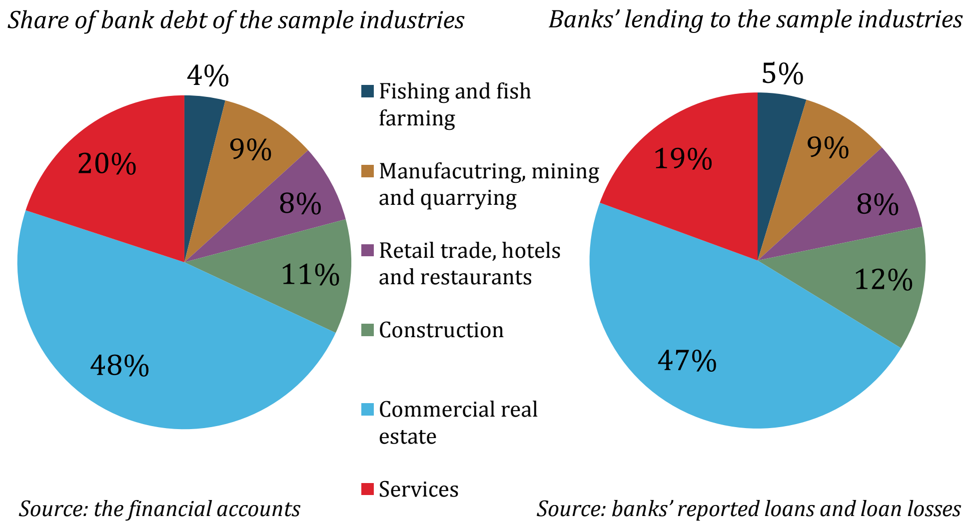
Figure 3: Share of firms in each credit rating category. 1999–2015



covering the period 2000–2015.

Norwegian banks' share of loans to each industry in our sample (Other industries excluded) is very similar to the share of bank debt in each industry in the financial accounts, see Figure 4. If we included Other industries, the discrepancies between the two data sources would be higher with regard to the share of loans to each industry.

Figure 4: Bank debt according to the financial accounts of the sample industries and banks' lending to the sample industries (Other industries excluded). Percent. As of 2014Q4



4 An econometric model of bankruptcy prediction

The probability of bankruptcy during the calendar year $t+1$ for firm i in industry S, given that the firm is active at the end of year t, is assumed to be given by a logit model:

$$\ln\left(\frac{p_{i,t+1}^S}{1-p_{i,t+1}^S}\right) = \beta^S x_{it} + \pi^S r_{it} + \mu^S + \lambda_{t+1}^S \tag{1}$$

where x_{it} is a vector of firm-specific time-varying variables obtained from the financial accounts, r_{it} is a vector of dummy variables indicating rating category, μ^S is the intercept and λ_{t+1}^S is a common year effect for all firms in the industry. Both x_{it} and r_{it} are dated end of year t. The corresponding coefficient vectors are β^S and π^S. We use the notation θ^S to denote the coefficient vector pertaining to industry S (for specifications of θ^S, see below).

The following variables are included in x_{it}:

1. Return on total assets (RoA) $= \frac{\text{Results before extraordinary items and taxes + Interest expenses}}{\text{Total assets}}$
2. Equity ratio (ER) $= \frac{\text{Equity}}{\text{Total assets}}$ (opening balance)
3. Real total assets[4] (RA) on logarithmic and squared logarithmic form

The financial accounting variables RoA and ER are subject to truncation: The lowest and highest 2 percent of the observations are replaced by the value of the 2nd and 98th percentile.

Our specification above includes just a small set of variables. Other potential measures are e.g. debt-servicing capacity, degree of interest burden and measures of liquidity. However, none of these measures adds much extra value in terms of increased goodness-of-fit and some of the variables were not robustly significant or did not have the expected sign across all industries. We have also tested other functional form assumptions, such as including interactions between x_{it} and r_{it} (allowing the impact of financial variables to

[4]Nominal prices are converted into fixed prices using the consumer price index.

differ across rating categories). Our maintained specification is driven by considerations of parsimony and robustness.

The rating, r_{it}, is published in real time and is based on financial indicators as well as information about overdue payments, audit remarks, etc. Because of the publication lag of about three quarters with regard to the financial accounts data, only *lagged* values of x_{it} are generally known to the rating agency when r_{it} is determined. Thus x_{it} contains *new* information relative to r_{it}.

We estimate two versions of the model: a benchmark model and a predictive model. In the benchmark model, λ_{t+1}^S is treated as a fixed parameter to be estimated for each t (with one zero-restriction for the reference year). In that case, the unknown coefficient vector is $\theta^S = (\beta^S, \pi^S, \mu^S, \lambda_2^S, ..., \lambda_T^S)$, where T is the last year with observations on the dependent variable, B_{it}, in the estimation sample. The coefficient λ_{t+1}^S captures all industry-wide effects in year t that are not captured by other variables in the model. The inclusion of time dummies implies that the model will perfectly fit annual bankruptcy rates in-sample at the aggregate (industry) level:

$$\sum_{i \in S} B_{i,t+1} = \sum_{i \in S} p_{i,t+1}^S(\widehat{\theta_T^S})$$

where $\widehat{\theta_T^S}$ is the logit estimate based on data on bankruptcies until year T $(T \geq t+1)$.

On the other hand, in the predictive model λ_{t+1}^S is assumed to be a function of (a vector of) economic indicators, z_{t+1}^S, measured at the industry level as follows:

$$\lambda_{t+1}^S = \rho^S z_{t+1}^S \tag{2}$$

where ρ^S is an unknown parameter or parameter vector to be estimated and z_t^S is a scalar or vector of economic indicators relevant to the industry. In this case, the unknown coefficient vector consists of far fewer parameters: $\theta^S = (\beta^S, \pi^S, \mu^S, \rho^S)$. Unlike the benchmark model, the specification (2) will not give perfect in-sample fit to aggregate bankruptcy rates at the industry level. To make real-time predictions, z_{t+1}^S must be predicted based on data available at the time of prediction.

4.1 Risk-weighted debt in the benchmark model

In Figure 5 we compare the estimated aggregate risk-weighted debt across all industries with the share of debt in bankruptcy accounts (see Table A.1 in Appendix A for the parameter estimates). The aggregate risk-weighted debt has been made by adding all risk-weighted debt per industry together.

Figure 5: Aggregated risk-weighted debt and bank debt in bankruptcy accounts as a share of total bank debt. Benchmark model. Percent. 2000–2014

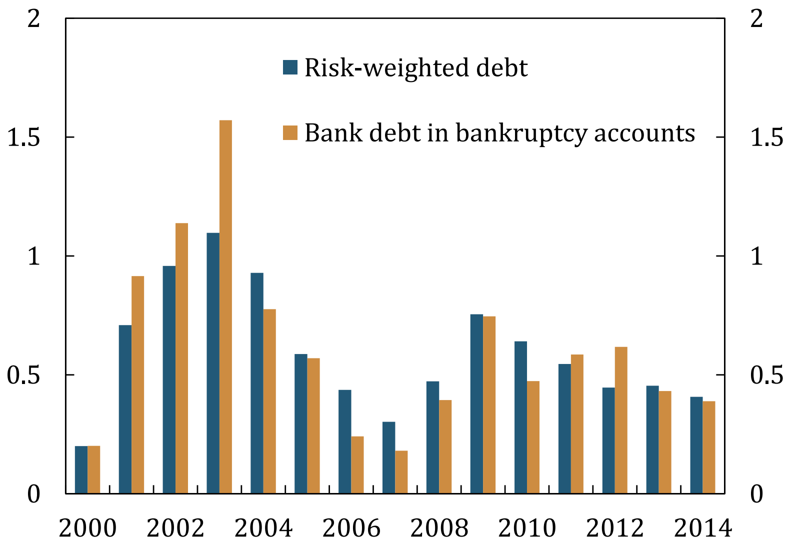
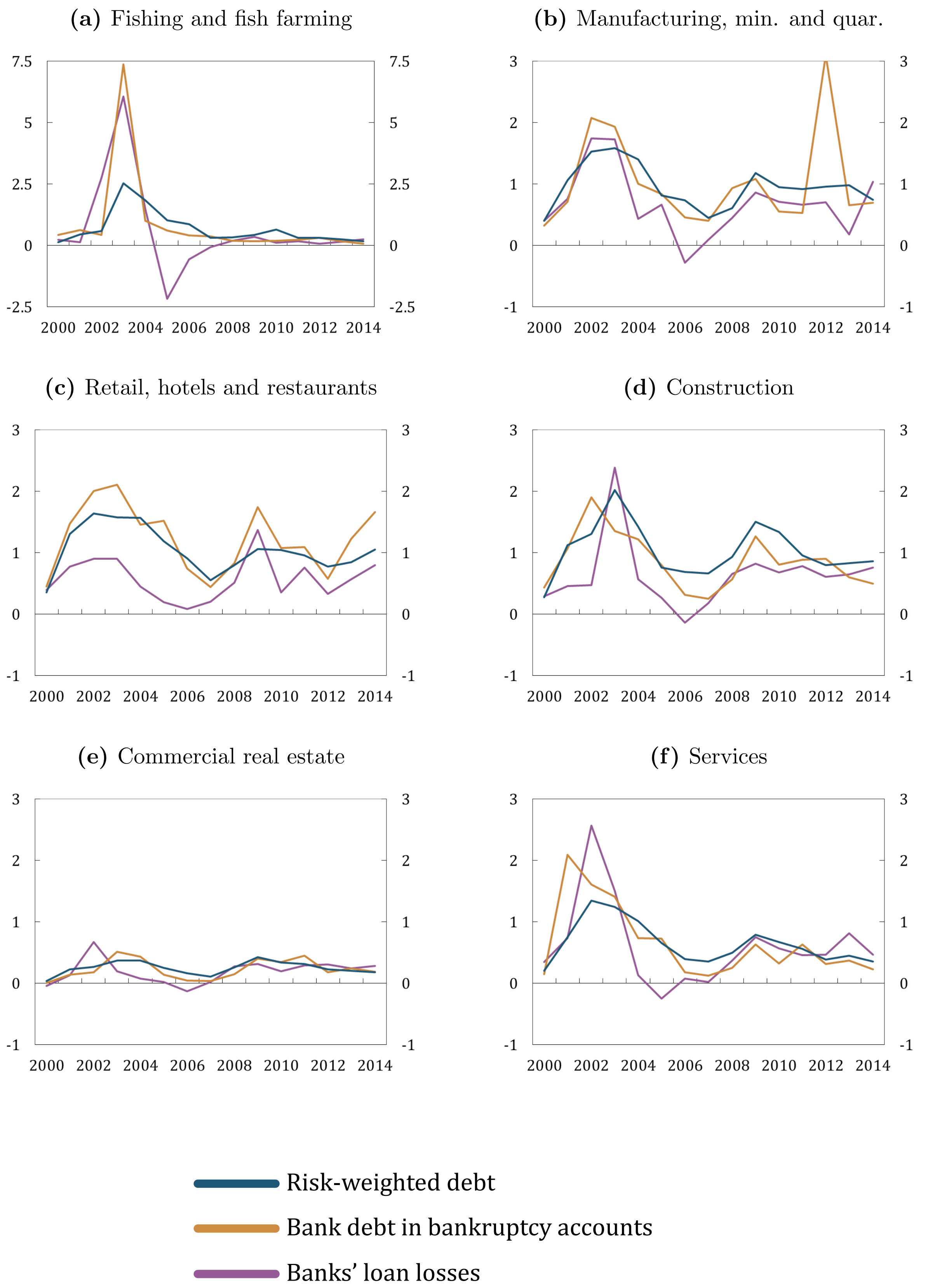


Figure 6, charts (a)–(f) shows the same data together with loan losses at the industry level. Estimated risk-weighted debt and the share of debt in bankruptcy accounts are fairly equal. One exception is Manufacturing, which has a big spike in bankruptcy debt in 2012 due to a single firm. Another exception is in 2003, related to the slump in the fish farming industry. In that case, a few firms with very high bank debt went bankrupt and caused large write-downs in the banking sector. In 2003 both bank losses and debt in bankruptcy accounts were fairly equal, whereas risk-weighted debt is much lower. However, we see that some of the original write-downs were reversed in 2003–2005 (when “losses” were negative). We observe a somewhat similar pattern for all of the other industries in 2001–2005.

Figure 6: Risk-weighted debt (benchmark model), bank debt in bankruptcy accounts and banks' loan losses. Percent. 2000–2014



4.2 Predictive model

We will now discuss the implementation of the prediction version of the bankruptcy model where *i)* we take into account when information is published in real time and *ii)* the common year effect λ_{t+1}^S is a function of real-time economic indicators as specified in Equation (2).

First, let us now assume that information is published quarterly for the highest-frequency variables. To capture this, the time variable – year – is allowed to take non-integer values, such that $t + q/4$ $(q = 1, ..., 4)$ denotes the q'th quarter of year $t + 1$ (that is, years passed since the base year, $t = 0$). For example, $t + 1/4$ is at the end of ("just after") the first quarter of the calendar year $t + 1$, and $t + 1$ is at the end of this year ("last day" of year $t + 1$).

For any variable X_t, let $\widehat{X}_{t|t'}$ denote the predicted value of X_t given the information set at time t'. For example, $\widehat{x}_{i,t+s|t+q/4}$ and $\widehat{z}_{t+s|t+q/4}^S$ (for $s = 0, 1, 2, ...$) denote the predicted value of $x_{i,t+s}$ and z_{t+s}^S given the information available just after $t+q/4$. In our predictive model, the probability of bankruptcy in year $t + s + 1$ given the information just after $t + q/4$ (for $q = 1, ..., 4$) is assumed to be given by the logit model:

$$\ln(\frac{p_{i,t+s+1|t+q/4}(\theta^S)}{1 - p_{i,t+s+1|t+q/4}(\theta^S)}) = \beta^S \widehat{x}_{i,t+s|t+q/4} + \pi^S \widehat{r}_{i,t+s|t+q/4} + \mu^S + \rho^S \widehat{z}_{t+s+1|t+q/4}^S \qquad (3)$$

where $\theta^S = (\beta^S, \pi^S, \mu^S, \rho^S)$. In practice, θ^S must be estimated. We discuss the importance of the estimation sample and return to a comparison of the in-sample and out-of-sample properties of the (predictive) model in Section 4.4. Until then, we will assume that θ^S is estimated as in the benchmark model, with the modification that the dummy coefficient λ_{t+1}^S is replaced with $\rho^S z_{t+1}^S$.

In this paper, we will only discuss two values of s: $s = 0$ (nowcast) and $s = 1$ (one year ahead forecast). Furthermore, we will present predictions given information dated either $t + 3/4$ ("just after" third quarter of year $t + 1$) or $t + 1$ ("last day" of year $t + 1$).

The real-time updating of the information about the explanatory variables can be described as follows:

$$\widehat{\text{RoA}}_{i,t|t+3/4} = \text{RoA}_{it},\ \widehat{\text{RA}}_{i,t|t+3/4} = \text{RA}_{it},\ \widehat{D}_{i,t|t+3/4} = D_{it},$$
$$\widehat{\text{ER}}_{i,t|t-1+3/4} = \text{ER}_{it},\ \widehat{r}_{i,t|t} = r_{it},\ \widehat{z}^S_{t|t} = z^S_t \text{ and } \widehat{B}_{i,t|t+1} = B_{it}$$

That is: the accounting data (x_{it}, D_{it}) are published with a three-quarter lag, except the equity ratio (ER_{it}) which is obtainable from the outgoing balance of the accounts of year $t-1$ (published "just after" $t-1+3/4$). The rating r_{it} and the economic indicator z^S_t are observed quarterly.[5] Finally, B_{it} is published at the end of $t+1$ (recall that $B_{it} = 1$ if $t-1$ is the last year the firm filed approved financial accounts and the firm is registered as bankrupt by the end of year $t+1$). Hence, for the calculation of $p_{i,t+1|t+3/4}$ (nowcast) all relevant firm-specific explanatory variables are observed ($\widehat{x}_{i,t|t+3/4} = x_{it}$ and $\widehat{r}_{i,t|t+3/4} = r_{it}$) whereas for a one year ahead forecast made at the same time, i.e., the calculation of $p_{i,t+2|t+3/4}$, we use the predictors $\widehat{x}_{i,t+1|t+3/4} = x_{it}$ (with the exception that $\widehat{\text{ER}}_{i,t+1|t+3/4} = \text{ER}_{i,t+1}$) and $\widehat{r}_{i,t+1|t+3/4} = r_{i,t+3/4}$. For the economic indicators, both $\widehat{z}^S_{t+1|t+3/4}$ and $\widehat{z}^S_{t+2|t+3/4}$ are forecasts as of Q3 of year $t+1$.

We define predicted risk-weighted (RW) debt as:

$$RW^S_{t+1+s|t+q/4}(\theta^S) = \frac{\sum_{i\in S} p_{i,t+s+1|t+q/4}(\theta^S)\widehat{D}_{i,t+s|t+q/4}}{\sum_{i\in S} \widehat{D}_{i,t+s|t+q/4}} \tag{4}$$

When nowcasting is done at the end of $t+1$ – after x_{it} is published – both the nowcast for $t+1$ (s=0) and the forecast for $t+2$ (s=1) is of interest (recall that it is not until the end of $t+2$ that we actually observe the realised bankruptcies of year $t+1$).

4.3 Parameter estimates and risk-weighted debt in the predictive model

Tables 3–4 depict estimation results and average estimated marginal effects for the predictive model. The marginal effect of an explanatory variable is the estimated average *percentage point* change in bankruptcy probability when the variable changes by one unit

[5] We ignore publication lags with respect to z^S_t. The importance of such lags in our context (e.g. with regard to quarterly GDP) is small compared to the timeliness problems of accounts data.

(see the notes to Table 4 for the unit of measurement for each variable).[6]

We first observe that the rating variables are highly significant across the industries. The reference category is firms with an AAA-rating, hence all parameter estimates regarding rating must be interpreted as the effect of changing the rating category from AAA to the rating category under consideration. We see that a lower rating comes with a significantly higher bankruptcy probability. The estimates are significant at the 1 percent level in all industries with regard to the two lowest rating categories. In three of the industries all rating categories are significant at the 1 percent level. The marginal effects in Table 4 show that, *cet. par.*, going from rating category AAA to C increases bankruptcy probability by 4–13 percentage points on average. The corresponding effects for category B are more modest: an increase of about 2–3 percentage points, except for Commercial real estate, where the estimated marginal effect is less than 1 percentage point.

In all industries return on assets (RoA) is significant at the 1 percent level. The effect of RoA is very similar across industries. From Table 4, we see that a one percentage point increase in returns on assets (RoA) decreases bankruptcy probability by 0.01–0.07 percentage points on average. The strongest effect is found in Retail trade, hotels and restaurants. The equity ratio (ER) is also a significant variable in all the industries, albeit the effect of this variable is smaller than for RoA: a one percentage point increase in the share of equity decreases bankruptcy probability by 0.01–0.02 percentage points, except in Commercial real estate, where the estimated marginal effect is less than 0.01 percentage point.

Table 3 shows that the impact of total real assets (RA) is statistically significant, except in Commercial real estate, but the size effect is non-monotone: the typical pattern is that moderately large firms have a higher probability of bankruptcy than very small ones, but when the firms' assets cross a certain threshold (extremal point), bankruptcy

[6]The estimates in Tables 3–4 can be compared to the corresponding estimates for the benchmark model reported in Tables A.1–A.2. The difference is that in the predictive model, economic indicators are included as explanatory variables instead of time dummies.

probability starts to decrease.[7] The first (positive) relation could reflect that a creditor has more to gain from bankruptcy proceeding in the case of an asset-rich firm compared to a firm with little assets. The second effect is the dominant one according to numerous empirical studies about bankruptcy and firm liquidation; some examples are Mata et al. (1995), Olley and Pakes (1996) and Foster et al. (2008). A negative relation could reflect that larger firms have more financial muscle to withstand temporary economic setbacks, or to re-negotiate debt conditions in times of crisis.[8]

Annual mainland GDP growth is the sole economic indicator used in most industries. The exceptions are: Commercial real estate, where we used annual growth in real rental prices for office premises and an interest rate swap, and Fishing and fish farming, where the economic indicator is a price index for salmon. The economic indicators are highly significant in all industries, with a p-value of less than 1 percent everywhere. The estimated marginal effects in Table 4 reveal that a one percentage point increase in GDP growth reduces the average estimated bankruptcy probability by 0.14–0.27 percentage points in Manufacturing, Retail trade, hotels and restaurants, Construction and Services, with the largest impact in Manufacturing.

We now discuss how well our predictive model predicts bankruptcies in-sample. To do this we construct ROC curves for each industry as follows (see Figure A.1, charts (a)–(f) in Appendix A).[9] First, we use the estimated model to calculate the probability that a firm will go bankrupt during the next year. Next, we choose a threshold probability p. Among firms that went bankrupt, the share with probability above p (classified as:

[7] The estimated extremal point (in 1000 NOK) can be derived from the estimates in Table 3: $exp(-1/2 \times \text{RA(log)}/\text{RA(logsq)})$, where the variable names refer to the corresponding coefficient estimate in Table 3. The estimates of the extremal point vary a lot: from below NOK 2 million to above NOK 35 million in total assets.

[8] Golombek and Raknerud (2015) show that in a theory model with adjustment costs of capital a higher stock of capital has two opposite effects: More capital will increase production, and therefore raise the value of the firm if it continues to operate – this tends to lower the liquidation probability. On the other hand, more capital increases the amount of money obtained if the firm sells its entire stock of capital – this tends to increase the liquidation probability. They show that with costly reversibility of investment, the first effect always dominates.

[9] ROC (Receiver Operating Characteristic) curves are common in medicine to assess how well a decision rule, using clinical results, predicts a disease (see, for example, van Erkel and Pattynama, 1998). To measure the degree of predictability of the test, the area below the ROC curve is calculated: it is 1 if the test is perfect and 0.5 if the test is worthless.

positive) is termed the true positive rate. Among firms that did *not* go bankrupt, the share with probability above the threshold p is termed the false positive rate. For each p the observed true and false positive rate represent one point on the ROC curve. By increasing p from 0 to 1 we construct the entire curve. The straight lines in Figure A.1 correspond to the case where the false positive rate always equals the true positive rate. The success of the model in predicting bankruptcy is measured by the area below the ROC curve (AUC). As a rule of thumb, if the area exceeds 0.9 the test is typically regarded as excellent. The results in Table 3 shows that the area below the ROC curve is slightly lower than 0.9 in all industries, indicating at least a very good in-sample fit.

Table 3: Predictive model: parameter estimates

	Fishing and fish farming			*Manufacturing, mining and quarrying*			*Retail trade, hotels and restaurants*		
Variable	Coeff.	z	95 CI	Coeff.	z	95 CI	Coeff.	z	95 CI
Economic indicator 1[1)]	-0.05***	-3.29	[-0.09,-0.02]	-13.17***	-9.85	[-15.80,-10.55]	-7.60***	-11.86	[-8.86,-6.34]
Rating: C	2.80***	4.54	[1.59,4.01]	4.61***	12.65	[3.89,5.32]	4.35***	17.18	[3.85,4.84]
Rating: B	1.43**	2.36	[0.24,2.61]	3.32***	9.16	[2.61,4.03]	3.00***	11.89	[2.51,3.50]
Rating: A	0.54	0.89	[-0.65,1.73]	2.40***	6.68	[1.70,3.11]	1.93***	7.64	[1.43,2.42]
Rating: AA	-0.06	-0.09	[-1.34,1.22]	1.40***	3.85	[0.69,2.12]	1.04***	4.05	[0.54,1.55]
RoA	-3.00***	-12.38	[-3.47,-2.52]	-2.33***	-26.14	[-2.51,-2.16]	-2.42***	-53.52	[-2.50,-2.33]
ER	-1.14***	-5.06	[-1.58,-0.70]	-0.60***	-6.92	[-0.76,-0.43]	-0.61***	-13.99	[-0.70,-0.53]
RA (log)	1.12***	2.80	[0.34,1.90]	0.82***	6.89	[0.58,1.05]	0.73***	7.98	[0.55,0.91]
RA (logsq)	-0.07***	-2.92	[-0.11,-0.02]	-0.04***	-6.46	[-0.06,-0.03]	-0.05***	-8.81	[-0.06,-0.04]
No. of obs.		14647			80540			246851	
Pseudo R^2		0.22			0.20			0.22	
AUC		0.88			0.86			0.86	

	Construction			*Commercial real estate*			*Services*		
Variable	Coeff.	z	95 CI	Coeff.	z	95 CI	Coeff.	z	95 CI
Economic indicator 1[2)]	-11.95***	-10.70	[-14.14,-9.76]	-1.46***	-7.19	[-1.86,-1.06]	-8.82***	-8.91	[-10.76,-6.88]
Economic indicator 2[3)]				14.99***	3.48	[6.56,23.43]			
Rating: C	5.20***	11.49	[4.31,6.08]	3.37***	11.14	[2.78,3.97]	3.80***	19.10	[3.41,4.19]
Rating: B	3.65***	8.08	[2.76,4.53]	1.14***	3.79	[0.55,1.73]	2.08***	10.52	[1.69,2.47]
Rating: A	2.61***	5.78	[1.73,3.50]	-0.04	-0.13	[-0.63,0.55]	1.10***	5.54	[0.71,1.49]
Rating: AA	1.92***	4.22	[1.03,2.82]	-0.58*	-1.79	[-1.22,0.06]	0.25	1.22	[-0.16,0.66]
RoA	-2.67***	-30.42	[-2.84,-2.50]	-3.11***	-22.77	[-3.38,-2.85]	-2.08***	-36.51	[-2.19,-1.97]
ER	-0.96***	-11.56	[-1.12,-0.80]	-1.27***	-10.87	[-1.50,-1.04]	-0.65***	-12.02	[-0.76,-0.55]
RA (log)	1.15***	8.35	[0.88,1.42]	-0.04	-0.27	[-0.36,0.27]	0.81***	7.28	[0.59,1.03]
RA (logsq)	-0.07***	-8.36	[-0.09,-0.05]	0.00	0.02	[-0.02,0.02]	-0.05***	-7.00	[-0.06,-0.04]
No. of obs.		122074			261994			193114	
Pseudo R^2		0.24			0.23			0.22	
AUC		0.86			0.88			0.86	

Notes: Estimation period: 2000-2014. The asterisks indicate significance levels at: * $p<0.1$, ** $p<0.05$ and *** $p<0.01$. AAA is the reference category for Rating.

1) Annual growth in mainland GDP except for Fishing and fish farming where a price index on salmon has been used.

2) Annual growth in mainland GDP except for Commercial real estate where growth in real rental prices for prime offices has been used.

3) Annual average for a 10-year swap rate is the second economic indicator for Commercial real estate.

Table 4: Predictive model: average estimated marginal effects[1)]

Variable	*Fishing and fish farming*			*Manufacturing, mining and quarrying*			*Retail trade, hotels and restaurants*		
	ME	z	95 CI	ME	z	95 CI	ME	z	95 CI
Economic indicator 1[2)]	-0.00***	-3.23	[-0.00,-0.00]	-0.27***	-9.65	[-0.33,-0.22]	-0.21***	-11.83	[-0.25,-0.18]
Rating: C	6.74***	6.82	[4.81,8.68]	11.45***	20.42	[10.35,12.55]	13.10***	44.98	12.53,13.67]
Rating: B	1.58***	4.02	[0.81,2.35]	3.51***	20.47	[3.17,3.84]	3.87***	40.89	[3.69,4.06]
Rating: A	0.37	1.10	[-0.29,1.03]	1.38***	14.63	[1.19,1.56]	1.25***	18.06	[1.12,1.39]
Rating: AA	-0.03	-0.09	[-0.70,0.64]	0.43***	6.13	[0.29,0.56]	0.40***	6.01	[0.27,0.53]
RoA	-0.05***	-10.73	[-0.06,-0.04]	-0.05***	-23.42	[-0.05,-0.04]	-0.07***	-49.32	[-0.07,-0.06]
ER	-0.02***	-5.05	[-0.03,-0.01]	-0.01***	-6.87	[-0.02,-0.01]	-0.02***	-13.95	[-0.02,-0.01]

Variable	*Construction*			*Commercial real estate*			*Services*		
	ME	z	95 CI	ME	z	95 CI	ME	z	95 CI
Economic indicator 1[3)]	-0.21***	-10.58	[-0.26,-0.18]	-0.01***	-7.06	[-0.01,-0.00]	-0.14***	-8.86	[-0.17,-0.11]
Economic indicator 2[4)]				0.01***	3.46	[0.02,0.09]			
Rating: C	10.94***	25.95	[10.11,11.77]	4.20***	15.03	[3.65,4.75]	10.16***	31.21	[9.52,10.80]
Rating: B	2.64***	25.56	[2.44,2.84]	0.35***	6.07	[0.23,0.46]	1.90***	21.55	[1.72,2.07]
Rating: A	0.92***	15.23	[0.80,1.04]	-0.01	-0.13	[-0.10,0.09]	0.56***	8.50	[0.43,0.69]
Rating: AA	0.43***	8.03	[0.33,0.54]	-0.07	-1.46	[-0.17,0.02]	0.08	1.32	[-0.04,0.20]
RoA	-0.05***	-28.57	[-0.05,-0.04]	-0.01***	-19.71	[-0.01,-0.01]	-0.03***	-33.31	[-0.04,-0.03]
ER	-0.02***	-11.46	[-0.02,-0.01]	-0.00***	-10.44	[-0.01,-0.00]	-0.01***	-11.91	[-0.01,-0.01]

Notes: Estimation period: 2000-2014. The asterisks indicate significance levels at: * p<0.1, ** p<0.05 and *** p<0.01. AAA is the reference category for Rating.

1) Average of estimated marginal effects for each observation in percentage points of a unit change in the explanatory variables. A unit change is from 0 to 1 for the rating variables and 1 percentage point for RoA and ER. For the economic indicators a unit change is 1 NOK for the salmon price index and 1 percentage point for GDP growth, rental price growth and the swap rate, see 2), 3) and 4).

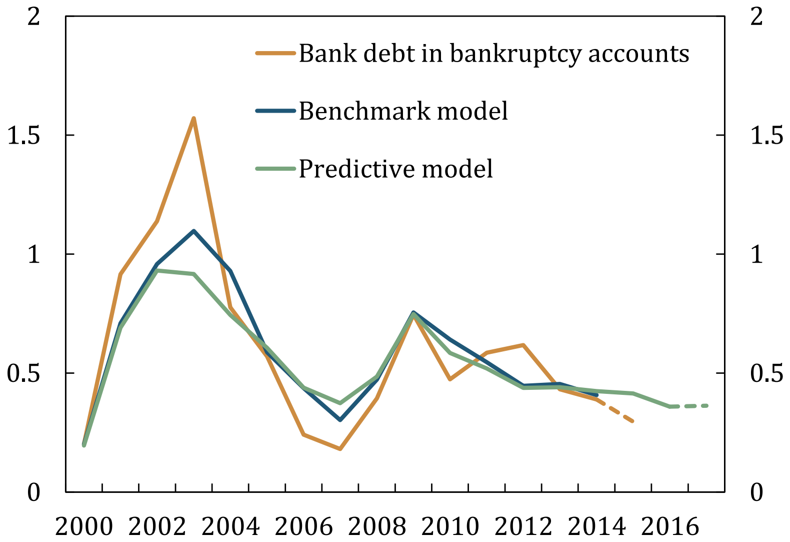
2) Annual growth in mainland GDP except for Fishing and fish farming where a price index for salmon has been used.

3) Annual growth in mainland GDP except for Commercial real estate where growth in rental prices for prime offices has been used.

4) Annual average for a 10-year swap rate is the second economic indicator for Commercial real estate.

Figure 7 compares aggregate in-sample risk-weighted debt in the benchmark model and the predictive model for 2000–2014, along with debt in bankruptcy accounts. Figure 7 also depicts – using the predictive model – the following: *i)* real-time nowcasts for 2015 based on information dated end of 2015, *ii)* nowcasts for 2016 based on information dated 2016Q3 and *iii)* forecast for 2017 based on information also dated 2016Q3. Figure 8, charts (a)–(f) displays the same information as Figure 7, but separately for each of the six industries. From these figures, we conclude that the benchmark and predictive model yield remarkably similar in-sample aggregate predictions.[10]

Figure 7: Aggregated risk-weighted debt (benchmark model and predictive model) and bank debt in bankruptcy accounts. Percent. 2000–2017[1)]



1) Projections (dotted lines)

Figure 9 depicts a decomposition of aggregate risk-weighted debt – calculated using the predictive model – into contributions from the six industries. Although Commercial real estate has very low risk-weighted debt, the industry's contribution to the aggregate is significant because of its high share of total bank debt. Note that the actual contribution to debt in bankruptcy accounts and risk-weighted debt differ substantially for some years and industries, e.g. Fishing and fish farming in 2003.

[10]Also, with regard to both the parameter estimates and the goodness-of-fit measures (Pseudo R^2 and AUC), there are no substantial differences between the benchmark model reported in Table A.1 and the predictive model reported in Table 3.

Figure 8: Risk-weighted debt (benchmark model and predictive model) and bank debt in bankruptcy accounts. Percent. 2000–2017. Dotted lines are projections

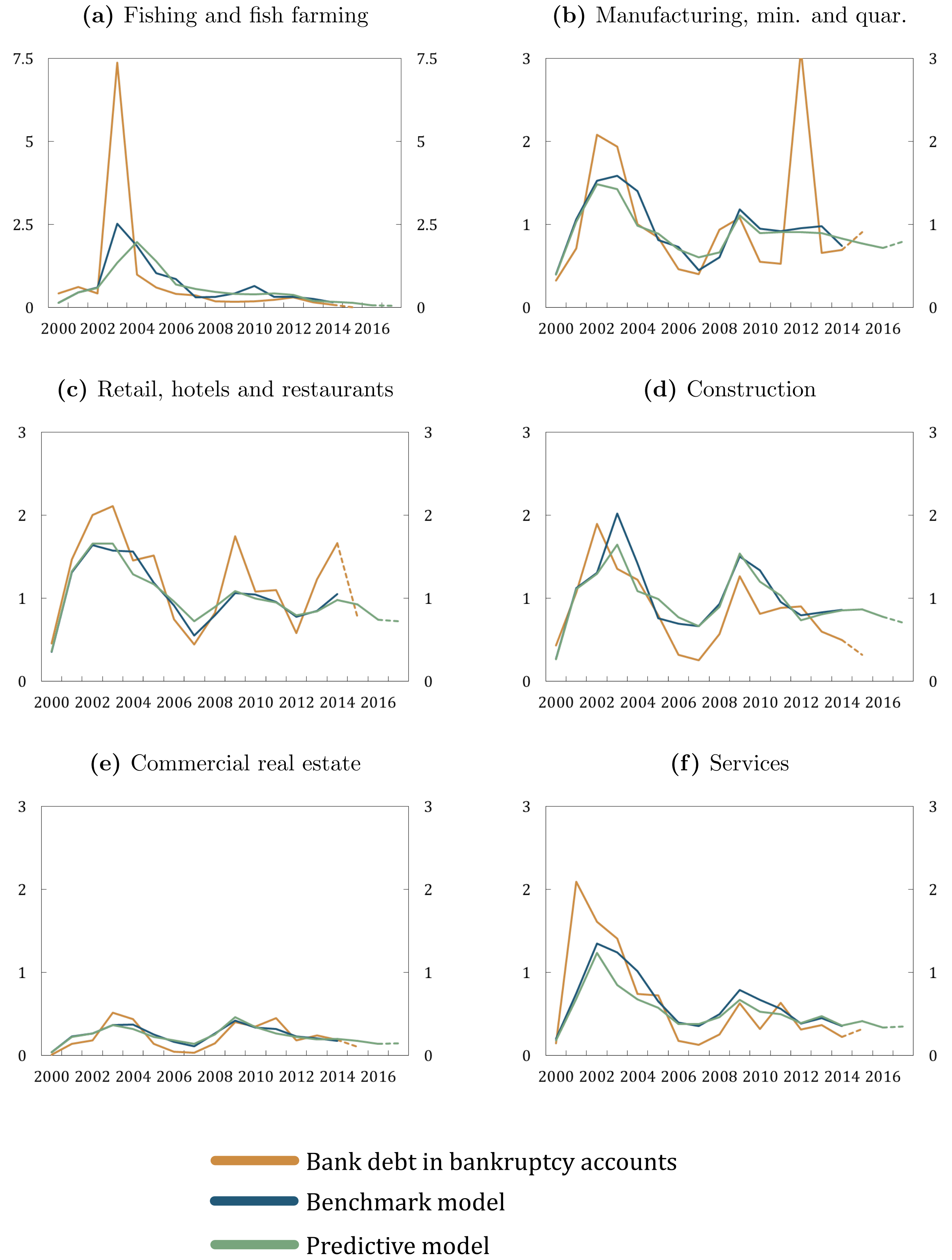
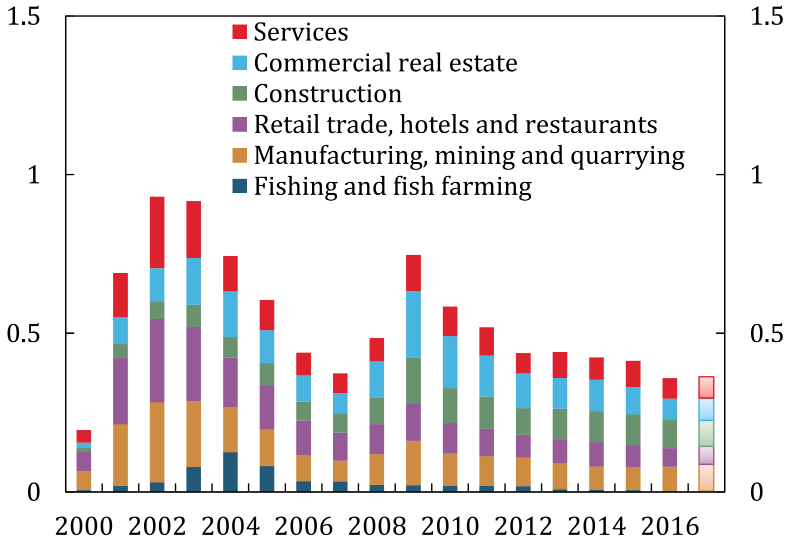


Figure 9: Contribution from each industry to aggregated risk-weighted debt. Predictive model. Percent. 2000–2017[1)]



1) Projections for 2017 (shaded)

4.4 Out-of-sample performance and alternative specifications

Out-of-sample prediction of risk-weighted debt We have demonstrated above that the in-sample performance of the predictive model with economic indicators is good compared to a benchmark model where year-dummies were used to fit annual aggregate bankruptcy rates (perfectly) at the industry level. However, good in-sample performance is no guarantee against poor real-time nowcasting or forecasting properties. If, for example, the estimated relations between the economic indicators at the industry level and the dependent variable are unstable, the model may quickly break down when used to make predictions out-of-sample.

In order to evaluate out-of-sample properties, let $\widehat{\theta}_t^S$ denote the estimate of θ^S (see (3)) based on firm-level bankruptcy data until year t, i.e., using $B_{i1}, ..., B_{it}$, with $t \leq T$. Given $\widehat{\theta}_t^S$, it is possible to use the estimated model to make out-of-sample predictions. Estimated risk-weighted debt, $RW^S_{t+1+s|t+1}(\widehat{\theta}_t^S)$, is a nowcast when $s = 0$ and a one-year ahead forecast when $s = 1$.

We now compare two measures of risk-weighted debt for a given year $t+1$: *i)* in-sample risk-weighted debt, $RW^S_{t+1|t+1}(\widehat{\theta^S_T})$, where data for 2000–2014 were used to estimate θ^S, and *ii)* real time-nowcasts, $RW^S_{t+1|t+1}(\widehat{\theta^S_t})$, where data up until year t is used to estimate θ^S. In both cases, explanatory variables included in the information set at the end of year $t+1$ were used to calculate risk-weighted debt.

We illustrate the results in Figures 10–11. Starting with $t = T = 2014$, which is currently (2016Q3) our last available year, the two curves corresponding to the predictive model are identical, as $RW^S_{t+1|t+1}(\widehat{\theta^S_T})$ by definition is a real-time nowcast in this case. Then we repeat the calculations for $t = T-1, T-2, \ldots$. The smallest t for which $RW^S_{t+1|t+1}(\widehat{\theta^S_t})$ is displayed is $t = 2007$. Then, to calculate the corresponding real-time nowcast of risk-weighted debt, only data for 2000–2006 were used in the estimation. If we use an even shorter estimation period, the estimated coefficients of the economic indicators become erratic and unstable. The main impression is that during the period 2007–2015, the agreement between in-sample and out-of-sample risk-weighted debt is good. The largest discrepancies are found during the financial crisis in the industries Construction and Services. The out-of-sample nowcasts of bankruptcy probabilities were in these instances somewhat higher than the actual bankruptcy outcome during the crisis.

Figure 10: Aggregated risk-weighted debt. In-sample benchmark model, in-sample predictive model and out-of-sample nowcasts. Percent. 2000–2015

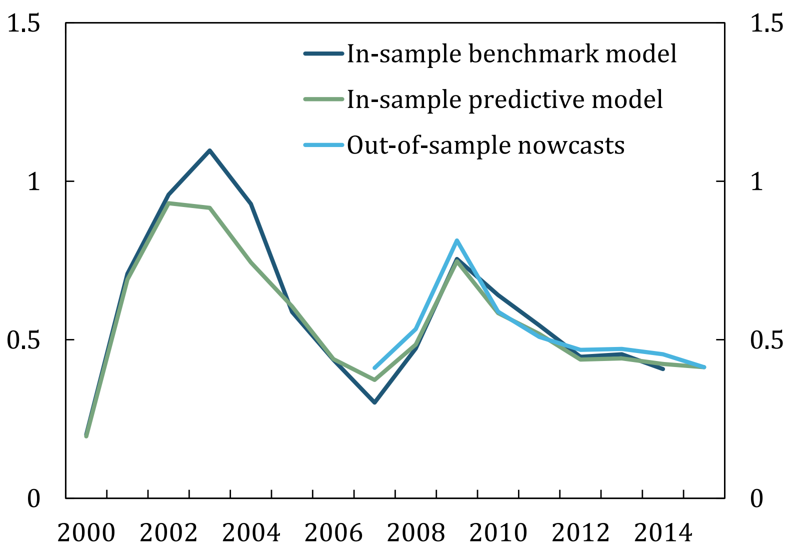
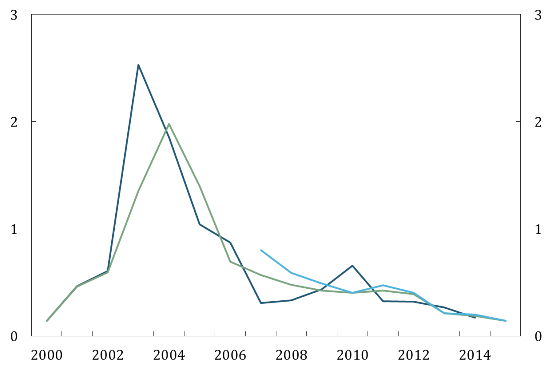
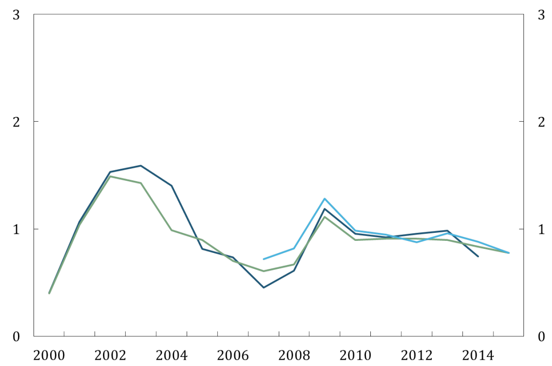


Figure 11: Risk-weighted debt. In-sample benchmark model, in-sample predictive model and out-of-sample nowcasts. Percent. 2000–2015

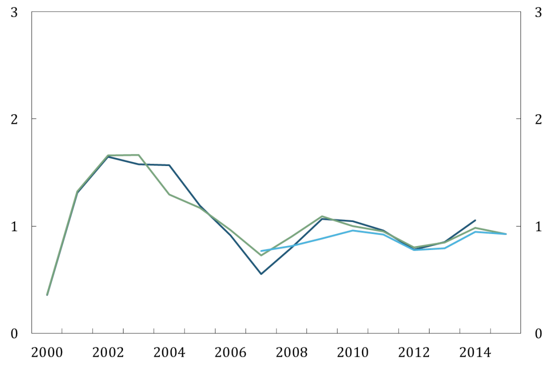
(a) Fishing and fish farming



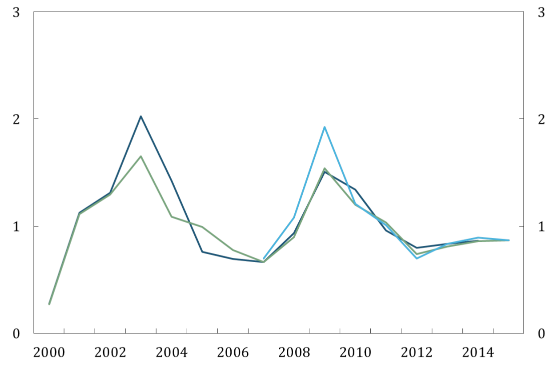
(b) Manufacturing, min. and quar.



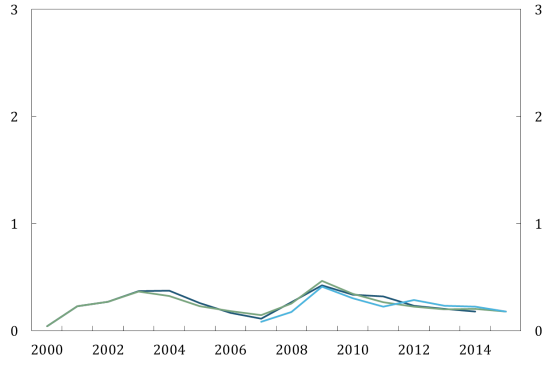
(c) Retail, hotels and restaurants



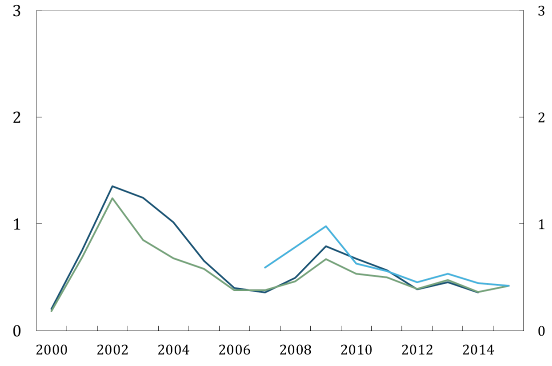
(d) Construction

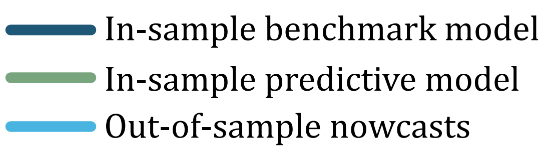


(e) Commercial real estate



(f) Services



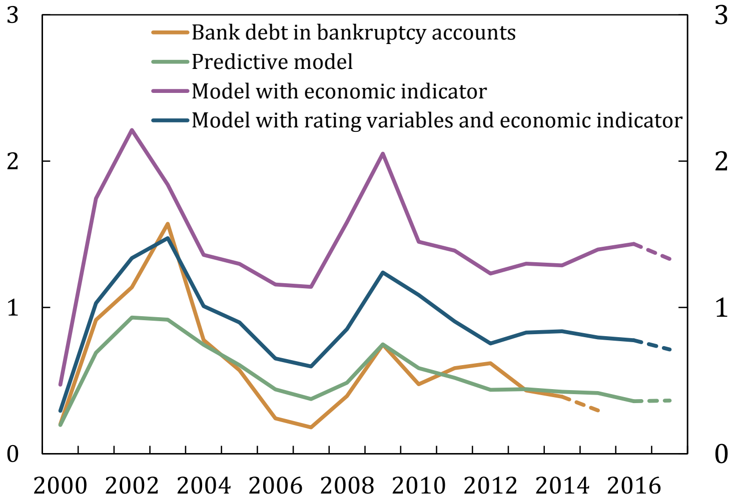


Alternative specifications To make further comparisons, we show in Figure 12 aggregated risk-weighted debt from our preferred (predictive) model together with risk-weighted debt from two alternative specifications where we *i)* only include economic indicators, or *ii)* include economic indicators together with rating variables. Thus, both alternative specifications exclude the financial accounting variables. We see that specification *i)* substantially overstates bankruptcy debt. Also *ii)* overstates bankruptcy debt, but less so than *i)*. The predictive model gives the best results compared to realised bankruptcy debt. To interpret these results, it is important to recognise that a model with only industry-wide variables accommodates no firm heterogeneity with respect to bankruptcy probability. That is, in a given year risk-weighted debt equals average bankruptcy probability. Including firm-specific information in the form of rating, as in specification *ii)*, gives lower risk-weighted debt, reflecting a *negative* correlation between (firm-specific) probabilities and shares of debt (recall that risk-weighted debt is the debt-weighted average bankruptcy probability; cf. Footnote 2). Our preferred predictive model discriminates even more between firms than do *ii),* leading to further lowered risk-weighted debt. The main reason is the impact of total assets: very large firms have lower bankruptcy probabilities *and* more debt than small firms and this relationship is not fully picked up by the rating categories.

Figure 12 reveals that both groups of firm-specific variables contribute substantially to predicted risk-weighted debt at the industry level and that excluding financial accounting variables and/or the rating variables from our model leads to serious omitted variable bias.

Further specifications issues are addressed in Appendix B.

Figure 12: Aggregated risk-weighted debt. Predictive model with different model specifications. Percent. 2000–2017[1)]



1) Projections (dotted lines)

5 Conclusions

In this paper we have proposed a method for assessing the risk of losses on bank lending to the non-financial corporate sector based on bankruptcy probability modelling. A strong link between bankruptcies and bank losses has been demonstrated in the previous literature, e.g. in Norges Bank's SEBRA model and in numerous theoretical papers based on option pricing theory. Moreover, bank losses according to bank statistics and debt in bankruptcy accounts display a similar pattern over time in most industries.

We have estimated bankruptcy models for different industries and attached a risk weight to each firm's debt in a given year. The risk weight is equal to the estimated probability of bankruptcy during the year given the information at the time of prediction. By summing all risk-weighted debt in an industry, we obtained a prediction of the share of debt in bankruptcy accounts in a given year.

We have discussed and proposed solutions to timeliness problems due to a considerable lag in the publication of bankruptcy and accounts data. A key feature of our approach is the inclusion of economic indicators at the industry level, observed in real time, as explanatory variables alongside standard financial accounting and real-time credit rating information in a predictive model which is useful for nowcasting and forecasting. We found that historically, during 2000–2014, there is good correspondence between our estimated measure of risk-weighted debt and actual debt in bankruptcy accounts.

To evaluate the out-of-sample properties of our approach, we have compared two measures of risk-weighted debt for any given year $(t+1)$: *i)* in-sample risk-weighted debt, where the whole data period (2000–2014) was used to estimate the model, and *ii)* real-time nowcasts, where only data up until year t were used. In both cases, risk-weighted debt was calculated using explanatory variables included in the information set at the end of year $t+1$. The out-of-sample nowcasts of the model are generally close to in-sample risk-weighted debt after 2006 (using at least seven years of data). The most notable discrepancies are found in some industries during the financial crisis, where out-of-sample nowcasts are higher than in-sample risk-weighted debt.

We have also compared risk-weighted debt from our preferred (predictive) model to risk-weighted debt from two alternative specifications were we *i)* included only industry-wide economic indicators, or *ii)* included these indicators together with rating variables (excluding financial accounting variables). Our analyses show that excluding financial accounting and/or rating variables from our model lead to a severely misspecified model due to omitted variable bias. The main source of the bias seems to be the negative correlation between firm-specific shares of debt and bankruptcy probabilities. Hence it is important to have a model that is able to predict bankruptcy probabilities at the firm level even if the main focus is on aggregate measures of risk-weighted debt.

There are issues that are either not discussed or not fully resolved in this paper that might be addressed in future work. One is the application of our framework for stress testing purposes. This would require projecting rating categories and other firm specific variables as functions of macroeconomic scenarios, which is challenging. Another issue is the utilization of real-time bankruptcy registrations (i.e., during the same year) to improve nowcasts, i.e. by combining preliminary data on bankruptcies with our (ex ante) statistical model. We envisage that this can be done in a way that is similar to Bayesian information updating.

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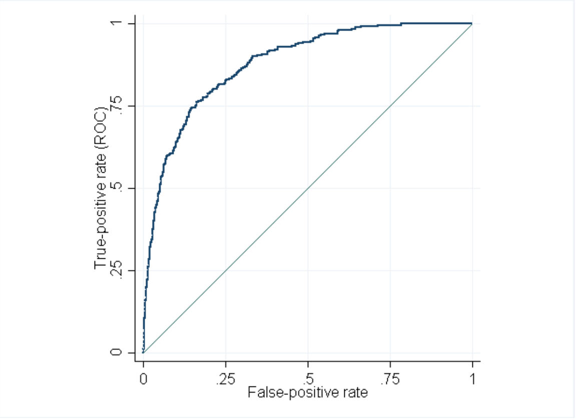
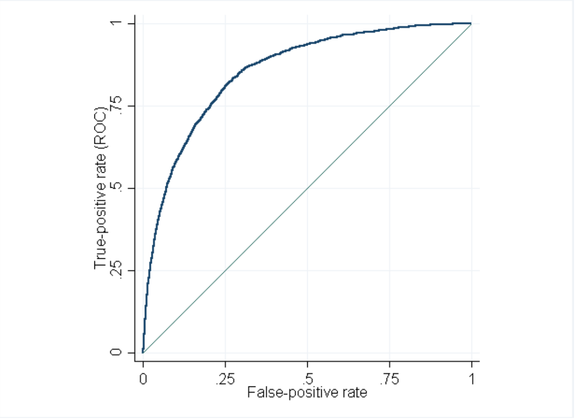
Appendix A: Supplementary tables and figures

Figure A.1: ROC curves - Predictive model

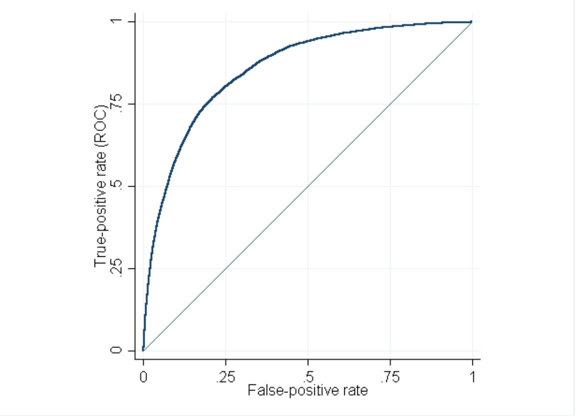
(a) Fishing and fish farming



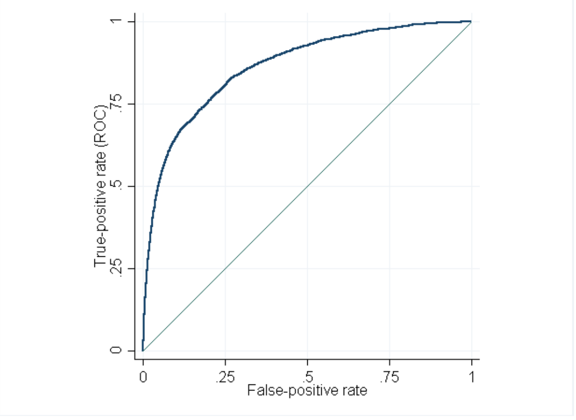
(b) Manufacturing, min. and quar.



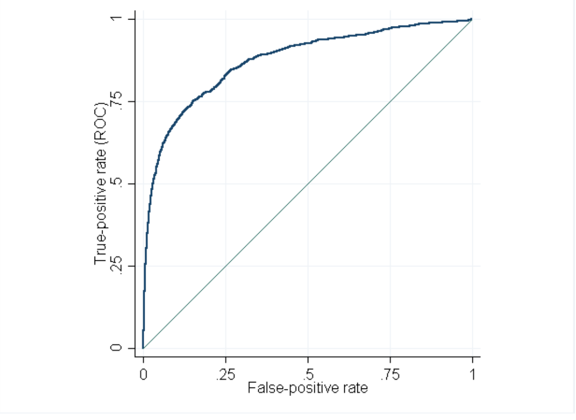
(c) Retail, hotels and restaurants



(d) Construction



(e) Commercial real estate



(f) Services

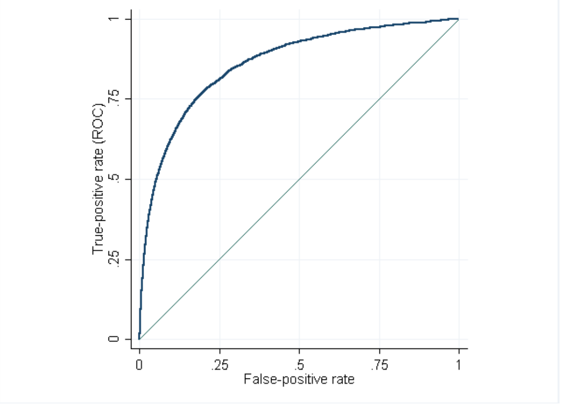


Table A.1: Benchmark model: parameter estimates

	Fishing and fish farming			*Manufacturing, mining and quarrying*			*Retail trade, hotels and restaurants*		
Variable	Coeff.	z	95 CI	Coeff.	z	95 CI	Coeff.	z	95 CI
Rating: C	2.79***	4.53	[1.58,4.00]	4.59***	12.59	[3.87,5.30]	4.34***	17.14	[3.84,4.83]
Rating: B	1.40**	2.32	[0.22,2.58]	3.31***	9.12	[2.60,4.02]	3.00***	11.88	[2.50,3.49]
Rating: A	0.46	0.77	[-0.72,1.65]	2.39***	6.62	[1.68,3.09]	1.93***	7.63	[1.43,2.42]
Rating: AA	-0.11	-0.17	[-1.39,1.17]	1.39***	3.82	[0.68,2.11]	1.04***	4.04	[0.54,1.55]
RoA	-2.97***	-11.92	[-3.46,-2.48]	-2.33***	-26.14	[-2.51,-2.16]	-2.42***	-53.55	[-2.51,-2.33]
ER	-1.18***	-5.06	[-1.63,-0.72]	-0.61***	-7.09	[-0.78,-0.44]	-0.62***	-14.16	[-0.70,-0.53]
RA (log)	0.96**	2.54	[0.22,1.70]	0.76***	6.37	[0.53,0.99]	0.78***	7.64	[0.58,0.98]
RA (logsq)	-0.06***	-2.65	[-0.10,-0.01]	-0.04***	-5.91	[-0.05,-0.03]	-0.06***	-8.32	[-0.07,-0.04]
No. of obs.		14647			80540			246851	
Pseudo R^2		0.24			0.21			0.22	
AUC		0.88			0.86			0.86	

	Construction			*Commercial real estate*			*Services*		
Variable	Coeff.	z	95 CI	Coeff.	z	95 CI	Coeff.	z	95 CI
Rating: C	5.20***	11.49	[4.31,6.08]	3.36***	11.07	[2.76,3.95]	3.77***	18.96	[3.38,4.16]
Rating: B	3.65***	8.08	[2.76,4.53]	1.12***	3.74	[0.54,1.71]	2.06***	10.39	[1.67,2.45]
Rating: A	2.61***	5.78	[1.72,3.49]	-0.05	-0.16	[-0.64,0.54]	1.06***	5.34	[0.67,1.45]
Rating: AA	1.92***	4.22	[1.03,2.81]	-0.59*	-1.82	[-1.22,0.05]	0.22	1.07	[-0.19,0.63]
RoA	-2.67***	-30.29	[-2.84,-2.49]	-3.13***	-22.68	[-3.40,-2.86]	-2.06***	-36.01	[-2.17,-1.95]
ER	-0.96***	-11.60	[-1.13,-0.80]	-1.26***	-10.84	[-1.49,-1.04]	-0.65***	-11.92	[-0.76,-0.54]
RA (log)	1.09***	7.58	[0.81,1.37]	-0.06	-0.35	[-0.37,0.26]	0.62***	6.04	[0.42,0.83]
RA (logsq)	-0.07***	-7.54	[-0.08,-0.05]	0.00	0.10	[-0.02,0.02]	-0.04***	-5.74	[-0.05,-0.02]
No. of obs.		122074			261994			193114	
Pseudo R^2		0.24			0.23			0.23	
AUC		0.86			0.87			0.86	

Notes: Estimation period: 2000-2014. The asterisks indicate significance levels at: * $p<0.1$, ** $p<0.05$ and *** $p<0.01$. AAA is the reference category for Rating.

Table A.2: Benchmark model: average estimated marginal effects[1)]

Variable	*Fishing and fish farming* ME	z	95 CI	*Manufacturing, mining and quarrying* ME	z	95 CI	*Retail trade, hotels and restaurants* ME	z	95 CI
Rating: C	6.83***	6.76	[4.85,8.81]	11.37***	20.44	[10.28,12.46]	12.99***	44.76	[12.42,13.56]
Rating: B	1.58***	3.91	[0.79,2.37]	3.50***	20.44	[3.17,3.84]	3.87***	40.73	[3.68,4.05]
Rating: A	0.32	0.92	[-0.36,1.00]	1.37***	14.52	[1.18,1.55]	1.26***	18.03	[1.12,1.39]
Rating: AA	-0.06	-0.16	[-0.75,0.63]	0.43***	6.07	[0.29,0.56]	0.40***	6.00	[0.27,0.53]
RoA	-0.05***	-10.5	[-0.06,-0.04]	-0.05***	-23.41	[-0.05,-0.04]	-0.07***	-49.36	[-0.07,-0.06]
ER	-0.02***	-5.02	[-0.03,-0.01]	-0.01***	-7.04	[-0.02,-0.01]	-0.02***	-14.12	[-0.02,-0.01]

Variable	*Construction* ME	z	95 CI	*Commercial real estate* ME	z	95 CI	*Services* ME	z	95 CI
Rating: C	10.94***	25.9	[10.11,11.77]	4.17***	14.94	[3.62,4.72]	10.15***	31.16	[9.51,10.79]
Rating: B	2.64***	25.55	[2.44,2.84]	0.34***	5.96	[0.23,0.46]	1.90***	21.26	[1.72,2.07]
Rating: A	0.92***	15.21	[0.80,1.04]	-0.01	-0.15	[-0.11,0.09]	0.54***	8.1	[0.41,0.67]
Rating: AA	0.043***	8.02	[0.33,0.54]	-0.07	-1.48	[-0.17,0.02]	0.07	1.15	[-0.05,0.20]
RoA	-0.05***	-28.47	[-0.05,-0.04]	-0.01***	-19.65	[-0.01,-0.01]	-0.03***	-32.94	[-0.04,-0.03]
ER	-0.02***	-11.5	[-0.02,-0.01]	-0.00***	-10.42	[-0.01,-0.00]	-0.01***	-11.82	[-0.01,-0.01]

Notes: Estimation period: 2000-2014. The asterisks indicate significance levels at: * p<0.1, ** p<0.05 and *** p<0.01. AAA is the reference category for Rating.

1) Average of estimated marginal effects for each observation in percentage points of a unit change in the explanatory variables. A unit change is from 0 to 1 for the rating variables and 1 percentage point for RoA and ER.

Appendix B: Further specification issues

The timing of the explanatory variables The latest available information at the end ("last day") of year t for predicting $p_{i,t+1}$ (forecasting) is $(x_{i,t-1}, r_{it}, \widehat{z}^S_{t+1|t})$ (for simplicity of notation we ignore now that we actually observe the ER from the outgoing balance of the previous year). Then, $p_{i,t+1|t}$ in (3) simplifies to:

$$\ln\left(\frac{p_{i,t+1|t}(\theta^S)}{1 - p_{i,t+1|t}(\theta^S)}\right) = \beta^S x_{i,t-1} + \pi^S r_{it} + \mu^S + \rho^S \widehat{z}^S_{t+1|t} \tag{A.1}$$

To obtain Equation (A.1) from (3): first set $q = 4$, then replace $t+1$ with t and finally set $\widehat{x}_{i,t|t} = x_{i,t-1}$. An alternative model would be to estimate Equation (A.1) directly instead of first estimating (1)–(2) and then replace x_{it} with the trivial prediction $\widehat{x}_{i,t|t} = x_{i,t-1}$. Furthermore, to use all available information for nowcasting suggests the regression equation:

$$\ln\left(\frac{p_{i,t|t}(\theta^S)}{1 - p_{i,t|t}(\theta^S)}\right) = \beta^{*S} x_{i,t-1} + \pi^{*S} r_{it} + \mu^{*S} + \rho^{*S} z^S_t \tag{A.2}$$

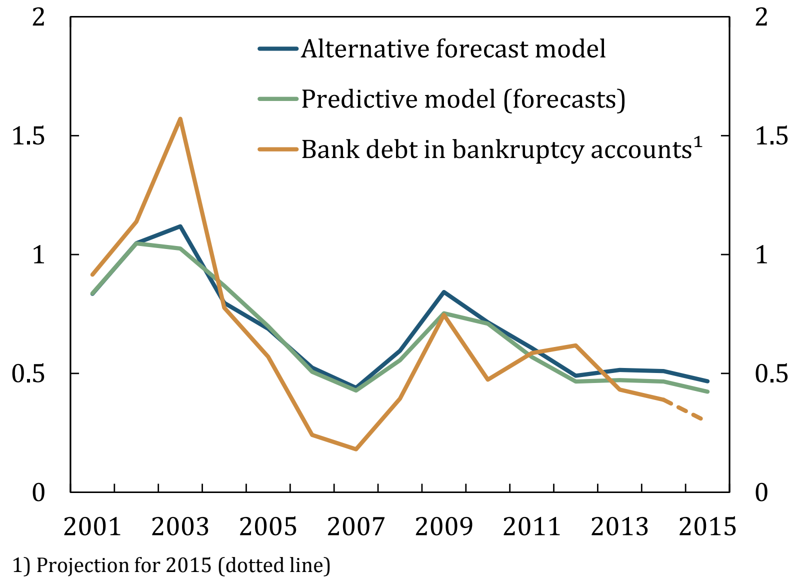
Arguably, Equation (A.1) is more intuitive than (3) for forecasting purposes and we will discuss its properties below.[11] On the other hand, Equation (A.2) is not a valid regression equation because of reverse causality: the rating of a firm dated the same year as the dependent variable could be a direct consequence of the latter. The reason is that a rating from the same year tells us that the firm is probably still active. Conversely, a firm on the verge of collapse may be downgraded if this is known to the rating agency. A bankruptcy could also be registered in the same year as it occurs. Recall that in our preferred model, bankruptcy probability is the conditional probability of bankruptcy in the *future* year for an active firm given firm-specific variables dated at the end of the current year.

To evaluate the properties of our original model against the alternative (A.1), we

[11] As discussed in Section 4, information about $x_{i,t-1}$ is already incorporated into the rating, r_{it}, whereas x_{it} includes new information. Thus our preferred specification accommodates more information and allows *any* information updating about x_{it} during year $t+1$, for example utilizing quarterly accounts from public limited companies. It is outside the scope of this paper to explore this possibility, but the point is that no new estimation is needed to accommodate an *arbitrary* information updating scheme.

estimate (A.1) directly and compare its in-sample implied forecasts of risk-weighted debt with $RW^S_{t+1|t}(\widehat{\theta^S_T})$ from our predictive model. To eliminate the uncertainty stemming from $\widehat{z}^S_{t+1|t}$, we assume for simplicity that $\widehat{z}^S_{t+1|t} = z^S_{t+1}$ ("perfect foresight"). To summarise the estimation results (not displayed): both Pseudo R^2 and AUC are decreased by 0.02–0.04 points in the alternative model compared to the original model. The impacts of the economic indicators and the rating variables (especially the dummy for rating category C), measured either by the magnitude of the average marginal effects or the z values, have increased moderately. On the other hand, the impact of the financial variables is reduced, with lower z values than in the original model, although they are still significant. Figure B.1 reveals that predicted risk-weighted debt from the alternative model is very similar to our preferred (predictive) model.

Figure B.1: Aggregated risk-weighted debt. Predictive model (forecasts), alternative forecast model and bank debt in bankruptcy accounts. Percent. 2000–2015



Utilizing early bankruptcy registration Overall, typically about 15–20 percent of the bankruptcies that we date $t+1$ is registered by $t+3/4$, i.e., Q3 of year $t+1$ (only by $t+2$ is the registration complete). This means that there is a share of the bankruptcies of year $t+1$ that are "revealed early". To see how this information can be utilised, it is convenient to frame our model using Bayesian terminology. First, the probability of bankruptcy from our predictive model, $\Pr(B_{i,t+1} = 1)$ ($B_{i,t+1}$ is the bankruptcy indicator as defined before), can be termed the *prior* probability. The information updating problem is then to calculate the *posterior* probability that $B_{i,t+1} = 1$ given the early bankruptcy registrations. Obviously, the posterior bankruptcy probability among the bankrupt firms is 1. Second, the posterior bankruptcy probability of the remaining firms must be altered relative to their prior probability to reflect that they were not (yet) registered as bankrupt. To study this information-updating problem formally, assume that

$$\chi_{i,t+1} = B_{i,t+1} U_{i,t+1}$$

where $U_{i,t+1}$ is an indicator which is 1 if the value of $B_{i,t+1}$ is revealed by $t + 3/4$. Then, $\chi_{i,t+1}$ is a binary variable which is 1 if and only if $B_{i,t+1} = 1$ *and* $U_{i,t+1} = 1$ (indicating an early bankruptcy registration). Assuming that $\alpha_{i,t+1} \equiv \Pr(U_{i,t+1} = 1 | B_{i,t+1} = 1)$ is a known probability, it is straightforward to show, using Bayes' formula, that

$$\Pr(B_{i,t+1} = 1 | \chi_{i,t+1} = 0) = \frac{\alpha_{i,t+1} \Pr(B_{i,t+1} = 1)}{\alpha_{i,t+1} \Pr(B_{i,t+1} = 1) + \Pr(B_{i,t+1} = 0)}$$

To apply this formula in practice, $\alpha_{i,t+1}$ could be estimated. However, using the historic average as a common estimator over all firms and years, our attempts to calculate $\Pr(B_{i,t+1} = 1 | \chi_{i,t+1} = 0)$ led to erratic and unreliable results. One could imagine a Bayesian version of this method, assuming a prior distribution on $\alpha_{i,t+1}$. Although interesting, it is outside the scope of this paper to pursue this approach further.