The I Theory of Money*

Markus K. Brunnermeier[†] and Yuliy Sannikov[‡]

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Abstract

A theory of money needs a proper place for financial intermediaries. Intermediaries create money by taking deposits from savers and investing them in productive projects. The money multiplier depends on the size of intermediary balance sheets, and their ability to take risks. In downturns, as lending contracts and the money multiplier shrinks, the value of money rises. This leads to a Fisher deflation that hurts borrowers and amplifies shocks. An accommodative monetary policy in downturns, focused on the assets held by constrained agents, can mitigate these destabilizing adverse feedback effects. We devote particular attention to interest rate cuts, and study the potential for such policies to create moral hazard.

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[†]Princeton University.

[‡]Princeton University.

1 Introduction

A theory of money needs a proper place for financial intermediaries. Financial institutions are able to create money, for example by accepting deposits backed by loans to businesses and home buyers. The amount of money created by financial intermediaries depends crucially on the health of the banking system and the presence of profitable investment opportunities. This paper proposes a theory of money and provides a framework for analyzing the interaction between price stability and financial stability. It therefore provides a unified way of thinking about monetary and macroprudential policy.

Intermediaries serve three roles. First, intermediaries *monitor* end-borrowers. Second, they *diversify* by extending loans to and investing in many businesses projects and home buyers. Third, they are active in maturity *transformation* as they issue short-term (inside) money and invest in long-term assets. Intermediation involves taking on some risk. Hence, a negative shock to end borrowers also hits intermediary levered balance sheets. Intermediaries' individually optimal response to an adverse shock is to lend less and accept fewer deposits. As a consequence, the amount of inside money in the economy shrinks. As the total demand for money as a store of value changes little, the value of outside money increases, i.e. deflation occurs.

More specifically, in our model the economy lives within two extreme polar cases. In one polar case the the financial sector is undercapitalized and cannot perform its functions. As the intermediation sector does not create any inside money, money supply is scarce and the value of money is high. Savers hold only outside money and direct claims from end-borrowers. The latter claims are risky, as savers are not equipped with an effective monitoring technology and cannot diversify. In the opposite polar case, intermediaries are well capitalized. Intermediaries mitigate financial frictions and channel funds from savers to productive projects. They lend and invest across in many loans and projects, exploiting diversification benefits and their superior monitoring technology. Intermediaries also create short-term (inside) money and hence the money multiplier is high. In this polar case the value of money is low as inside money supply supplements outside money.

As intermediaries are exposed to end-borrowers' risk, an adverse shock also lowers the financial sector's risk bearing capacity. It moves the economy closer to the first polar regime with high value of money. In other words, a negative productivity shock leads to deflation of Fisher (1933). Financial institutions are hit on both sides of their balance sheets. On the asset side, they are exposed to productivity shocks of end-borrowers. End-borrowers' fire sales depress the price of physical capital and liquidity spirals further erode intermediaries' net worth (as shown in Brunnermeier and Sannikov (2010)). On the liabilities side, they are hurt by the Fisher deflation. As intermediaries cut their lending and create less inside money, the money multiplier collapses and the real value of their nominal liabilities expands. The Fisher deflation spiral amplifies the initial shock and the asset liquidity spiral even further.

Monetary policy can work against the adverse feedback loops that precipitate crises, by affecting the prices of assets held by constrained agents and redistributing wealth. Since monetary policy softens the blow of negative shocks and helps the reallocation of capital to productive uses, this wealth redistribution is not a zero-sum game. It can actually improve welfare.

Simple interest rate cuts in downturns improve economic outcomes only if they boost prices of assets, such as long-term government bonds, that are held by constrained sectors. Wealth redistribution towards the constrained sector leads to a rise in economic activity and an increase in the price of physical capital. As the constrained intermediary sector recovers, it creates more (inside) money and reverses the deflationary pressure. The appreciation of long-term bonds also mitigates money demand, as long-term bonds can be used as a store of value as well. From an ex-ante perspective long-term bonds provide intermediaries with a hedge against losses due to negative macro shocks, appropriate monetary policy can serve as an insurance mechanism against adverse events.

Like any insurance, "stealth recapitalization" of the financial system through monetary policy creates a moral hazard problem. However, moral hazard problems are less severe as the moral hazard associated with explicit bailouts of failing institutions. The reason is that monetary policy is a crude redistributive tool that helps the strong institutions more than the weak. The cautions institutions that bought long-term bonds as a hedge against downturns benefit more from interest rate cuts than the opportunistic institutions that increased leverage to take on more risk. In contrast, ex-post bailouts of the weakest institutions create strong risk-taking incentives ex-ante.

Related Literature. Our approach differs in important ways from both the prominent New Keynesian approach but also from the monetarist approach. The New Keynesian approach emphasizes the interest rate channel. It stresses role of money as unit of account and price and wage rigidities are the key frictions. Price stickiness implies that a lowering nominal interest rates also lowers the real interest rate. Households bring consumption forward and investment projects become more profitable.

In contrast, our I Theory stresses the role of money as store of value and a redistributional channel of monetary policy. Financial frictions are the key friction. Prices are fully flexible. This monetary transmission channel works primarily through capital gains, as in the asset pricing channel promoted by Tobin (1969) and Brunner and Meltzer (1972). As assets are not held symmetrically in our setting, monetary policy redistributes wealth and thereby mitigate debt overhang problems. In other words, instead of emphasizing the substitution effect of interest rate changes, in the I Theory wealth/income effects of interest rate changes are the driving force.

Like in monetarism (see e.g. Friedman and Schwartz (1963)), an endogenous reduction of money multiplier (given a fixed monetary base) leads to deflation in our setting. However, in our setting outside money is only an imperfect substitute for inside money. Intermediaries, either by channeling funds through or by underwriting and thereby enabling firms to approach capital markets directly, enable a better capital allocation and more economic growth. Hence, in our setting monetary intervention should aim to recapitalize undercapitalized borrowers rather than simply increase the money supply across the board. A key difference to our approach is that we focus more on the role of money as a store of value instead of the transaction role of money.

Instead of the "money view" our approach is closer in spirit to the "credit view" à la Gurley and Shaw (1955), Patinkin (1965), Tobin (1969, 1970), Bernanke (1983) Bernanke and Blinder (1988) and Bernanke, Gertler and Gilchrist (1999).[1]

As in Samuelson (1958) and Bewley (1980), money is essential in our model in the sense of Hahn (1973). In Samuelson households cannot borrow from future not yet born generations. In Bewley and Scheinkman and Weiss (1986) households face explicit borrowing limits and cannot insure themselves against idiosyncratic shocks. Agent's desire to self-insure through precautionary savings creates a demand for the single asset, money. In our model households can hold money and physical capital. The return on capital is risky and its risk profile differs from the endogenous risk profile of money. Financial institutions create inside money and mitigate financial frictions. In Kiyotaki and Moore (2008) money and capital coexist. Money is desirable as it does not suffer from a resellability constraint as physical capital does. Lippi and Trachter (2012) characterize the trade-off between insurance and production incentives of liquidity provision. Levin (1991) shows that monetary policy is more effective than fiscal policy if the government does not know which agents are productive. More recently, Cordia

[1]The literature on credit channels distinguishes between the bank lending channel and the balance sheet channel (financial accelerator), depending on whether banks or corporates/households are capital constrained. Strictly speaking our setting refers to the former, but we are agnostic about it and prefer the broader credit channel interpretation.

and Woodford (2010) introduced financial frictions in the new Keynesian framework. The finance papers by Diamond and Rajan (2006) and Stein (2010) also address the role of monetary policy as a tool to achieve financial stability.

More generally, there is a large macro literature which also investigated how macro shocks that affect the balance sheets of intermediaries become amplified and affect the amount of lending and the real economy. These papers include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), who study financial frictions using a log-linearized model near steady state. In these models shocks to intermediary net worths affect the efficiency of capital allocation and asset prices. However, log-linearized solutions preclude volatility effects and lead to stable system dynamics. Brunnermeier and Sannikov (2010) also study full equilibrium dynamics, focusing on the differences in system behavior near the steady state, and away from it. They find that the system is stable to small shocks near the steady state, but large shocks make the system unstable and generate systemic endogenous risk. Thus, system dynamics are highly nonlinear. Large shocks have much more serious effects on the real economy than small shocks. He and Krishnamurthy (2010) also study the full equilibrium dynamics and focus in particular on credit spreads. In Mendoza and Smith's (2006) international setting the initial shock is also amplified through a Fisher debt-deflation that arises from the interaction between domestic agents and foreign traders in the equity market. In our paper debt deflation is due to the appreciation of inside money. For a more detailed review of the literature we refer to Brunnermeier et al. (2010).

This paper is organized as follows. Section 2 sets up the model and derives first the solutions for two polar cases. Sect ion 3 presents computed examples and discusses equilibrium properties, including capital and money value dynamics, the amount of lending through intermediaries, and the money multiplier for various parameter values. Section 4 introduces long-term bonds and studies the effect of interest-rate policies as well as open-market operations. Section 5 showcases a numerical example of monetary policy. Section 6 concludes.

2 The Baseline Model Absent Policy Intervention

We build a dynamic but simple model of money and intermediation.[2]

Agents. The economy is populated by households and intermediaries. Some households are end-borrowers and some are savers. End borrower value physical capital more since they

[2]The following model is a product of multiple iterations, streamlined to leave out inessential technical ingredients and to include several parameters of interest in a straightforward manner.

are more productive than savers. For simplicity, we assume that the total wealth of end-borrowers is zero, i.e., all household wealth is concentrated in the hands of savers. Absent intermediaries, savers can simply rent out physical capital to end-borrowers. Intermediaries open another funding channel. Savers can deposit some of their funds with the intermediary sector which finances the projects of end-borrowers.

Technology. Physical capital k_t produces output at rate $(a - \iota_t)k_t$ and grows deterministically according to

$$dk_t = (\Phi(\iota_t) - \delta)k_t\, dt,$$

where ι_t is the rate of investment and Φ is a standard investment function with adjustment costs, such that $\Phi' > 0$ and $\Phi'' \leq 0$.

Shocks. Macro shocks hit the economy with Poisson intensity λ. In the event of a shock, a fraction $\underline{\phi}$ of end-borrowers can steal physical capital and become regular savers. Savers who rented out their capital lose their entire capital with the idiosyncratic probability of $\underline{\phi}$. Intermediaries have two advantages. First, they can diversify across (a continuum of) end-borrowers. Second, their superior monitoring technology allows them to reduce the probability to $\phi < \underline{\phi}$. Both advantages together imply that in the case of a macro shock each intermediaries loses a fixed fraction ϕ of their capital.

Importantly, these macro shocks affect the net worth of intermediaries but not the total quantity of capital in the economy, which grows according to

$$dK_t/dt = (\Phi(\iota_t) - \delta)K_t. \tag{2.1}$$

Preferences. Both households and intermediaries have logarithmic utility. Households discount their utility at rate r, while intermediaries, at rate $\rho > r$.

Money. In the baseline model, there is a fixed amount of fiat money in the economy that pays zero interest. Money is backed by taxes. The government taxes all output at a fixed rate $\tau \in [0, 1)$, and uses the proceeds (in real output) to purchase money at the current market price.

When we consider monetary policy in Section 3, we allow the central bank to pay interest on money deposits and introduce long-term government bonds.

Agents' Portfolios. Intermediaries and saver households choose to allocate their wealth between money and capital. Intermediaries hold risky capital and rent it out to end-borrowers. They can (and will in equilibrium) put a negative portfolio weight on money by accepting money deposits from savers. They can borrow up to the amount that they can guarantee to repay for sure, even if hit by macro shock. Importantly, *the deposits are denominated in money*, i.e. intermediaries are obligated to repay savers money, whatever is its market price. Hence, the inside money created by the banking sector is a perfect substitute for outside money (which is (partially backed by tax revenues). A saver can hold money either directly (outside money) or by making a deposit with an intermediary (inside money). Holding only physical capital and renting it out is very risky for savers as with intensity $\lambda\underline{\phi}$ each saver loses its entire capital stake. Because a savers would get an expected utility of $-\infty$ if it held only capital, there is a demand for money.

The value of money depends on money demand and supply. The latter is determined by the amount of inside money that intermediaries create by accepting deposits to invest in capital projects. In turn, the intermediaries' ability and willingness to create money depends their risk-taking capacity, determined by their net worth. We measure the values of all assets in the units of *output*. In these units, denote the equilibrium price of one unit of physical capital by q_t, and the price of money (per unit of capital) by p_t. Therefore, the total wealth in the economy is

$$q_t K_t + p_t K_t.$$

The prices of money and capital are fluctuating, and capital may be stolen. In addition, capital generates dividends in the form of net output (after investment).

The economy is subject to financial frictions as intermediaries can issue only debt and not equity, i.e. the must entirely absorb all losses from their projects.[3]

2.1 Two Benchmark Cases

As a start, we identify two important benchmarks. In equilibrium, the economy will fluctuate between the two. First, the *money regime* benchmark arises when intermediaries cannot function, for example when they have zero net worth. With zero loss absorption capacity intermediaries can not any inside money. As a consequence, saving households hold only outside money or inefficiently rented capital. Second, the *frictionless* benchmark, in which the economy behaves as if no (redistributional) shocks (i.e. $\lambda = 0$). In this case, money creation by intermediaries is unimpaired as intermediaries do not need net worth buffers to absorb risk. For a start we consider for simplicity the case in which capital grows at a

[3]Allowing intermediaries to issue outside equity to savers would not qualitatively change the insights of this model.

constant rate, $g = \Phi(0) - \delta$, independent of the investment rate and produces output at a rate of rate a.

The Money Regime. If intermediaries have zero net worth and $\lambda > 0$, then all capital in the economy must be permanently held by saver households. Denote by $\underline{p}$ and $\underline{q}$ the prices of money and capital in the steady state of this regime. Money earns the return of

$$r_t^M = \frac{\tau a}{\underline{p}} + g$$

From physical capital, savers get the return of

$$r_t^H = \frac{(1-\tau)a}{\underline{q}} + g$$

until with intensity $\lambda\underline{\phi}$ they lose all capital. If a savers puts weight $\underline{x}$ on capital and $1 - \underline{x}$ on money, then the optimal choice of $\underline{x}$ maximizes

$$\underline{x}\, r_t^H + (1-\underline{x})\, r_t^M + \lambda\underline{\phi} \ln(1-\underline{x}),$$

because with intensity $\lambda\underline{\phi}$ the household loses a fraction $\underline{x}$ of wealth. Differentiating with respect to $\underline{x}$ and plugging in the expressions for r_t^M and r_t^H, we get

$$\left(\frac{(1-\tau)a}{\underline{q}} + g\right) - \left(\frac{\tau a}{\underline{p}} + g\right) - \lambda\underline{\phi}\frac{1}{1-\underline{x}} = 0.$$

Market clearing for capital implies $\underline{x} = \underline{q}/(\underline{q} + \underline{p})$. In terms of output, savers want to consume a fraction r of their wealth $(\underline{q} + \underline{p})K$, while total output is aK. Market clearing implies $r(\underline{q} + \underline{p}) = a$. Putting the two market clearing conditions together with savers' first order condition leads to

$$\underline{q} = \frac{(1-\tau)a}{r + \lambda\underline{\phi}} \quad \text{and} \quad \underline{p} = \frac{a}{r}\frac{\tau r + \lambda\underline{\phi}}{r + \lambda\underline{\phi}}. \tag{2.2}$$

Money becomes more valuable when the intensity of individual shocks to capital $\lambda\underline{\phi}$ is greater. Even though money generates a lower dividend yield, households prefer to hold it because it is safe. Of course, the money multiplier in this extreme case is 1, as there is no inside money but only outside money.

Interestingly, money can have positive value not only if $\tau = 0$, but also if τ is negative (as long as $\tau > -\lambda\underline{\phi}/r$). That is, money can have value in equilibrium even if the government perpetually prints money to finance transfers to individual agents, as long as the rate of transfers is not too large. Of course, if $\tau \le 0$, then there is also another equilibrium in which money is worthless.[4]

A Frictionless Economy. In a frictionless economy agents can perfectly insure themselves against redistributional λ shocks and it is as if $\lambda = 0$ in our setting. In this case intermediaries are not constrained by financial frictions to channel funds from savers to end-borrowers. Let's denote by $\bar{p}$ and $\bar{q}$ the prices of money and capital at the steady state of the frictionless economy. If so, then capital and money are both risk-free. Their returns must equal. Capital earns the return of

$$\frac{(1-\tau)a}{\bar{q}} + g$$

from its dividend yield and its capital gains rate. At the same time, all money in the economy, with value $\bar{p}K_t$ that grows at rate g, generates "dividends" at rate $\tau a K_t$. Thus, the return on money is

$$\frac{\tau a}{\bar{p}} + g.$$

Equating the two returns, we get

$$\frac{1-\tau}{\bar{q}} = \frac{\tau}{\bar{p}}.$$

Because households and intermediaries earn the same returns, but the less patient intermediaries consume at a higher rate, at the steady state the share of net worth that belongs to the intermediaries is $\bar{\eta} = 0$. Household demand for consumption goods is $r(\bar{p} + \bar{q})K_t$, proportionate to their wealth. Equating demand and supply, we get

$$r(\bar{p} + \bar{q})K_t = aK_t \quad \Rightarrow \quad \bar{p} + \bar{q} = \frac{a}{r}, \quad \bar{p} = \frac{\tau a}{r} \quad \text{and} \quad \bar{q} = \frac{(1-\tau)a}{r}. \tag{2.3}$$

Note that we could have simply set $\lambda = 0$ in the price equations of the "money regime" to obtain the same result. In this frictionless benchmark money has value only when it is backed by taxes, i.e. $\tau > 0$.

Intermediaries create the maximal amount of money if they borrow to hold all capital in

[4]Technically, the households would get utility $-\infty$ in this equilibrium, since they lose their entire wealth with positive probability. Thus, the equilibrium in which money is worthless is more meaningful in a model where households can diversify as intermediaries.

the economy. In this case, the value of all deposits (inside money) is

$$\frac{(1-\tau)a}{r}K_t.$$

Because outside money is worth $\frac{\tau a}{r}K_t$, the money multiplier (total money to outside money) is $1/\tau$.

2.2 Returns and Equilibrium Conditions in the Dynamic Model

The full model lives between the two benchmark cases. After a negative macro shock, he economy collapses towards the money regime. As intermediaries' net worth declines, their capacity to mitigate financial frictions declines and the price of capital falls towards $\underline{q}$, the price of money rises towards $\underline{p}$ and the money multiplier collapses to 1. As the economy recovers, intermediaries create more money, the money multiplier expands and the value of outside money falls. Upon recovery, the economy gets closer to the frictionless benchmark described above, although that benchmark is never reached. In booms, intermediaries need net worth buffers to manage risks, and there is always a strictly positive chance of lapsing back into a crisis.

To characterize the equilibrium formally, we need to understand returns that households and intermediaries earn from capital and money, and combine these agents' portfolio optimization conditions with market clearing.

In equilibrium, asset prices p_t and q_t do not stay fixed - instead they change with time and with shocks. To characterize equilibrium, we need to find prices and allocations, such that all agents maximize utility and markets clear. We have to find these quantities for any history $\{t_1 < t_2 \ldots < t_n \leq t\}$, where $t_1, \ldots t_n$ are the times of all previous shocks, and t is the current time. Denote by $(x_t, 1-x_t)$, $x_t \geq 0$, the intermediaries' portfolio allocations to capital and money at time t, and by $(\underline{x}_t, 1-\underline{x}_t)$, $x_t \in [0,1]$, the households' portfolio allocations. Let N_t and $(p_t+q_t)K_t - N_t$ be the aggregate net worths of intermediaries and households, and C_t and $\underline{C}_t$, their aggregate consumption rates, respectively. Formally:

Equilibrium Definition. *An equilibrium is a map from histories $\{t_1 < t_2 \ldots < t_n \leq t\}$ to prices p_t and q_t, allocations $(x_t, 1-x_t)$, $(\underline{x}_t, 1-\underline{x}_t)$ and $(C_t, \underline{C}_t)$, as well as wealth levels $(N_t, (p_t+q_t)K_t - N_t)$, such that*

(i) all markets, for capital, money and consumption goods, clear

(ii) all agents choose portfolio allocations and consumption rates to maximize utility

(iii) and the agents' wealth levels satisfy their budget constraints.

Returns. Denote by

$$\frac{dp_t/dt}{p_t} = \mu_t^p \quad \text{and} \quad \frac{dq_t/dt}{q_t} = \mu_t^q \tag{2.4}$$

the endogenous evolution of prices in the absence of shocks, and by $\breve{p}_t$ and $\breve{q}_t$ the new price levels in the event that a shock arrives at time t.

The returns that intermediaries and households earn on money and capital are as follows. Since the value of all money in the economy is $p_t K_t$, where K_t follows (2.1) and p_t follows (2.4), and since the flow of payments to all money holders equals $\tau a K_t$, the return on money is

$$r_t^M = \underbrace{\frac{\tau(a-\iota_t)}{p_t}}_{\text{dividend yield}} + \underbrace{\mu_t^p + \Phi(\iota_t) - \delta}_{\text{capital gains rate}} \tag{2.5}$$

in the absence of shocks. Following a shock, the value of money changes by a factor of $\breve{p}_t/p_t$.

Likewise, an investment in capital of $q_t k_t$ earns the return of

$$r_t^K = \underbrace{\frac{(1-\tau)(a-\iota_t)}{q_t}}_{\text{dividend yield}} + \underbrace{\mu_t^q + \Phi(\iota_t) - \delta}_{\text{capital gains rate}} \tag{2.6}$$

in the absence of shocks. Following a shock, the change in value of a capital stake depends on who holds it. The value of capital held by an intermediary changes by a factor of $(1-\phi)\breve{q}_t/q_t$. The value of capital held by a household changes by a factor of $\breve{q}_t/q_t$ with probability $1-\underline{\phi}$, and drops in value to zero with probability $\underline{\phi}$.

The optimal equilibrium rate of investment ι_t maximizes the return on capital, i.e. it solves

$$\max_{\iota} \Phi(\iota) - \frac{(1-\tau)\iota}{q_t} \quad \Rightarrow \quad \Phi'(\iota_t) = \frac{1-\tau}{q_t}. \tag{2.7}$$

Optimal Portfolio Choice Conditions. With logarithmic utility, optimal portfolio weights x_t and $1-x_t$ that any intermediary puts on capital and money, solve

$$\max_x \underbrace{x\, r_t^K + (1-x)\, r_t^M}_{\text{return if no shocks}} + \lambda \underbrace{\log\left(x(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x)\frac{\breve{p}_t}{p_t}\right)}_{\text{utility drop if shock}}. \tag{2.8}$$

Thus, x_t must satisfy the first-order condition

$$r_t^K - r_t^M + \lambda \frac{(1-\phi)\frac{\breve{q}_t}{q_t} - \frac{\breve{p}_t}{p_t}}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\frac{\breve{p}_t}{p_t}} = 0. \tag{2.9}$$

Note that the allocation to capital x_t that solves the intermediaries' problem automatically guarantees that their wealth stays positive after a negative shock, since their utility function is defined only on the positive domain.

Similarly, households choose portfolio weights $(\underline{x}_t, 1-\underline{x}_t)$ to solve

$$\max_{x\geq 0}\; x\, r_t^K + (1-x)\, r_t^M + \lambda(1-\underline{\phi})\, \log\left(x\frac{\breve{q}_t}{q_t} + (1-x)\frac{\breve{p}_t}{p_t}\right) + \lambda\underline{\phi}\log\left((1-x)\frac{\breve{p}_t}{p_t}\right). \tag{2.10}$$

Thus, $\underline{x}_t$ must satisfy the first-order condition

$$r_t^K - r_t^M + \lambda(1-\underline{\phi})\, \frac{\frac{\breve{q}_t}{q_t} - \frac{\breve{p}_t}{p_t}}{\underline{x}_t\,\frac{\breve{q}_t}{q_t} + (1-\underline{x}_t)\,\frac{\breve{p}_t}{p_t}} - \lambda\underline{\phi}\,\frac{1}{1-\underline{x}_t} = 0. \tag{2.11}$$

Market Clearing Conditions. Equating the total value of capital held by intermediaries and households to the value of all capital in the economy, we obtain the market-claring condition for capital

$$x_t N_t + \underline{x}_t((q_t + p_t)K_t - N_t) = q_t K_t \tag{2.12}$$

This condition automatically implies the market-clearing condition for money, because portfolio weights add up to 1.

Due to logarithmic utility, the optimal consumption rates of intermediaries and households are ρN_t and $r((q_t+p_t)K_t - N_t)$ respectively. Equating demand and supply, we get the market-clearing condition for output

$$\rho N_t + r((q_t + p_t)K_t - N_t) = (a - \iota_t)K_t. \tag{2.13}$$

The Budget Constraints. The law of motion of intermediaries' net worth, given their portfolio allocations and consumption choices, is

$$dN_t/dt = (x_t\, r_t^K + (1-x_t)\, r_t^M - \rho)\, N_t \tag{2.14}$$

when there are no shocks. In the event of a shock, the intermediaries' net worth drops to

$$\breve{N}_t = \left(x_t(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\frac{\breve{p}_t}{p_t} \right) N_t. \tag{2.15}$$

Given (2.14) and (2.15), the net worth of households is automatically given by $(q_t+p_t)K_t - N_t$.

2.3 The State Variable

Because our setting is scale-invariant, the severity of financial frictions in our economy is quantified by the fraction of wealth that belongs to intermediaries

$$\eta_t = \frac{N_t}{(q_t+p_t)K_t}. \tag{2.16}$$

That is, we expect two economies, in which η_t is the same, to look like scaled versions of one another. Therefore, we look for a Markov equilibrium with the state variable η_t, in which prices (p_t, q_t) and allocations $(x_t, \underline{x}_t)$ are functions of η_t.

Using (2.14) and (2.1), the law of motion of η_t in the absence of shocks can be written as

$$\frac{d\eta_t/dt}{\eta_t} = \underbrace{x_t\frac{(1-\tau)(a-\iota_t)}{q_t} + (1-x_t)\frac{\tau(a-\iota_t)}{p_t} + (x_t-\theta_t)(\mu_t^q - \mu_t^p) - \rho}_{\mu_t^\eta}, \tag{2.17}$$

where

$$\theta_t = \frac{q_t}{p_t+q_t} \tag{2.18}$$

is the fraction of the economy's wealth that is in capital. From (2.15), in the event of a shock, η_t drops to

$$\breve{\eta}_t = \eta_t \left(x_t(1-\phi)\frac{\breve{\theta}_t}{\theta_t} + (1-x_t)\frac{1-\breve{\theta}_t}{1-\theta_t} \right) \tag{2.19}$$

It is useful to summarize all equilibrium conditions in a proposition. Below, we express the equilibrium conditions in the form that includes only scaled variables such as η_t, q_t and x_t, and not aggregate variables such as N_t or K_t.

Proposition 1. *In a Markov equilibrium, the law of motion of the state variable η_t is given by (2.17) and (2.19), where $q_t = \theta_t(q_t+p_t)$, $p_t = (1-\theta_t)(q_t+p_t)$, and ι_t, q_t+p_t satisfy*

$$\theta_t(q_t+p_t)\Phi'(\iota_t) = 1 \quad \text{and} \quad (\rho\eta_t + r(1-\eta_t))(q_t+p_t) = a - \iota_t. \tag{2.20}$$

Functions $\theta_t = \theta(\eta_t)$, $x_t = x(\eta_t)$ and $\underline{x}_t = \underline{x}(\eta_t)$ are jointly determined by the conditions

$$x_t\eta_t + \underline{x}_t(1-\eta_t) = \theta_t,$$

$$\frac{(1-\tau)(a-\iota_t)}{q_t} - \frac{\tau(a-\iota_t)}{p_t} + \mu_t^q - \mu_t^p + \lambda\left((1-\phi)\frac{\breve{\theta}_t}{\theta_t} - \frac{1-\breve{\theta}_t}{1-\theta_t}\right)\frac{\eta_t}{\breve{\eta}_t} = 0 \quad \text{and}$$

$$\frac{(1-\tau)(a-\iota_t)}{q_t} - \frac{\tau(a-\iota_t)}{p_t} + \mu_t^q - \mu_t^p + \lambda(1-\underline{\phi})\,\frac{\frac{\breve{\theta}_t}{\theta_t} - \frac{1-\breve{\theta}_t}{1-\theta_t}}{\underline{x}_t\frac{\breve{\theta}_t}{\theta_t} + (1-\underline{x}_t)\frac{1-\breve{\theta}_t}{1-\theta_t}} - \lambda\underline{\phi}\,\frac{1}{1-\underline{x}_t} \le 0,$$

with equality if $\underline{x}_t > 0$, where it is convenient to express

$$\mu_t^q - \mu_t^p = \theta'(\eta)\frac{\mu_t^\eta\eta_t}{\theta_t(1-\theta_t)}. \tag{2.21}$$

Proof. The formulas follow directly from (2.7), (2.13), (2.12), (2.9), (2.11), and the definitions of η_t and θ_t. Regarding (2.21), note that

$$\mu_t^q = \left(\frac{\theta'(\eta)}{\theta(\eta)} + \frac{p'(\eta)+q'(\eta)}{p(\eta)+q(\eta)}\right)\mu_t^\eta\eta_t \quad \text{and} \quad \mu_t^p = \left(\frac{-\theta'(\eta)}{1-\theta(\eta)} + \frac{p'(\eta)+q'(\eta)}{p(\eta)+q(\eta)}\right)\mu_t^\eta\eta_t,$$

so

$$\mu_t^q - \mu_t^p = \left(\frac{\theta'(\eta)}{\theta(\eta)} + \frac{\theta'(\eta)}{1-\theta(\eta)}\right)\mu_t^\eta\eta_t = \frac{\theta'(\eta)}{\theta_t(1-\theta_t)}\mu_t^\eta\eta_t$$

□

Proposition 1 implies a first-order ordinary differential equation for the function $\theta(\eta)$, which can be solved numerically to find equilibrium for any parameter values. The derivative $d\theta/d\eta$ depends on the values of function θ on the interval $[0,\eta^*]$. In the Appendix, we outline the algorithm that we use to compute examples.

Next we provide several examples to explain equilibrium dynamics, and to explore how it depends on model parameters. The Appendix provides details of the algorithm we used to compute solutions.

2.4 Examples

First, let us discuss the general properties of equilibria. Figure 1 provides information about the equilibrium with a degenerate investment function $\Phi : \{0\} \to \mathbb{R}$ that leads to growth

$g = \Phi(0) - \delta = 4\%$, and parameters $a = 0.1$, $r = 5\%$, $\rho = 6\%$, $\lambda = 1$, $\phi = 0.002$, $\underline{\phi} = 0.02$ and $\tau = 0.1$.

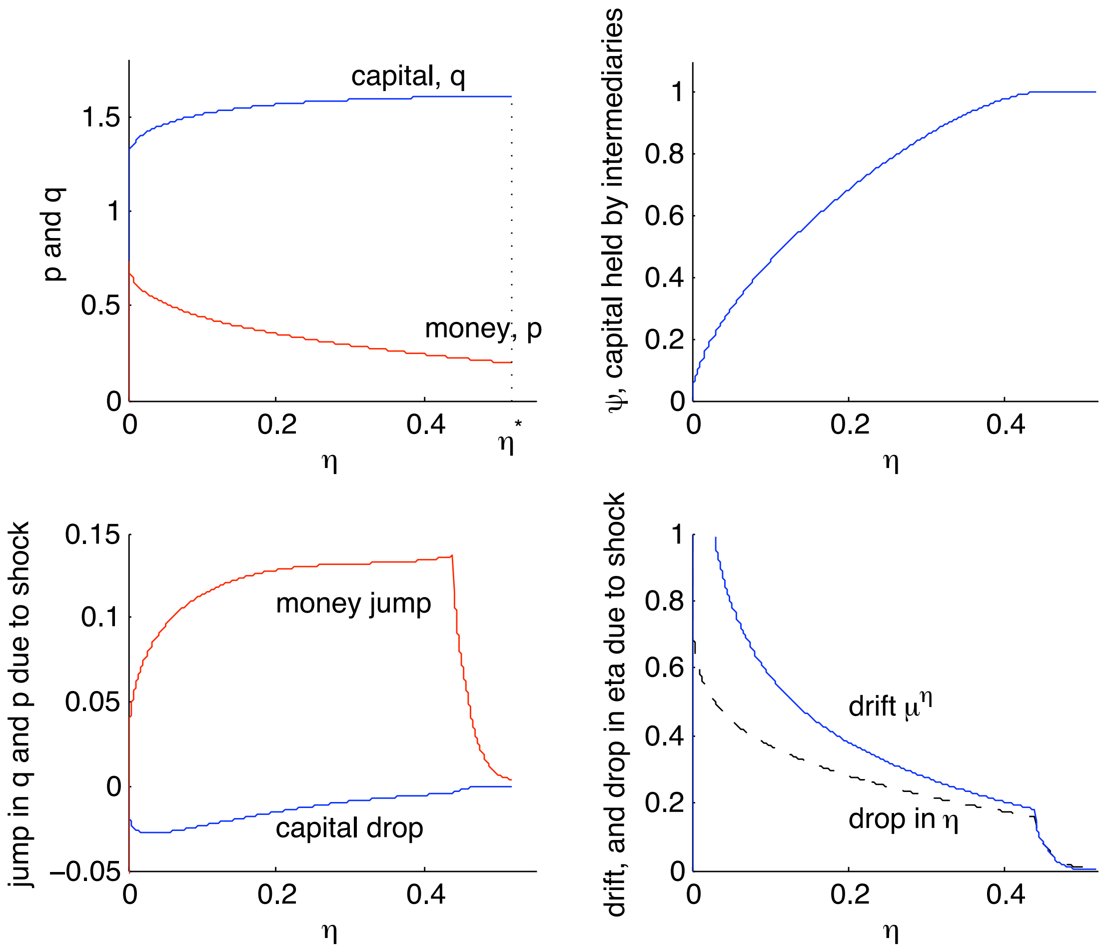

Figure 1: Equilibrium: quantities related to money are in blue, and capital, in red.

The top right panel shows the equilibrium allocation $\psi_t = \eta_t x_t/\theta_t$ of capital to intermediaries as a function of intermediaries' wealth share η. For large enough η 100 percent of the physical capital is held (or financed) by the intermediary sector. As the wealth share η drops intermediaries fire-sell physical capital to the less productive savers.

In the absence of shocks, the equilibrium state variable η_t has positive drift μ_t^η given by (2.17) and shown as a solid line in the bottom right panel. The drift is decreasing monotonically over the interval $[0, \eta^*]$ towards 0. Point η^* plays the role of the stochastic steady state: when the state variable η_t is at η^*, then it stays there when there are no shocks. If a shock arrives, η_t jumps down, as some of capital financed by intermediaries gets stolen and due to amplification through prices. The size of the jump is shown on the bottom right panel in solid color. Note that jumps, which arrive with intensity $\lambda = 1$, are smaller than the magnitude of the drift, so in expectation the system tends to come back to η^* as well.

The model exhibits the traditional amplification channel on the asset side of the intermediary balance sheets: as the price of physical capital $q(\eta)$, shown on the top left panel of Figure 1 in red, drops following a shock. In addition, shocks hurt borrowers on the liability sides of the balance sheet through the Fisher deflationary spiral. As we can observe from red money price curve $p(\eta)$ on the top left panel the figure, money appreciates following a negative shock. The sizes of jumps in capital and money values that occur after a shock are shown at the bottom left panel. These endogenous jumps become much higher in the region where savers hold capital.

The reason for the deflationary pressure after a shock is as follows. As intermediaries suffer losses, they contract their balance sheets. Thus, they take fewer deposits and create less inside money.[5] The total supply of money (inside and outside) shrinks, but the demand for money does not change significantly since saving households still want to allocate a portion of their savings to safe money. As a result, the value of money goes up. Figure 2 shows that the money multiplier, the ratio of total money to outside money, decreases towards 1 as η decreases to 0.[6]

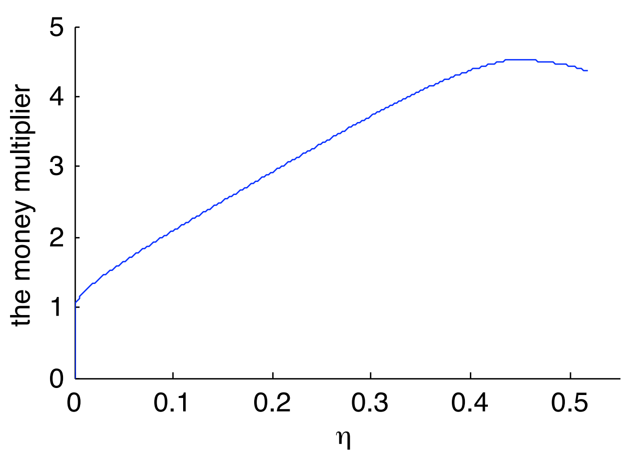

Figure 2: The money multiplier.

[5]In reality, rather than turning savers away, financial intermediaries might still issue demand deposits and simply park the proceeds with the central bank as excess reserves.

[6]It may look strange that the money multiplier decreases in η_t near η^*. This happens because when intermediaries already finance all capital in the economy, i.e. $\psi_t = 1$, the amount of money they create is decreasing in their need for outside funding, i.e. decreasing in η.

The model exhibits nonlinear amplification effects. Near the steady state intermediaries are least constrained: they hold all capital in the economy and prices respond little to shocks. As a result, there is little endogenous risk near η^*. In contrast, when intermediaries sell some of their capital holdings to savers to reduce risk, prices become much more sensitive to shocks. Savers are worse at monitoring capital, and they cannot diversify, so they attach a much lower valuation to capital than intermediaries. The deeper the crisis, the more capital is held by savers (and for longer periods of time, until the economy recovers), the lower the price of capital. Capital and money both change in value much more when savers directly invest in capital than near the steady state.

2.5 The Impact of Parameters on Equilibrium.

Next, we explore how model parameters affect equilibrium. In particular, we want to understand conceptually the effects of exogenous shocks, liquidity and fiscal policy on equilibrium. In the examples below, we change one parameter at a time from our baseline example in Figure 1.

Volatility Paradox. Brunnermeier and Sannikov (2012) coined the term the *volatility paradox* to refer to the phenomenon that endogenous risk persists even for very small levels of exogenous risk. To explore whether the volatility paradox holds up in our setting, we focus on the parameter ϕ, the fraction of capital managed by intermediaries that gets lost in the event of the macro shock. This parameter quantifies the level of exogenous risk that intermediaries face. From our baseline level of $\phi = 0.002$, we increase ϕ to 0.01 and decrease it to 0.0005 in Figure 3.

Lower exogenous risk does make the system *appear* more stable near the steady state η^*; the state variable η_t drops by less when a shock arrives (see bottom left panel). However, even while exogenous risk drops towards 0, endogenous risk persists (and may even get slightly higher) in the region of fire-sales, when $\psi_t < 1$ and households hold a positive amount of capital. Moreover, the distance from the steady state to the point of firesales shrinks as exogenous risk decreases. The top right panel of Figure 3 illustrates endogenous risk capital and money, and the bottom left panel illustrates the drop in η_t following a shock. In the region of firesales, a shock leads to a higher endogenous drop in η_t when exogenous risk is lower.

The explanation of the volatility paradox has two parts. The first part is that the stochastic steady state η^* is endogenous: it is determined by the relative rates of intermediaries'

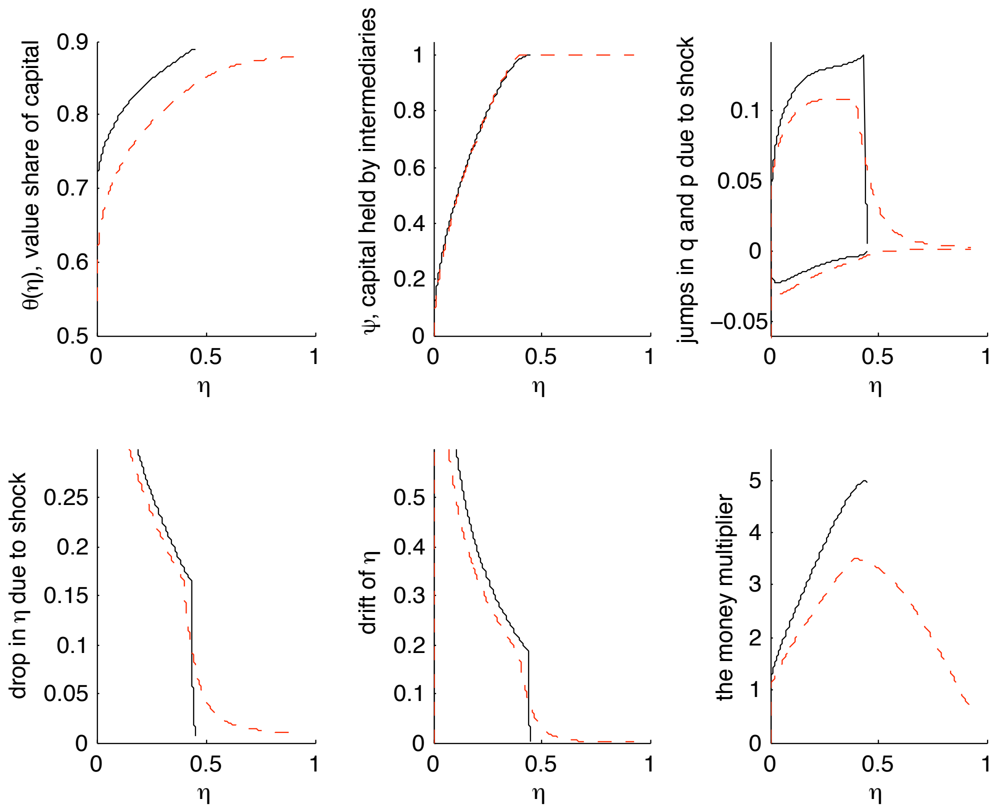

Figure 3: Equilibrium for $\phi = 0.01$ (dashed) and 0.0005 (solid).

earnings and consumption payouts. Earnings in turn depend on risk premia, which go down as soon as crisis probability falls sufficiently. When exogenous risk ϕ is lower, intermediaries can sustain much higher leverage and risk premia come down at a much lower level of η^*. The second part is backward induction: when shocks arrive, forward-looking agents do not assume that the economy recovers soon for sure, but instead assess the possibility that it gets stuck in the worst states, with severe capital misallocation and a high value of money. It is this uncertainty over future outcomes that lead to large endogenous risk in deep crisis states even when exogenous risk is low.

Liquidity. Market liquidity in our setting can be measured by the difference between savers' and intermediaries valuations of capital. Lower market liquidity gives rise to higher endogenous risk.

Figure 4 illustrates comparative statics on $\underline{\phi}$. Physical capital is more liquid when savers have a smaller disadvantage in monitoring end-borrowers relative to intermediaries. Formally,

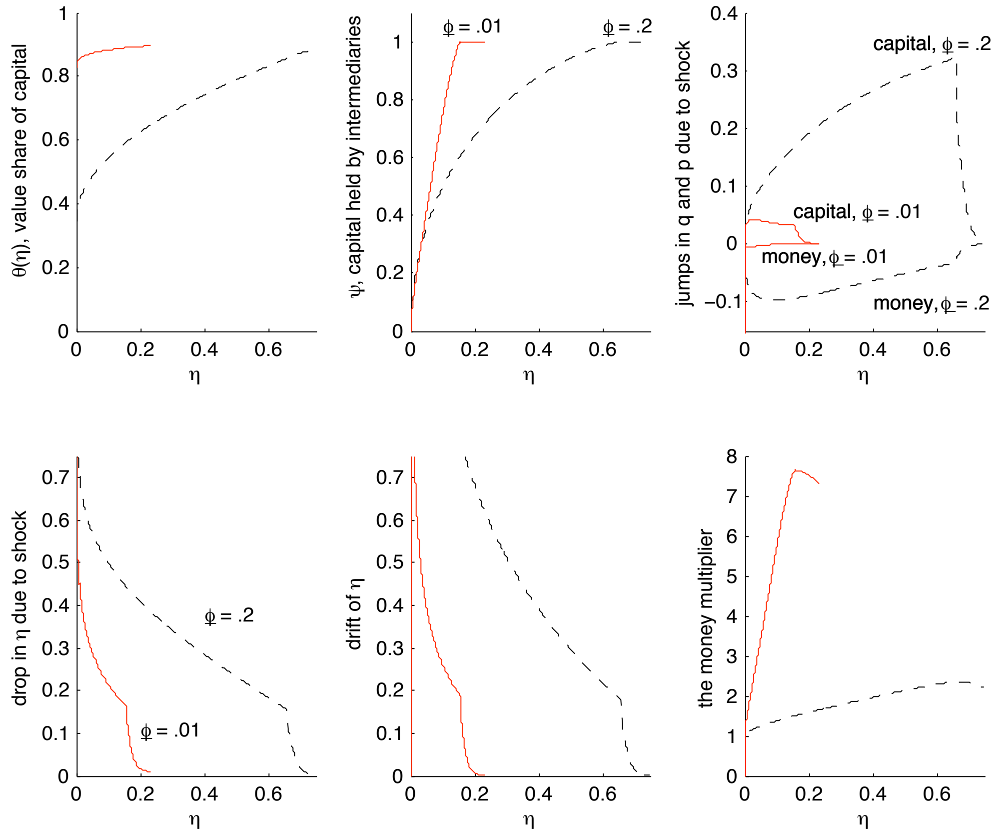

Figure 4: Higher market liquidity ($\underline{\phi} = 0.01$, solid) and lower liquidity ($\underline{\phi} = 0.2$, dashed).

lowering the probability $\underline{\phi}$ reduce the friction savers face. Hence, as the second-best (savers') valuation of capital is higher and closer to first best. It can be seen from the top right panel that both money and capital react to shocks less when physical capital can be more effectively by savers directly. This leads to a higher price of capital and a lower price of money at the steady state, and higher equilibrium leverage. As the intermediary sector is less afraid of liquidity problems, the money multiplier is higher.

This example highlights that, even when exogenous risk is low, endogenous risk on the asset side and deflationary spiral on the liability side of intermediary balance sheets can create huge inefficiencies.

Fiscal Backing of Money. As the tax rate τ goes to 0, the value of money in the frictionless economy benchmark goes to 0, while its value in the "money regime" without intermediaries stays positive. As a result, lower values of τ lead to higher amplification on the

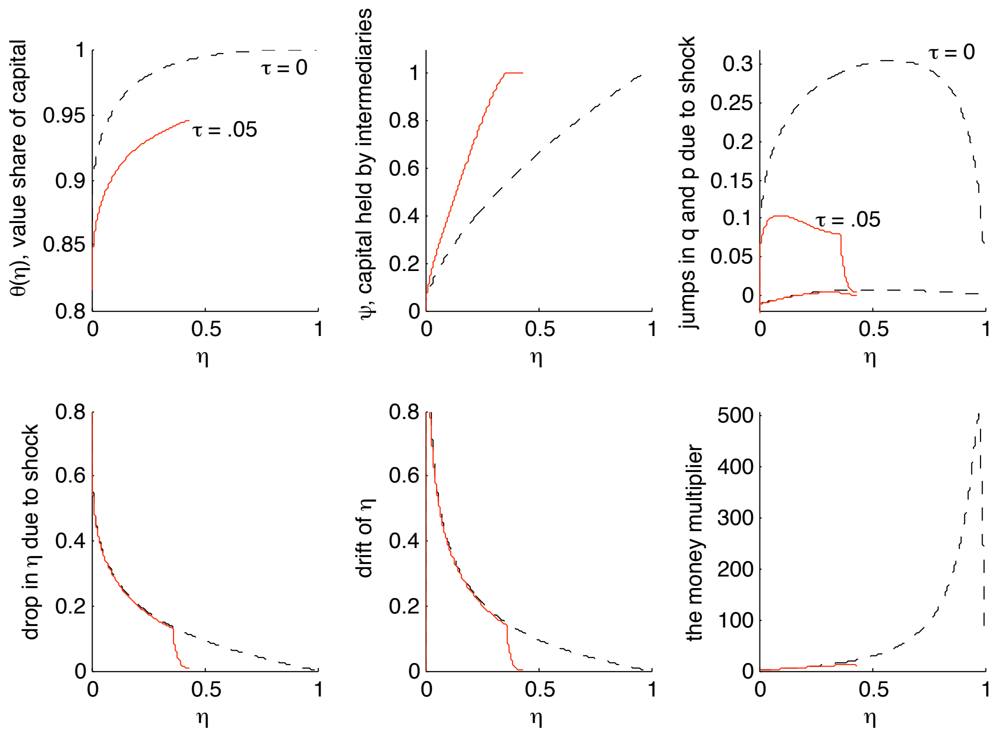

Figure 5: Equilibrium $\tau = 5\%$ (solid) and 0% (dashed).

liabilities side of the intermediaries' balance sheets, through the Fisher deflationary spiral. When money is backed future tax revenues, the equilibrium is much more stable. Otherwise, money is a pure "flight to safety" asset, which has value only because it appreciates and serves a role of storage in severe downturns.[7]

Figure 5 shows that money reacts to shocks much more in an environment where the fundamental value of money in the frictionless economy is lower. As a result, intermediaries accumulate more wealth at the steady state, and start deleveraging (by selling capital to households) at a higher level of η_t. Moreover, when the fundamental value of money in the frictionless economy is lower, then the money multiplier can be significantly higher. In fact, in the bottom right panel of Figure 5, money multiplier becomes as large as 500 in an economy where money is not backed by tax revenues.

[7]This logic suggests that gold, which does not produce any cash flows, appreciates much more in downturns than government debt, which is backed by future tax revenue. Note also that monetary policy can mitigate, or even reverse the Fisher deflationary spiral, as we discuss in the next sections.

3 Monetary and Macro-prudential Policy

Financial frictions lead to a number of inefficiencies in equilibrium. They limit risk sharing, as intermediaries and households who rent out capital are forced to absorb the fundamental risks of their projects. They lead to resource misallocation, away from natural buyers of capital who have a superior monitoring technology. Also, importantly, prices and deflationary pressures amplify shocks create endogenous risk on both sides of the intermediaries' balance sheets. Endogenous risk calls for higher net worth cushions to be maintained to perform any given level of financing, and limits the level of economic activity. In downturns, endogenous risk redistributes wealth away from productive sectors towards less productive agents.

Policy has the potential to mitigate some of these inefficiencies. It can undo some of the endogenous risk by redistributing wealth towards compromised sectors. It can control the creation of endogenous risk by affecting the path of deleveraging. It can also work to prevent the build-up of systemic risk in booms.

Policies affect the equilibrium in a number of ways, and can have unintended consequences. Interesting questions include: How does a policy affect equilibrium leverage? Does the policy create moral hazard? Does the policy inflated asset prices in booms? What happens to endogenous risk? How does the policy affect the frequency of crises, i.e. episodes characterized by resource misallocation and loss of productivity?

We focus on several monetary policies in this section. These policies can be divided in several categories. Traditional monetary policy sets the short-term interest rate. It affects the yield curve through the expectation of future interest rates, as well as through the expected path of the economy, with the supply and demand of credit. When the zero lower bound for the short-term policy rate becomes a constraint, forward guidance is an additional policy tool that is often employed in practice. The use of this tool depends on central bank's credibility, as it ties the central bank's hands in the future and leaves it less room for discretion. In this paper we assume that the central bank can perfectly commit to contingent future monetary policy and hence the interest rate policy incorporates some state-contingent forward guidance.

Several non-conventional policies have also been employed. The central bank can directly purchases assets to support prices or affect the shape of the yield curve. The central bank can lend to financial institutions, and choose acceptable collateral as well as margin requirements and interest rates. Some of these programs work by transferring tail risk to the central bank, as it suffers losses (and consequently redistributes them to other agents) in the event that the value of collateral becomes insufficient and the counterparty defaults. Other policies include direct equity infusions into troubled institutions. At some point, the distinction between monetary, macro-prudential and fiscal policies becomes fuzzy.

The classic "helicopter drop of money" has in reality a strong fiscal component as money is typically paid out via a tax rebate. Importantly, the helicopter drop also leads to redistributional effects. As the money supply expands, the nominal liability of financial intermediaries and hence the household's nominal savings are diluted. The redistributional effects are even stronger if the additional money supply is not equally distributed among the population but targeted to specific impaired (sub)sectors in the economy.

Instead of analyzing fiscal policy, we focus in this paper on conventional and non-conventional monetary policy. For example, a change in the short-term policy interest rate redistributes wealth through the prices of nominal long-term assets. The redistributive effects of monetary policy depend on who holds these assets.[8] In turn, asset allocation depends on the anticipation of future policy, as well as the demand for insurance. Specifically, we introduce a perpetual long-term bond, and allow the monetary authority to both set the interest rate on short-term money, and affect the composition of outstanding government liabilities (money and long-term bonds) through open-market operations.

3.1 Extended Model with Long-term Bond

Money and Long-Term Bonds. We extend our baseline model in two ways: we allow money to pay the floating rate interest and we introduce perpetual bonds, which pay interest at a fixed rate in money. Monetary policy sets interest $i_t \geq 0$ on money and controls the value $b_t K_t$ of all perpetual bonds outstanding. These policies are independent of fiscal policy: the fiscal authority taxes output at rate τ as before, and uses it to redeem money and long term bonds. The monetary authority takes deposits of outside money, and finances interest payments as well as open market operations by printing money.

We now denote by $p_t K_t$ the supply of all outstanding nominal assets: outside money and perpetual bonds. Also, let B_t be the endogenous equilibrium price of a single perpetual bond, which follows

$$\frac{dB_t}{dt} = \mu_t^B \, B_t \tag{3.1}$$

[8]Brunnermeier and Sannikov (2012) discuss the redistributional effects in a setting in which several sectors' balance sheets can be impaired. Forward guidance not to increase the policy interest rate in the near future has different implications than a further interest rate cut, since the former narrows the term spread while the latter widens it.

in the absence of shocks, and jumps to $\breve{B}_t$ when a shock arrives. Note that i_t and b_t are policy instruments, while B_t is the endogenous equilibrium process.

Returns. Capital earns return of the same form as before: it follows (2.6) in the absence of shocks, and jumps in value by $(1-\phi)\breve{q}_t/q_t$ for intermediaries and $\breve{q}_t/q_t$ or 0 for households in the event of a shock.

The real returns on money r_t^M and bonds r_t^B can be derived as follows. First, someone who borrows a dollar of money to invest in bonds earns

$$r_t^B - r_t^M = \frac{1}{B_t} + \mu_t^B - i_t, \tag{3.2}$$

since he gets the current yield and appreciation of the bond (relative to money) and has to pay nominal interest i_t to borrow. Second, the world portfolio of money and bonds earns

$$\left(1 - \frac{b_t}{p_t}\right) r_t^M + \frac{b_t}{p_t} r_t^B = \frac{\tau a}{p_t} + \mu_t^p + \Phi(\iota_t) - \delta. \tag{3.3}$$

From (3.2) and (3.3), the returns on money and bonds in the absence of shocks are given by

$$r_t^M = \frac{\tau(a-\iota_t)}{p_t} + \mu_t^p + \Phi(\iota_t) - \delta - \frac{b_t}{p_t}\left(\frac{1}{B_t} + \mu_t^B - r_t\right) \quad \text{and} \tag{3.4}$$

$$r_t^B = \frac{\tau(a-\iota_t)}{p_t} + \mu_t^p + \Phi(\iota_t) - \delta + \left(1 - \frac{b_t}{p_t}\right)\left(\frac{1}{B_t} + \mu_t^B - r_t\right). \tag{3.5}$$

A shock causes the value of the bond relative to money to change by $\breve{B}_t/B_t$, and the value of the world portfolio of money and bonds, by $\breve{p}_t/p_t$. Therefore, in the event of a shock, money and bonds change in value by factors[9]

$$\breve{M}_t \equiv \frac{\breve{p}_t}{p_t} \frac{1}{1 - \frac{b_t}{p_t} + \frac{b_t}{p_t}\frac{\breve{B}_t}{B_t}} \qquad \text{and} \qquad \breve{M}_t \frac{\breve{B}_t}{B_t}, \tag{3.6}$$

respectively.

[9]Note that the change in value of long-term bonds in the event of a shock is typically not equal to $\breve{b}_t/b_t$, as this fraction takes into account the change in value of *outstanding* bonds due to open-market operations immediately following the shock.

Equilibrium Conditions. Intermediaries solve

$$\max_{x,y}\; x\, r_t^K + (1-x)\, r_t^M + y(r_t^B - r_t^M) + \lambda \log\left(x(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x)\, \breve{M}_t + y\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t \right).$$

The first-order conditions with respect to portfolio weights x_t and y_t are

$$r_t^K - r_t^M + \lambda \frac{(1-\phi)\frac{\breve{q}_t}{q_t} - \breve{M}_t}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t} = 0, \tag{3.7}$$

$$\frac{1}{B_t} + \mu_t^B - r_t + \lambda \frac{\breve{M}_t\left(\frac{\breve{B}_t}{B_t} - 1\right)}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t} = 0. \tag{3.8}$$

Likewise, the households' first-order condition with respect to $\underline{x}$ is

$$r_t^K - r_t^M + \lambda(1-\underline{\phi}) \frac{\frac{\breve{q}_t}{q_t} - \breve{M}_t}{\underline{x}_t\,\frac{\breve{q}_t}{q_t} + (1-\underline{x}_t)\,\breve{M}_t} - \lambda\underline{\phi}\,\frac{1}{1-\underline{x}_t} \geq 0 \tag{3.9}$$

with equality if $\underline{x}_t \geq 0$, where we implicitly assumed that households choose to hold no long-term bonds, i.e. $\underline{y}_t = 0$. That is, only intermediaries hold long-term bonds to insure themselves against shocks.[10] The market-clearing conditions for output and capital are the same as before,

$$\rho\eta_t + r(1-\eta_t) = \frac{a-\iota_t}{p_t+q_t} \quad \text{and} \quad x_t\eta_t + \underline{x}_t(1-\eta_t) = \theta_t.$$

The market-clearing condition for bonds is

$$y_t\eta_t = \frac{b_t}{p_t+q_t}. \tag{3.10}$$

The Law of Motion of η_t. From the laws of motion of N_t and $(q_t+p_t)K_t$, as well as the equilibrium conditions, we get the following lemma. Its proof is in Appendix B.

[10] Households refrain from holding long-term bonds if

$$\frac{1}{B_t} + \mu_t^B - r_t + \lambda(1-\underline{\phi}) \frac{\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}}{\underline{x}_t\,\frac{\breve{q}_t}{q_t} + (1-\underline{x}_t)\,\breve{M}} + \lambda\underline{\phi}\,\frac{\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}}{(1-\underline{x}_t)\,\breve{M}} \leq 0.$$

Lemma 1. *The law of motion of η_t in the absence of shocks is*

$$\frac{d\eta_t/dt}{\eta_t} = (r-\rho)(1-\eta_t) + \lambda\left(\frac{\eta_t}{\breve{\eta}_t}(1-\phi\breve{\theta}_t) - 1\right) \quad \text{where} \tag{3.11}$$

$$\frac{\breve{\eta}_t}{\eta_t} = x_t\,(1-\phi)\frac{\breve{\theta}_t}{\theta_t} + (1-x_t)\,\tilde{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\tilde{M}_t, \quad \text{and} \quad \tilde{M}_t = \frac{1-\breve{\theta}_t}{1-\theta_t}\frac{1}{1-\frac{b_t}{p_t}+\frac{b_t}{p_t}\frac{\breve{B}_t}{B_t}}. \tag{3.12}$$

Equation (3.11) is instructive. It suggests that the impact of the policy on equilibrium dynamics works primarily by moderating the impact of shocks on the intermediary sector, i.e. lowering $\eta_t/\breve{\eta}_t$. While the economy becomes more resilient to shocks, the speed of the recovery $d\eta_t/dt$ gets lower as risk premia decrease, particularly if the policy stimulates the price of capital relative to money θ_t.

3.1.1 Cash Flows to Bonds and Open Market Operations

We already derived all equations that we need to characterize equilibrium with bonds. In this subsection we focus on interpretations. Specifically, we look at the open-market operations implied by the policy, to help interpret long-term bonds as an asset that ensures the financial system in downturns.

In the absence of shocks, while the return on bonds is given by r_t^B, the value of outstanding bonds in the market evolves according to

$$\frac{d(b_tK_t)/dt}{b_tK_t} = \mu_t^b + \Phi(\iota_t) - \delta.$$

Therefore,

$$r_t^B(b_tK_t) - d(b_tK_t)/dt = \frac{b_t}{p_t}\tau(a-\iota_t)K_t + (\mu_t^p - \mu_t^b)b_tK_t + \left(1-\frac{b_t}{p_t}\right)\left(\frac{1}{B_t} + \mu_t^B - r_t\right)b_tK_t$$

is the continuous cash flow that bondholders receive at time t. Of this cash flow, b_tK_t/B_t arrives through interest payments, and the rest, through open-market operations.

Money holders receive the rest of tax revenues,

$$\left(1-\frac{b_t}{p_t}\right)\tau(a-\iota_t)K_t - (\mu_t^p - \mu_t^b)b_tK_t - \left(1-\frac{b_t}{p_t}\right)\left(\frac{1}{B_t} + \mu_t^B - r_t\right)b_tK_t.$$

In the event of a shock, a position in bonds jumps by $\breve{M}_t \breve{B}_t / B_t$, while the value of all bonds outstanding changes by $\breve{b}_t / b_t$. Therefore, the cash flow to bonds in the event of a jump is given by

$$\left(\frac{\breve{b}_t}{b_t} - \breve{M}_t \frac{\breve{B}_t}{B_t} \right) b_t K_t.$$

The cash flow to money holders is minus that.

3.2 Monetary Policy: An Example

This section provides an example of how monetary authority can affect asset prices, in particular prices of long-term bonds, through monetary policy. Without long-term bonds, interest rate policy alone would not have any real effect on equilibrium in our model: it would affect only the nominal return on money and the rate that intermediaries pay to depositors. Likewise, with constant interest rate, bonds and money are perfect substitutes as the price of bonds in terms of money is fixed (and so open market operations alone do not have any effect on the economy).

In the following example, the central bank sets the short-term interest rate to $i_t = 0.25\% + \eta_t * 10\%$, and targets the ratio of the value of outstanding bonds to the total value of nominal assets of $b_t/p_t = 0.5$. Note that the short-term rate is lowered when η_t drops, causing long-term bonds to appreciate. This recapitalizes intermediaries, which hold long-term bonds, and mitigates the impact of shocks on the economy. Figure 6 applies this policy to our baseline example with parameter $\underline{\phi}$ raised to 0.2, to make the inefficiencies more pronounced without the policy, and compares equilibria with and without the policy.

The policy works through multiple channels. First, it mitigates the deflationary spiral: for any level of η_t, the value of money is higher with the policy. As bonds appreciate when η_t goes down, they fill in some of the demand for money, and as a result outside money appreciates less. Intermediaries can also create more inside money in downturns by borrowing against bonds.[11] Second, bonds provide intermediaries with a partial hedge against shocks. Because bonds, which intermediaries hold on the asset side of balance sheets, appreciate after shocks, intermediary net worth η_t drops by less with the policy than without.

The location of the steady state η^* is endogenous, and it drops significantly with the

[11] Note that the graph for p on the top left panel of Figure 6 may be deceptive: the rise in p_t in downturns reflects for the most part the appreciation of bonds rather than deflation. Note that bonds appreciate by a factor of about 4 in downturns, while p_t appreciates by a factor of only 3 (and bonds comprise one half of the value of all nominal assets).

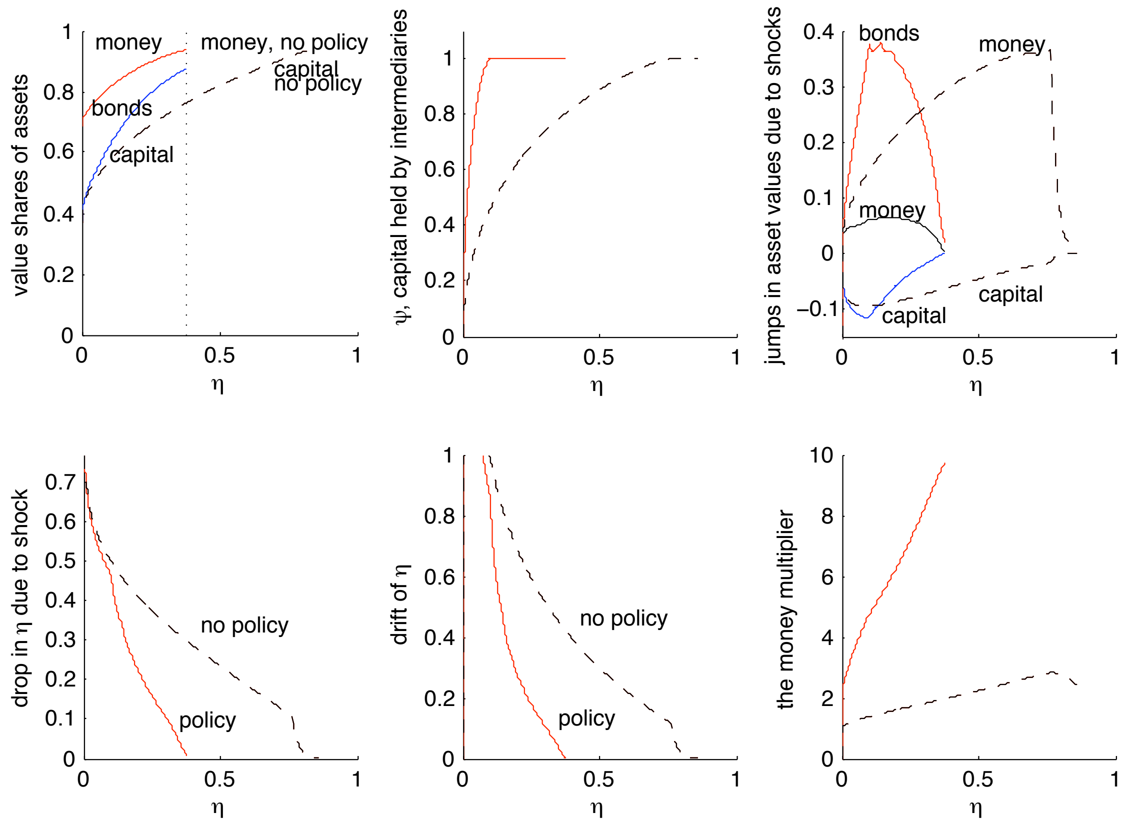

Figure 6: Equilibrium in the baseline model without policy (black) and with (red).

introduction of the policy. As risk premia driven by endogenous risk become lower, intermediaries' earnings fall and their wealth at the steady state becomes lower. The bottom middle panel shows the drift of η_t with and without policy. Despite lower net worth, intermediaries are able to finance all capital near the steady state, as the risk on the liabilities side of their balance sheets is diminished.

Even though the policy leads to higher equilibrium leverage, it does not necessarily create moral hazard. The reason is that the "stealth recapitalization" through bond prices does not reward the weak institutions that took risk and suffered losses, but the cautious institutions that bought bonds to insure themselves against downturns. Indeed, endogenous risk of capital is comparable to that without policy, while the endogenous risk of money is significantly lower.

The money multiplier is much greater with the policy, as intermediaries can create money by borrowing against bonds, which also serve as a natural hedge.

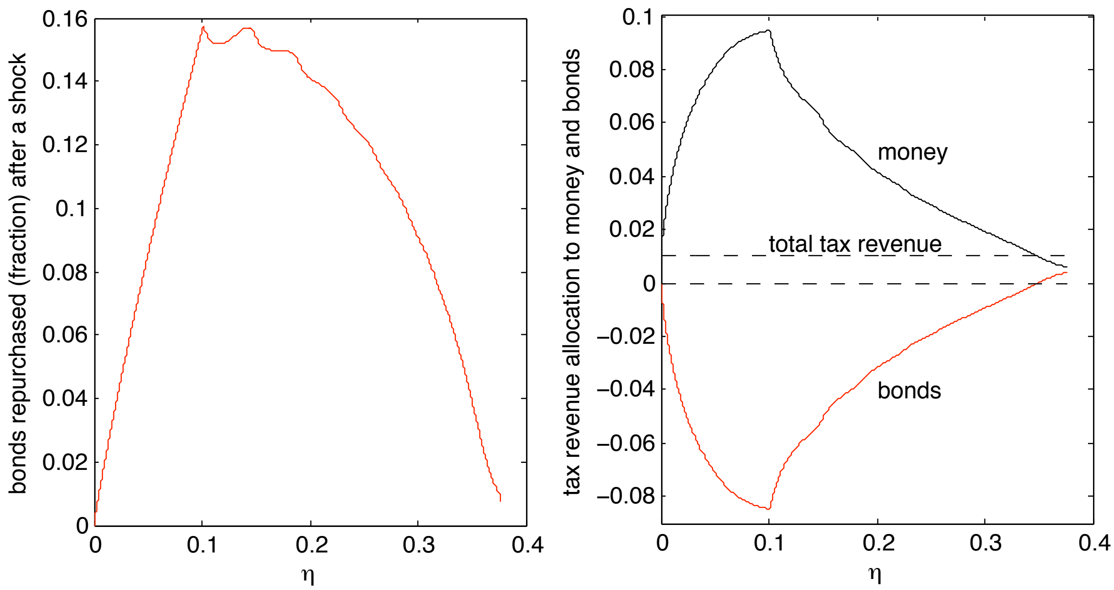

Figure 7: Bond repurchases after shocks. The distribution of tax revenues to money and bonds.

Long-Term Bonds as Insurance. Effectively, under this policy long-term bonds recapitalize the financial system. When a shock occurs, bonds appreciate relative to money but the ratio of outstanding bonds to money in the market stays constant at $b_t/p_t = 0.5$. To maintain the ratio, the central bank has to perform open market operations, i.e. purchase bonds and sell money. The left panel in Figure 7 illustrates the percentage of bonds repurchased following a shock. The world supply of bonds gets a positive cash flow.

As the economy recovers, the opposite happens: the cash flow to the world supply of bonds tends to be negative, as illustrated in the bottom right panel of Figure 7. As bonds depreciate, the central bank sells back some of the bonds it purchased, to maintain a constant ratio of outstanding bonds to money.

4 Conclusion

We consider an economy in which some agents are savers who look for ways to store wealth, while others are entrepreneurs who look to borrow against profitable projects. There is a role for financial intermediation and a role for money. Savers have the option to hold *outside* money or make deposits with intermediaries, i.e. hold *inside* money. Thus, intermediation and money are complementary. A shock to intermediaries causes them to shrink balance sheets and create less inside money; such a shock leads to a rising demand for outside money, i.e. deflation. This deflationary spiral amplifies shocks, as it hurts borrowers who owe nominal debt. It works on the liabilities side of the intermediary balance sheets, while the liquidity spiral that hurts the price of capital works on the asset side. Importantly, in this economy the money multiplier is endogenous: it depends on the health of the intermediary sector.

Endogenous risk, manifested in the deflationary and liquidity spirals, exacerbates the effect of shocks and leads to inefficiencies. Endogenous risk depresses the price of productive assets particularly when they are illiquid, i.e. their value differs significantly in first and second-best use. Endogenous risk in money is particularly large when the money multiplier in boom is large, i.e. the intermediary sector provides significant amounts of credit.

Monetary policy can work against the deflationary spiral. For example, interest rate cuts in downturns can lead to appreciation of long-term nominal assets, recapitalizing institutions that hold these assets and increasing the supply of the safe asset. Such a policy also, indirectly, can reduce endogenous risk due to the liquidity spiral, as it makes the system more stable.

Of course, any policy that provides insurance against downturns creates some moral hazard. However, these drawbacks are mitigated by the crudeness of the policy: it does not redistribute towards the weakest failing institutions, but rather to stronger and more cautions institutions that took care to hedge the downturn risk.

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A Numerical Procedure to find Equilibrium.

Computation to solve the differential equations and characterize equilibrium poses several challenges. First, the location of the steady state η^* is unknown. Second, the equations have singularities at $\eta = 0$ and η^*. Third, the derivative $\theta'(\eta)$ depends on the value of the function $\theta(\eta)$ not only at point η, but at other points of the domain (in particular, $\breve{\eta} \in (0, \eta)$, to which η_t transitions in the event of a macro shock).

To resolve these challenges, we start by defining

$$\theta(\eta) = \underline{\theta} + \epsilon_1$$

on an interval $[0, \epsilon_2]$. Since ϵ_1 and ϵ_2 are small constants, we match the boundary condition $\theta(0) = \underline{\theta}$ approximately. We fix ϵ_2, but search for ϵ_1 to match an appropriate boundary condition at the steady state η^*. Since we always know the value of the function θ on the entire interval $[0, \eta]$, we are always able to find $\breve{\eta}$ to compute $\theta'(\eta)$.

Formally, the procedure we employ is as follows. We use the Euler method. This method can be refined to a higher-order method, such as Runge-Kutta, if necessary. We set $\epsilon_L = 0$, $\epsilon_R = 0.4$, define $\epsilon_1 = (\epsilon_L + \epsilon_R)/2$ and set

$$\theta(\eta) = \theta(0) + \epsilon_1, \quad \text{for } \eta = \{\eta(1) = 0,\ \eta(2),\ \ldots\ \eta(m) = \epsilon_2\}.$$

Then to solve the differential equation for $\eta(n)$, $n = m, m+1, \ldots$, we perform the following sequence of steps. We set $\breve{n} = 1$ for $n = m$.

(i) For each $n \geq m$, we want to identify the value of $\breve{\eta}$ to which η jumps from $\eta(n)$ in the event of a shock. To identify $\breve{\eta}$, we try $\breve{\eta} = \eta(k)$ for $k = \breve{n},\ \breve{n}+1, \ldots$ and in each case perform the following sequence of steps.

We calculate intermediary leverage x required to sink η to $\breve{\eta}$ using formula (2.19), which implies

$$x = \frac{\frac{1-\breve{\theta}_t}{1-\theta_t} - \frac{\breve{\eta}_t}{\eta_t}}{\frac{1-\breve{\theta}_t}{1-\theta_t} - \frac{\breve{\theta}_t}{\theta_t}(1-\phi)}, \tag{A.1}$$

where $\theta_t = \theta(n)$ and $\breve{\theta}_t = \theta(k)$. Note that a higher level of $\breve{\eta}_t$ corresponds to lower leverage, i.e. lower x.

Then compute

$$\psi(k) = \frac{x\eta_t}{\theta_t} \quad \underline{x} = \frac{1-\psi(k)}{1-\eta_t}\theta_t, \quad q_t = \theta_t \frac{a - \iota(q_t)}{r}. \tag{A.2}$$

To check if intermediary leverage x is appropriate, note that (2.9) and (2.11) (together with (2.19)) imply that

$$A(k) = \left(\frac{\breve{\theta}_t}{\theta_t}(1-\phi) - \frac{1-\breve{\theta}_t}{1-\theta_t}\right)\frac{\eta_t}{\breve{\eta}_t} + \underline{\phi}\frac{1}{1-\underline{x}} - (1-\underline{\phi})\frac{\frac{\breve{\theta}_t}{\theta_t} - \frac{1-\breve{\theta}_t}{1-\theta_t}}{\underline{x}\frac{\breve{\theta}_t}{\theta_t} + (1-\underline{x})\frac{1-\breve{\theta}_t}{1-\theta_t}} \geq 0,$$

with strict inequality only if $\psi = 1$. Therefore, if $A(k) < 0$ (or $A(k) > 0$ but $\psi(k) > 1$) then intermediary leverage is still too large and $\breve{\eta}$ too low. Therefore, we keep raising k until $A(k) > 0$ and $\psi(k) < 1$, and then we stop. We know at this point that $\breve{\eta}$ has to be between $\eta(k)$ and $\eta(k-1)$ and proceed to step (ii).

(ii) If $\psi(k-1) > 1$, then we identify $\breve{\eta} \in [\eta(k-1), \eta(k)]$ that leads to $\psi = 1$ according to a linear interpolation formula.[12] Let

$$w(n) = \frac{\psi(k-1)-1}{\psi(k-1)-\psi(k)},$$

and

$$\breve{\eta} = w(n)\eta(k) + (1-w(n))\,\eta(k-1), \quad \breve{\theta} = w(n)\,\theta(k) + (1-w(n))\,\theta(k-1). \tag{A.3}$$

If $\psi(k-1) < 1$, then we want to identify $\breve{\eta} \in [\eta(k-1), \eta(k)]$ that leads to $A = 1$. Let

$$w(n) = \frac{A(k-1)}{A(k-1)-A(k)},$$

and then set $\breve{\eta}_t$ and $\breve{\theta}_t$ again according to (A.3).

Thereafter, compute x, ψ_t, $\underline{x}$ and q_t using (A.1) and (A.2), and let $p_t = (1-\theta_t)/\theta_t q_t$. To find $d\theta/d\eta$, we proceed as follows.

Use (2.9) to calculate

$$\mu_t^q - \mu_t^p = \frac{\tau(a-\iota)}{p_t} - \frac{(1-\tau)(a-\iota)}{q_t} - \lambda\left(\frac{\breve{\theta}_t}{\theta_t}(1-\phi) - \frac{1-\breve{\theta}_t}{1-\theta_t}\right)\frac{\eta_t}{\breve{\eta}_t}.$$

Use (2.17) to calculate

$$\mu_t^\eta = x\frac{(1-\tau)(a-\iota)}{q_t} + (1-x)\frac{\tau(a-\iota)}{p_t} + (x-\theta_t)(\mu_t^q - \mu_t^p) - r.$$

Then

$$\frac{d\theta_t}{d\eta_t} = \theta_t(1-\theta_t)\frac{\mu_t^q - \mu_t^p}{\mu_t^\eta \eta_t}.$$

Let[13]

$$\theta(n+1) = \theta(n) + \frac{d\theta_t}{d\eta_t}(\eta(n+1)-\eta(n)).$$

(iii) Perform several checks. First, it could be that $\theta(n+1) > 1$. If so, then our guess of ϵ_1 was too high - we go back to the beginning to revise ϵ_1 down by setting $\epsilon_R = \epsilon_1$. Second,

[12]Technically, we need $A > 0$ at $\breve{\eta}$, but the error from ignoring this issue is small since $A(k) > 1$.

[13]This is the Euler method for solving the ODE. It can be replaced by a more precise Runge-Kutta method, if necessary.

it could be that $d\theta_t/d\eta_t < 0$ at some point ϵ_t. If so, we revise ϵ_1 up by setting $\epsilon_L = \epsilon_1$. We also interrupt integration and revise ϵ_1 down if $\mu_t^\eta(n+1) > \mu_t^\eta(n)$, a condition that has been identified experimentally.

We then go back to step (i), and iterate until convergence.

B Proofs.

Proof of Lemma 1. In the event of a shock, N_t and $(q_t + p_t)K_t$ change by the factors of

$$x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t \quad \text{and} \quad \theta_t\frac{\breve{q}_t}{q_t} + (1-\theta_t)\frac{\breve{p}_t}{p_t}$$

respectively. Therefore, η_t drops to

$$\breve{\eta}_t = \eta_t \frac{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t}{\theta_t\frac{\breve{q}_t}{q_t} + (1-\theta_t)\frac{\breve{p}_t}{p_t}} \tag{B.1}$$

Multiplying the numerator and denominator of (B.1) by $\frac{q_t+p_t}{\breve{q}_t+\breve{p}_t}$, we get (3.12).

In the absence of shocks,

$$\frac{dN_t/dt}{N_t} = x_t\,r_t^K + (1-x_t)\,r_t^M + y_t\,(r_t^B - r_t^M) - \rho$$

and

$$\frac{d((p_t+q_t)K_t)/dt}{(p_t+q_t)K_t} = \Phi(\iota) - \delta + (1-\theta_t)\mu_t^p + \theta_t\mu_t^q.$$

Therefore, $(d\eta_t/dt)/\eta_t$ equals

$$x_t\,r_t^K + (1-x_t)\,r_t^M + y_t\,\left(\frac{1}{B_t} + \mu_t^B - r_t\right) - \rho - (\Phi(\iota) - \delta + (1-\theta_t)\mu_t^p + \theta_t\mu_t^q). \tag{B.2}$$

To get from this expression to (3.11), we have to perform some algebra.

Taking $\theta_t - x_t$ times (3.7) and subtracting $y_t - (1-\theta_t)\frac{b_t}{p_t}$ times (3.8), we get

$$(\theta_t - x_t)(r_t^K - r_t^M) - \left(y_t - (1-\theta_t)\frac{b_t}{p_t}\right)\left(\frac{1}{B_t} + \mu_t^B - r_t\right) + \tag{B.3}$$

$$\lambda \underbrace{\frac{(\theta_t - x_t)(1-\phi)\frac{\breve{q}_t}{q_t} - (\theta_t - x_t)\breve{M}_t - \left(y_t - (1-\theta_t)\frac{b_t}{p_t}\right)\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t}}_{A} = 0,$$

Furthermore, since $\breve{M}_t\left(1 + \frac{b_t}{p_t}\left(\frac{\breve{B}_t}{B_t} - 1\right)\right) = \frac{\breve{p}_t}{p_t}$, we have

$$A = \frac{\theta_t(1-\phi)\frac{\breve{q}_t}{q_t} + (1-\theta_t)\breve{M}_t + (1-\theta_t)\frac{b_t}{p_t}\breve{M}_t\left(\frac{\breve{B}_t}{B_t} - 1\right)}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t} - 1 =$$

$$\frac{\theta_t(1-\phi)\frac{\breve{q}_t}{q_t} + (1-\theta_t)\frac{\breve{p}_t}{p_t}}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t} - 1 = \underbrace{\frac{\theta_t(1-\phi)\frac{\breve{q}_t}{q_t} + (1-\theta_t)\frac{\breve{p}_t}{p_t}}{\theta_t\frac{\breve{q}_t}{q_t} + (1-\theta_t)\frac{\breve{p}_t}{p_t}}}_{1-\phi\breve{\theta}_t}\frac{\eta_t}{\breve{\eta}_t} - 1.$$

Adding (B.3) to (B.2), we get

$$\frac{d\eta_t/dt}{\eta_t} = \theta_t\frac{(1-\tau)(a-\iota_t)}{q_t} + (1-\theta_t)\frac{\tau(a-\iota_t)}{p_t} + \lambda\left((1-\phi\breve{\theta}_t)\frac{\eta_t}{\breve{\eta}_t} - 1\right) - \rho$$

Furthermore, since $\theta_t/q_t = (1-\theta_t)/p_t = 1/(p_t+q_t)$ and $(a-\iota_t)/(p_t+q_t) = \rho\eta_t + r(1-\eta_t)$, we have

$$\frac{d\eta_t/dt}{\eta_t} = (r-\rho)(1-\eta_t) + \lambda\left((1-\phi\breve{\theta}_t)\frac{\eta_t}{\breve{\eta}_t} - 1\right).$$

$\square$