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Transaction Costs, the Value of Convenience, and the Cross-Section of Safe Asset Returns*

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Abstract

We study the cross-section of equilibrium returns on safe assets using a tractable asset pricing model with a micro-founded demand for liquidity, and multiple safe assets with heterogeneous transaction costs. A key feature of our model is the "value of convenience" which is an equilibrium object that measures the level of liquidity risk-sharing in the economy. Changes in asset supply or the transaction cost of a single safe asset affect aggregate liquidity and the returns of all assets. The model features a pecuniary externality, which investors fail to internalize when forming their portfolios, and which impacts equilibrium welfare. Therefore, policies that increase the payoff on the most liquid asset or liquid asset supply management improve welfare in the competitive equilibrium. We test the main predictions of our theory using a novel measure of relative (in)convenience yields in the US Treasury market.

Keywords: pecuniary externality, convenience yield, Treasury-OIS spread, quantitative easing

JEL Codes: G12, E44

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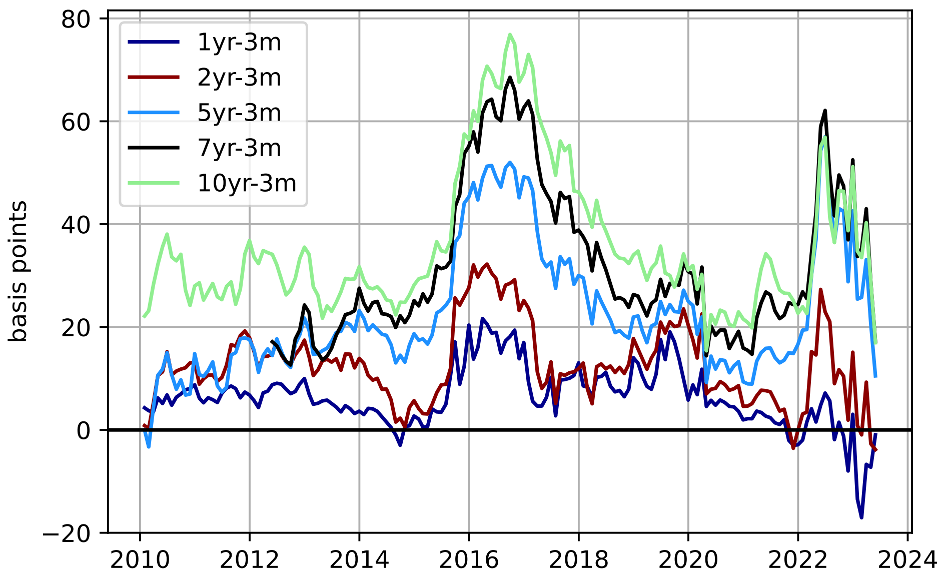
1 Introduction

Safe assets, such as government bonds are not just risk-free assets. Many investors hold these assets for the liquidity services they provide, since they either have a short enough maturity to match investors' liquidity needs or can be readily converted for cash in liquid secondary markets with little price impact. Therefore, safe assets tend to command an additional premium, often referred to as a liquidity premium or convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012). However, there is substantial variation in safe assets' convenience yields. To illustrate this, Figure 1 plots the spreads between Treasuries and (maturity-matched) overnight indexed swaps (OIS) – a common proxy of Treasury convenience – *relative* to the 3-months Treasury-OIS spread.[1] This difference in spreads, or relative spread, is a proxy for the relative (in)convenience yields of U.S. Treasuries with different maturities after neutralizing the duration risk.

Figure 1: Treasury-OIS relative spreads



Notes: This figure shows the US Treasury-overnight indexed swaps (OIS) spread across different maturities relative to the 3-months Treasury-OIS spread. Source: Bloomberg and authors calculations.

Motivated by this striking cross-sectional pattern, in this paper we develop a highly tractable theory of convenience yields with multiple assets with heterogeneous transaction costs, agent heterogeneity and aggregate risk. We use the model to illustrate a pecuniary externality that

[1] The OIS is a derivative, which is available with different maturities. Two parties swap a fixed rate for a floating overnight rate. Combining a Treasury with a maturity-matched OIS contract effectively equalizes the duration across tenors and ensures that we compare safe assets with similar interest rate sensitivity. Since OIS is a derivative that requires no principal investment while Treasuries require upfront capital, the Treasury-OIS spread measures the convenience yield, i.e., the return investors willingly forgo to hold actual Treasury securities.

arises in the presence of heterogeneous transaction costs and analyze different welfare-improving policies.

In our model, agents with recursive (Epstein and Zin, 1989) preferences face uninsurable idiosyncratic liquidity risk modeled via interim impatience shocks. This idiosyncratic risk creates a demand for liquidity. Specifically, agents self-insure against the risk of needing to consume early by holding and trading assets. Assets differ both in their exposure to an aggregate payoff risk factor but also in terms of their transaction costs when traded. Transaction costs are assumed to be proportional to the value of assets sold. We refer to the transaction cost on an asset as that asset's illiquidity. A larger transaction cost incurred during sale implies greater illiquidity. We assume there are multiple risk-free assets that differ in terms of their transaction costs and relative supply, as well as a single illiquid risky asset (a market portfolio).

The intensity of idiosyncratic shocks that agents are exposed to, combined with *the whole* asset and transaction cost structure, determine the equilibrium liquidity scarcity in this economy, which is fully summarized by a single equilibrium object. We refer to this important equilibrium object as *the value of convenience*. The value of convenience is defined as the ratio of equilibrium marginal utilities of consumption of an impatient versus a patient investor and as such reflects the degree of liquidity risk-sharing that is attained in equilibrium. Intuitively, a higher value of convenience implies a lower level of liquidity risk-sharing between investors. Transaction costs play a key role in determining the value of convenience, since the total transaction cost expenditure incurred by asset sellers creates a wedge between the marginal utilities of patient and impatient investors. With proportional transaction costs, such as bid-ask spreads and price impact costs, asset prices also impact this wedge, giving rise to a pecuniary externality that investors fail to internalize when making their portfolio decisions. To examine the cross-section of safe asset returns, we characterize *relative* inconvenience yields, defined as a return on an asset relative to the most liquid (zero transaction cost) safe asset in the economy. This relative inconvenience yield depends not only on an asset's illiquidity but also on the value of convenience. A higher value of convenience implies higher relative inconvenience yields and a wider cross-section of safe asset returns. Therefore, the value of convenience acts as a pricing factor for assets in the economy.

We use our framework to analyze how a change in asset supply or in the illiquidity of *a single* asset affects the returns on *all* assets via general equilibrium effects operating through the value of convenience. For example, substituting less liquid with more liquid safe assets one-for-one purifies the pool of safe assets, decreases the value of convenience, and compresses relative inconvenience yields. We also show that when safe assets are on average more liquid than the risky market portfolio – a natural assumption – an increase in aggregate risk or risk aversion *decreases* the value of convenience, narrowing the relative inconvenience yields across safe assets. The reason for this surprising outcome is a revaluation effect, due to a "flight to safety" adjustment in agents' portfolios. Intuitively, *ceteris paribus*, higher equilibrium valuations of the relatively more liquid risk-free safe assets (and, respectively, lower valuations of the less liquid risky assets) after a "flight to safety" episode shrink the transaction cost expenditure of impatient investors and lower the value of convenience.

In addition to serving as a pricing factor, the value of convenience fully summarizes equilibrium welfare. The higher the value of convenience, the lower is equilibrium welfare. In our framework there is a pecuniary externality that matters for aggregate welfare. Specifically, price-taking investors do not internalize how their asset demand affects asset prices and ultimately the equilibrium wedge between patient and impatient investors' marginal utilities. We show in a simple two-asset version of our model that distorting the asset demand of investors towards the more liquid asset and away from the less liquid asset improves social welfare. Intuitively, ex post patient investors, who are not affected by the transaction costs treat all assets as perfect substitutes. However, from a social optimum perspective, the more liquid asset is more valuable as it carries a lower transaction cost. Therefore, the distortion in asset demand brings closer together the private and social value of the two assets and increases aggregate welfare.

Consequently, there are welfare-improving policy interventions. For example, building directly on the two-asset intuition above, a revenue-neutral set of taxes and subsidies that distort safe asset payoffs by increasing the relative payoff on the most liquid asset increases aggregate welfare. An alternative welfare-improving policy involves substituting illiquid for liquid assets. Through the lens of our simple theory, such policies broadly resemble central bank balance sheet policies as well as government debt management policies.

Finally, we show a new set of stylized facts, which are consistent with our theoretical predictions. We compute relative spreads as the difference between a Treasury-OIS spread of a given maturity (1 year to 10 years) and the 3-months Treasury-OIS spread, as shown in Figure 1 above. We argue that these relative spreads measure relative inconvenience yields on Treasuries. We use plausibly exogenous debt ceiling episodes to show that a decrease in the growth of total outstanding government debt decreases relative spreads by around 1.5 basis points – a fact that our model can help rationalize. Finally, we show that relative inconvenience yields on Treasuries co-move positively with the MOVE Index, which measures yield volatility of US Treasuries, and is thus closely correlated with illiquidity and transaction costs in the Treasury market (Duffie et al., 2023). However, they co-move *negatively* with the VIX Index, consistent with revaluation effects of safe assets during "flight to safety" episodes. This finding resonates with the hedging perspective of safe assets offered by Acharya and Laarits (2025), who show that Treasuries have higher convenience yields in periods when they provide a good hedge against equity risk.

Related literature Our paper contributes to the large and growing literature on safe assets and convenience yields, spurred by the seminal contribution of Krishnamurthy and Vissing-Jorgensen (2012) (see Gorton et al. (2012), Caballero and Farhi (2013), Krishnamurthy and Vissing-Jorgensen (2015), Sunderam (2015), Caballero et al. (2016), Nagel (2016), Azzimonti and Yared (2019), He et al. (2019), Gorton (2020), Ahnert and Macchiavelli (2021), Christensen et al. (2021), Diamond and Van Tassel (2021), Jiang et al. (2021), Kacperczyk et al. (2021), Acharya and Dogra (2022), Barro et al. (2022), Brunnermeier et al. (2024), Gorton and Ordonez (2022), Van Binsbergen et al. (2022), Engel and Wu (2023), Krishnamurthy and Li (2023), Lenel (2023), Herrenbrueck and Wang (2025), Mota (2023), among others).[2] Following Krishnamurthy and Vissing-Jorgensen (2012), much of this literature models convenience demand via a reduced-form money-in-the-utility function. Our contribution to this literature is to introduce a micro-founded general equilibrium model which endogenizes convenience yields and liquidity scarcity as an equilibrium outcome of liquidity risk-sharing in the presence of transaction costs. Moreover, our model, due to its microfoundations of liquidity demand, allows for analyzing welfare and the welfare effects of policy interventions.

Our emphasis on transaction costs brings our paper close to models of equilibrium asset prices with transaction costs and trading frictions (Amihud and Mendelson (1986), Aiyagari and Gertler (1991), Heaton and Lucas (1996), Vayanos (1998), Vayanos and Vila (1999), Holmström and Tirole (2001), Huang (2003), Lo et al. (2004), Vayanos (2004), Acharya and Pedersen (2005), Duffie et al. (2005)).[3] As in our framework, in these models, transaction costs imply a liquidity premium for assets with different transaction costs or trading frictions.[4] Closest to our framework, Vayanos and Vila (1999) analyze a model with two risk-free assets, one without a transaction cost and one with a positive transaction cost. They consider an OLG framework with 3-period-lived agents that trade the assets for life-cycle reasons, with agents at different stages of their life-cycles demanding different types of assets. Relative to their model we propose

[2]See Brunnermeier and Haddad (2014), Caballero et al. (2017), Golec and Perotti (2017), and Gorton (2017) for a review of the literature.

[3]See Amihud et al. (2006) for a review of the literature. See also Constantinides (1986), Duffie and Sun (1990), Davis and Norman (1990), Grossman and Laroque (1990), Dumas and Luciano (1991) for partial equilibrium portfolio allocation models in the presence of transaction costs.

[4]In addition to differences in transaction costs or liquidity across assets, the liquidity premium may arise due to liquidity risk. We also provide a microfoundation for the transaction costs arising due to a specific form of liquidity risk – a correlation between the aggregate liquidity conditions in the economy and an asset's returns, similar to Holmström and Tirole (2001) and Acharya and Pedersen (2005).

a framework where liquidity demand and pricing arises from the combination of uninsured idiosyncratic liquidity shocks and transaction costs. Our framework can accommodate multiple risk-free assets in addition to a risky asset (the market portfolio).

More recently, Brunnermeier et al. (2024) emphasize a retrading perspective on safe assets convenience yields – the ability to retrade safe assets at a low transaction cost and at a predictable value whenever needed. They introduce an equilibrium price of service flows, which is conceptually related to the value of convenience in our framework. Relative to their model we provide a framework with a large set of assets, which can be used to understand the effects of changes to the composition of safe assets in an economy. Our emphasis on risk-sharing inefficiencies arising from pecuniary externalities and our optimal policy results also complement the insights of Brunnermeier et al. (2024). The central role of transaction cost expenditures, which are endogenously determined in equilibrium, for aggregate liquidity conditions also brings our theory close to models of endogenous liquidity, such as in Malherbe (2014), Kurlat (2013), and Bigio (2015). Unlike these models, which endogenize per unit transaction costs via asymmetric information frictions, we assume that the per-unit transaction costs are exogenous, and rather focus on the endogenous determination of relative prices for assets with different transaction costs.

Finally, our empirical measurement of convenience yields follows a recent wave of literature applying the OIS rate as a proxy for the risk-free rate and the corresponding Treasury-OIS spread as a proxy for the convenience yield embedded in Treasuries (Filipovic and Trolle (2013), He et al. (2022), Klingler and Sundaresan (2023), Du et al. (2023), Fleckenstein and Longstaff (2024), among others). Du et al. (2023) and Klingler and Sundaresan (2023) relate the increase in the Treasury-OIS spread since the Global Financial Crisis (GFC) to the increase in the supply of Treasuries.[5] Specifically, Du et al. (2023) argue that Treasury supply is a significant driver of the Treasury-OIS spreads and as a result contributed to the regime shift where dealers have positive net positions in Treasuries. In contrast to these articles, our primary empirical interest lies in the relative convenience yield across Treasuries. Our analysis, therefore, complements recent work by Mota (2023) which also focuses on understanding relative convenience yields. However, while she focuses on US corporate bonds and the relative bond-CDS basis, our focus is on government bonds.

[5] In an earlier contribution, Greenwood and Vayanos (2014) also analyze pricing effects of relative supply of Treasuries.

2 Theory

In this section we present our theoretical framework. We start by informally discussing the main features of our model. Agents in our model have standard Epstein-Zin (Epstein and Zin, 1989) preferences and form portfolios over risk-free and risky assets at an initial period.[6] They may then have interim trading needs, which arise due to uninsurable idiosyncratic shocks as in the asset-pricing with uninsured idiosyncratic risk literature (e.g. Bewley (1979), Aiyagari and Gertler (1991), Constantinides and Duffie (1996), Heaton and Lucas (1996), Heaton and Lucas (2000), Constantinides (2002), Di Tella (2020), Brunnermeier et al. (2024)). In our framework, these idiosyncratic shocks will take the form of shocks to marginal utility, making the agent more impatient to consume in the period. Assets in our framework will each have a proportional transaction cost that is borne by the seller of the asset in the interim period. As in Acharya and Pedersen (2005), we interpret this transaction cost much more broadly than simple fees or bid-ask spreads and instead let this cost encompass liquidity considerations related to price impact and trading delays. In addition, in the Appendix we provide a microfoundation where the transaction cost also incorporates liquidity risk – the covariance between the price of an asset and the aggregate liquidity needs in the economy. Finally, we assume that there is always a risk-free asset with zero transaction cost. This is consistent with theories of security design, in which a riskless security has the lowest transaction cost (see Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), Dang et al. (2009)).[7]

2.1 Model set-up

The model has 3 periods: $t = 0, 1, 2$. The state at $t = 2$ is uncertain and described by the realization z of a continuous random variable Z. There is a unit measure of 3-period lived investors. Investors have non-storable endowments Y_0 at $t = 0$ and Y_1 at $t = 1$. A measure λ of investors are subject to a liquidity/impatience shock at the beginning of $t = 1$. They are denoted by type $s = \overline{S}$ and are referred to as impatient. The remaining investors are denoted by $s = \underline{S}$ and are referred to as patient. Investors consume in all three periods, $c_0, c_{1,s}, c_{2,s}$, for $s \in \left\{\underline{S}, \overline{S}\right\}$. The liquidity shock makes investors value consumption at $t = 1$ more and not

[6] Using Epstein-Zin preferences allows for a separation of the elasticity of intertemporal substitution (EIS) from the coefficient of relative risk aversion. In our setting this would allow us to separately consider the effects of changes in payoff risk or risk aversion on asset prices and relative convenience yields, holding the EIS fixed at unity.

[7] Note that our framework only models the demand for convenience via liquidity needs, and we do not explicitly model any additional demand for safety, for example due to limited participation in markets for risky assets (Vissing-Jorgensen, 2002) or because of special properties of using safe and informationally-insensitive assets as collateral (Dang et al., 2010). Our model can be easily modified to accommodate the latter channel as borrowing against collateral for liquidity purposes (i.e. funding liquidity) and liquidating an asset (i.e. market liquidity) are closely related concepts.

value consumption at $t = 2$ anymore.

There are two types of assets. The first type is risk-free assets, and there are N such assets. We denote the set of these assets by $\mathbb{I} = \{1, 2, ..., N\}$. The second type is risky assets, and for simplicity we assume there is only one such asset, which we refer to as the market portfolio and denote it by m.

The risk-free asset i has a payoff of 1 in each state z at $t = 2$. In Section 4 we allow for more general payoffs for the risk-free assets. For the market portfolio, we denote the payoff by $\varphi^m(z)$. We further assume that $\log \varphi^m(z)$ is distributed according to $\log \varphi^m(z) \sim \mathrm{N}(\mu - \sigma^2/2, \sigma^2)$.[8] The total supply of all risk-free assets is denoted by Q^f, while for the market portfolio the total supply is Q^m. The supply of each risk-free asset i is Q^i, and therefore, $Q^f = \sum_{i \in \mathbb{I}} Q^i$. The investors are equally endowed with these assets at the start of $t = 0$.

Since investors are *ex ante* symmetric at $t = 0$, we denote the quantity of risk-free asset $i \in \mathbb{I}$ held by an investor at the end of $t = 0$ by X_1^i, while the quantity of the market portfolio is denoted by X_1^m. The quantity of risk-free asset i held by an investor type $s \in \{\underline{S}, \overline{S}\}$ at the end of $t = 1$ is $X_{2,s}^i$, and analogously, for the market portfolio holdings. The prices of the risk-free assets are denoted by P_t^i, and the price of the risky asset by P_t^m.

Investors who sell an asset at $t = 1$ pay a proportional transaction cost τ^i for risk-free asset i and τ^m for the market portfolio.[9] The assumption that transaction costs (interpreted broadly as any form of liquidation cost) are proportional to the market value of assets is an important feature of our model and subsequent analysis. The assumption is, however, empirically relevant in view of bid-ask spreads and price impact of trading. We assume that for risk-free asset $i = 1$, $\tau^1 = 0$. Therefore, there is one risk-free asset without transaction costs in this economy. Otherwise $\tau^i > 0$ for all $i \neq 1$.

Lastly, investors are assumed to have recursive Epstein-Zin (Epstein and Zin, 1989) preferences given by:

$$U_0 = \log(c_0) + E\left[\chi_s \log(c_{1,s}) + \beta_s \log(U_{2,s})\right],$$

where $U_{2,s} = \left(E\left[c_2^{1-\gamma}\right]\right)^{1/(1-\gamma)}$, and γ measures the degree of risk aversion. To model the $t = 1$ liquidity shock of the investors, we assume that

$$\chi_s = \begin{cases} \underline{\chi}, & s = \underline{S} \\ \overline{\chi}, & s = \overline{S} \end{cases}, \text{ with } 1 = \underline{\chi} < \overline{\chi}, \text{ and } \beta_s = \begin{cases} 1, & s = \underline{S} \\ 0, & s = \overline{S}. \end{cases}$$

In what follows, we focus on the case where $\overline{\chi} = 2$. This is a convenient parametrisation, since

[8] Subtracting $\sigma^2/2$ from the mean ensures that the expected value of $\log \varphi^m$ is μ and does not depend on σ^2.
[9] In the Appendix we also allow for buyer-specific proportional transaction costs or subsidies.

given the number of periods and the assumption of no discounting between periods, it implies that there are no total utility differences between the patient and impatient investors, which gives us tractability at the cost of little loss in generality. We can write the $t = 0$ preferences of an investor of type s recursively as

$$U_0 = \log(c_0) + E\left[U_{1,s}\right], \tag{1}$$

with $U_{1,s} = \chi_s \log(c_{1,s}) + \beta_s \log(U_{2,s})$ and $U_{2,s} = \left(E\left[c_{2,s}^{(1-\gamma)}\right]\right)^{1/(1-\gamma)}$. At $t = 1$ we have

$$U_{1,s} = \begin{cases} 2\log(c_{1,\overline{S}}), & s = \overline{S} \\ \log(c_{1,\underline{S}}) + \log(U_{2,\underline{S}}), & s = \underline{S}. \end{cases} \tag{2}$$

The $t = 0$ budget constraint of an investor is given by

$$c_0 + \sum_{i\in\mathbb{I}} P_0^i X_1^i + P_0^m X_1^m = Y_0 + \sum_{i\in\mathbb{I}} P_0^i Q^i + P_0^m Q^m. \tag{3}$$

We conjecture that all risk-free assets in period 1 have the same price P_1^f and verify this conjecture in the Appendix. Therefore, the $t = 1$ budget constraint for a type-s investor is given by

$$\begin{aligned} c_{1,s} + A_{2,s} = Y_1 + A_1 - \sum_{i\in\mathbb{I}} \tau^i P_1^f \max\left\{0, X_{1,s}^i - X_{2,s}^i\right\} \\ - \tau^m P_1^m \max\left\{0, X_{1,s}^m - X_{2,s}^m\right\}, \end{aligned} \tag{4}$$

where the last two terms on the right-hand side reflect the transaction costs incurred from selling assets, and A_1 denotes the asset holdings the investor has at the beginning of $t = 1$, namely $A_1 \equiv P_1^f X_{1,s}^f + P_1^m X_{1,s}^m$, with $X_{1,s}^f \equiv \sum_{i\in\mathbb{I}} X_{1,s}^i$. $A_{2,s}$ denotes the asset holdings of the investor at the end of $t = 1$, with $A_{2,\underline{S}} \equiv P_1^f X_{2,\underline{S}}^f + P_1^m X_{2,\underline{S}}^m$, where $X_{2,s}^f \equiv \sum_{i\in\mathbb{I}} X_{2,s}^i$. Finally, the period 2 budget constraint of a type-s investor in state z is given by

$$c_{2,s}(z) = X_{2,s}^f + \varphi^m(z) X_{2,s}^m. \tag{5}$$

Next, we define an equilibrium for this economy.

Definition 1 (Equilibrium). *An equilibrium for this economy consists of period 0 asset and consumption choices of the investors,* $\left\{X_1^i\right\}_{i\in\mathbb{I}}$, X_1^m, *and* c_0; *period 1 asset and period 1 and 2 consumption choices for type-s investors,* $\left\{X_{2,s}^i\right\}_{i\in\mathbb{I}}, X_{2,s}^m$, $c_{1,s}$, *and* $c_{2,s}(z)$; *and period 0 and 1 asset prices,* $\left\{P_0^i\right\}_i, P_0^m$, P_1^f, P_1^m, *such that* *a) given prices, the consumption allocations, and asset holdings solve the period 0 and period 1 problems of the investor (i.e. Eq. (1) subject to the period $t = 0$ budget constraint in Eq. (3), as well as Eq. (2) subject to the period $t = 1$ budget constraint in Eq. (4), as well as the period 2 budget constraint in Eq. (5) which must hold for all z);*

b) given the period 0 and period 1 asset holdings of investors, the asset markets clear in both $t = 0$ and $t = 1$, that is

$$X_1^i = Q^i, \ \forall i \in \mathbb{I}$$
$$X_1^m = Q^m$$

and

$$\lambda X_{2,\overline{S}}^i + (1-\lambda) X_{2,\underline{S}}^i = Q^i, \ \forall i \in \mathbb{I}$$
$$\lambda X_{2,\overline{S}}^m + (1-\lambda) X_{2,\underline{S}}^m = Q^m.$$

2.2 Characterisation

We solve the model backwards in time. We first look at the $t = 1$ problems and market clearing conditions given the asset holdings of the investors as of the beginning of $t = 1$ and then move to the $t = 0$ problem and market clearing conditions given some anticipated $t = 1$ prices. Imposing rational expectations and given the $t = 0$ choices of agents this approach characterizes the equilibrium of this economy.

2.2.1 $t = 1$ characterisation

From $t = 1$ onwards, there are two types of investors: patient and impatient. An impatient investor sells all her assets because she derives no utility from consumption in future periods. A patient investor's problem can be split into a consumption-saving problem and a portfolio choice problem.[10] In the consumption-saving problem, she decides how much to consume in the current period and how much to save for the next period. In the portfolio choice problem, she decides how to allocate her savings over assets. Specifically, below we denote by w the portfolio share invested in the market portfolio. Market clearing implies that patient investors hold all assets at the end of $t = 1$. The following Lemma summarises the $t = 1$ characterisation.

Lemma 1 ($t = 1$ **characterisation**). *Let $A_1 = \sum_{i \in \mathbb{I}} P_1^f Q_1^i + P_1^m Q_1^m$ denote the value of an investor's financial asset holdings at $t = 1$. The following characterises the equilibrium prices*

[10] This is implied by the assertion that in equilibrium she only increases her asset holdings at $t = 1$, as we verify in the Appendix.

and allocations at $t = 1$ *and* $t = 2$.

a) *Investors'* $t = 1$ *consumption and saving decisions:*

$$c_{1,\overline{S}} = \frac{2}{1+\lambda} Y_1 - \sum_{i \in \mathbb{I}} \tau^i P_1^f Q^i - \tau^m P_1^m Q^m$$
$$c_{1,\underline{S}} = \frac{1}{1+\lambda} Y_1$$

and

$$A_{2,\overline{S}} = 0$$
$$A_{2,\underline{S}} = \frac{Y_1 + A_1}{2}.$$

b) *Patient investors'* $t = 2$ *portfolio allocations solve* $P_1^f X_{2,\underline{S}}^f = (1 - w) A_{2,\underline{S}}$ *and* $P_1^m X_{2,\underline{S}}^m = w A_{2,\underline{S}}$, *where* w *solves:*

$$V_2(A_{2,\underline{S}}) = \max_w \left(E \left[c_{2,\underline{S}} (z)^{(1-\gamma)} \right] \right)^{1/(1-\gamma)}$$

s.t.

$$c_{2,\underline{S}} (z) = \left[(1 - w) \frac{1}{P_1^f} + w \frac{\varphi^m (z)}{P_1^m} \right] A_{2,\underline{S}}, \ \forall z.$$

c) *Prices are determined by:*

$$P_1^f Q^f = (1 - w) \frac{1 - \lambda}{1 + \lambda} Y_1$$
$$P_1^m Q^m = w \frac{1 - \lambda}{1 + \lambda} Y_1,$$

where w, *with* $0 \leq w \leq 1$, *is the equilibrium portfolio share invested by patient investors in the market portfolio. Moreover,* $P_1^i = P_1^f$, $\forall i \in \mathbb{I}$.

Proof. See Appendix A.2. □

2.2.2 $t = 0$ characterisation

At $t = 0$, investors choose their current consumption and their asset portfolio. The problem is characterised by a standard Euler equation. The marginal utility of today's consumption must equal the expected marginal utility of tomorrow's consumption times the return on each asset earned by holding it from the first to the second period, where the expectation is taken over the idiosyncratic investor-specific state at $t = 1$. All uninvested resources are consumed, and there is asset market clearing, with all assets held by the investors. Lemma 2 summarises the equilibrium equations that characterise the economy at $t = 0$.

Lemma 2 ($t = 0$ **characterisation**). *Given anticipated period $t = 1$ asset prices $P_1^i, \forall i \in \mathbb{I}$ and P_1^m, the following list of equations characterises the economy at $t = 0$.*

a) Prices are determined by the Euler equations with respect to every asset:

$$\frac{\partial U_0}{\partial c_0} = \lambda \frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}} \frac{P_1^j}{P_0^j}(1 - \tau^j) + (1 - \lambda) \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \frac{P_1^j}{P_0^j}, \ \forall j \in \mathbb{I} \cup m.$$

b) The portfolio allocations, or asset holdings, are:

$$X_1^i = Q^i, \ \forall i \in \mathbb{I}$$
$$X_1^m = Q^m.$$

c) Consumption is given by:

$$c_0 = Y_0.$$

Proof. See Appendix A.2. □

2.3 Approximating the period $t = 1$ portfolio choice problem

For some of our theoretical results below, we will simplify the $t = 1$ portfolio choice problem of the patient investor by applying a log-normal approximation for the equilibrium asset returns, as in Iachan et al. (2021), which in turn is based on the log-normal portfolio choice approximation in Campbell and Viceira (2002). Under that approximation, the portfolio choice problem of the patient investor simplifies to a mean-variance portfolio choice problem, given normally distributed log asset returns.

To set-up the simplified problem, we define the log returns $r_2^m(z) \equiv \log R_2^m = \log\left(\frac{\varphi^m(z)}{P_1^m}\right)$ and $r_2^f \equiv \log R_2^f = \log\left(\frac{1}{P_1^f}\right)$. Also, we define the log of the certainty-equivalent return on the portfolio, R_2^{ce}, by $r_2^{ce} \equiv \log(R_2^{ce})$, and the log of the risk premium on the risky asset by

$$\pi \equiv \log E[R_2^m] - \log R^f = \log E[\varphi^m(z)] + \log \frac{P_1^f}{P_1^m}.$$

The approximate solution of the portfolio choice problem is then given by

$$r_2^{ce} - r_2^f = \max_w w\pi - \frac{\gamma}{2} w^2 \sigma^2.$$

The first-order condition gives the Merton share (Merton, 1969):

$$w = \frac{\pi}{\gamma \sigma^2}, \tag{6}$$

that is the share invested in the risky asset is proportional to the Sharpe ratio, π/σ, where the coefficient of proportionality depends inversely on the risk aversion coefficient γ. The certainty equivalent log return of her portfolio is then

$$r_2^{ce} = r_2^f + \frac{1}{2} \frac{\pi^2}{\gamma \sigma^2}. \tag{7}$$

This approximation requires a minor adjustment in our equilibrium definition as well. In particular, the investor is assumed to act according to Eqs. (6) and (7) when forming $t = 1$ portfolios and making her $t = 0$ and $t = 1$ consumption-saving decisions.

Finally, note that

$$\pi \approx \log E[\varphi^m(z)] + \frac{P_1^f - P_1^m}{P_1^m}, \tag{8}$$

whenever $\frac{P_1^f - P_1^m}{P_1^m}$ is close to zero, so that $\log \frac{P_1^f}{P_1^m} \approx \frac{P_1^f - P_1^m}{P_1^m}$. This will be the case whenever the expected payoff of the market portfolio $E[\varphi^m(z)]$ is close to the terminal payoff on the risk-free assets and the risk premium is relatively small. We regard this as empirically plausible, given the observed risk premia on risky assets in the macroeconomy, and will therefore use this approximation for the log risk premium whenever we derive results involving a positive supply of risky assets below.

We point out that from our main theoretical results below only Proposition 6 and the second case in Proposition 5 depend on this log-approximation. The other results are independent of it.

2.4 Asset maturity and transaction costs

While the transaction costs can be interpreted literally as such, in the Appendix, we provide additional microfoundations for these costs. We first show two examples where we adjust our framework to incorporate assets with different maturities, a feature we are particularly interested in, in light of the stylized empirical facts we present in Section 5. In both examples we show that long maturity assets are less convenient to accommodate agents' interim liquidity needs and can, therefore, be associated with carrying higher transaction costs. In the first example, the reason for that is that there is a mismatch between the maturity of the long maturity asset and the horizon of liquidity needs. In the second example, we introduce liquidity risk by introducing additional shocks, such that the $t = 1$ return on the long maturity asset is negatively correlated with the marginal utility of consumption.

3 Theoretical results

3.1 The value of convenience

An important equilibrium object that is central to our theoretical results is *the value of convenience.* To define this object, let $m_{1,s} \equiv \frac{\partial U_{1,s}}{\partial c_{1,s}} / \frac{\partial U_0}{\partial c_0}$ denote the ratio of marginal utilities of consumption for an investor of type $s \in \{\underline{S}, \overline{S}\}$. Define

$$\eta \equiv \frac{m_{1,\overline{S}}}{m_{1,\underline{S}}} = \frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}} / \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}}. \tag{9}$$

Therefore, η captures the degree to which period $t = 1$ marginal utilities of consumption between patient and impatient investors are aligned in equilibrium. Indeed, as we show in Section 4 below, in the first-best full liquidity risk-sharing case $\eta = 1$, and there is full equalization of marginal utilities across idiosyncratic states at $t = 1$. Moreover, the higher is η, the further away this economy is from the full-insurance benchmark. In our decentralised economy, generally $\eta \geq 1$, with the value of η determined in equilibrium. Therefore, η can be thought of as reflecting the equilibrium liquidity scarcity in this economy. For this reason we refer to η as the *value of convenience.*

Proposition 1 (**Value of Convenience**)**.** *The value of convenience η is given by*

$$\eta = \frac{\frac{2}{1+\lambda} Y_1}{\frac{2}{1+\lambda} Y_1 - \Omega} \tag{10}$$

with $\Omega \equiv \widetilde{\tau} P_1^f Q^f + \tau^m P_1^m Q^m$ *and* $\widetilde{\tau} \equiv \sum_i \frac{Q^i}{Q^f} \tau^i$.

Proof. We use the investors' $t = 1$ consumption specified in Lemma 1 and insert it in the derivative of the logarithmic utility function given by $\frac{1}{c_{1,s}}$ for $s = \overline{S}, \underline{S}$. Taking the ratio according to equation (9) yields equation (10). □

The value of convenience is a function of model primitives and asset prices. $\widetilde{\tau}$ is the (weighted) average transaction cost on risk-free assets in this economy. Ω gives the total transaction cost expenditure that an impatient seller incurs in equilibrium. This expenditure creates a wedge in the marginal utilities of patient and impatient investors. The larger is the expenditure, the larger is the wedge and the higher is the value of convenience η. Since the transaction cost expenditure depends on asset prices, changes in these prices would impact the wedge in marginal utilities and the degree of risk-sharing in equilibrium.

We can further simplify η by using the equations from equilibrium prices from Lemma 2 above. Specifically, we have that the equilibrium transaction cost expenditure is $\Omega = [\widetilde{\tau}(1-w) + \tau^m w]\,(1-\lambda)Y_1/(1+\lambda)$. Substituting into Eq. (10) and simplifying, we get that

$$\eta = \frac{1}{1-(1-\lambda)[\widetilde{\tau}(1-w)+\tau^m w]/2}. \tag{11}$$

Therefore, the value of convenience depends on a weighted average of the (relative-supply weighted) transaction costs of the risk-free assets and the transaction cost on the risky asset, where the weight is the equilibrium portfolio share invested in risky assets, w. If $Q^m = 0$, this equation further simplifies to

$$\eta = \frac{1}{1-(1-\lambda)\widetilde{\tau}/2}. \tag{12}$$

In that case η depends only on the average transaction costs $\widetilde{\tau}$ and the share of impatient investors λ. Specifically, η increases in $\widetilde{\tau}$. Intuitively, a higher average transaction cost increases the total transaction cost expenditure and decreases the self-insurance possibilities, leading to a higher value of η. η also decreases with the share of impatient investors, λ. The larger the share of impatient investors, the lower the $t = 1$ price of the safe assets, P_1^f, due to the larger supply pressure and cash-in-the-market pricing from the patient investors. However, a lower $t = 1$ price actually *mitigates* the impact of the transaction costs, as it reduces the total transaction cost expenditure. This improves liquidity insurance and leads to a lower η.

3.2 Relative inconvenience yields

We compare the return of a specific asset *relative* to the return on the *most liquid* risk-free asset in the economy. A higher relative return means that the most liquid asset commands a premium (a higher convenience yield) over that specific asset.[11] Below, for brevity, we will use the term relative *in*convenience yield when referring to the (positive) relative return of a specific asset and the most liquid risk-free asset.

We define the period $t = 1$ gross return on an asset $i \in \mathbb{I} \cup \{m\}$ as $R_1^i \equiv \frac{P_1^i}{P_0^i}$ and show the following result.

[11] Note that we are consistent with standard measures of convenience yields, which are always expressed in terms of *relative* returns or spreads.

Proposition 2 (**Relative inconvenience yields**). *Define* $\psi^j \equiv \frac{R_1^j}{R_1^1}$ *as the return on asset j relative to the return on the most liquid asset in the economy, risk-free asset 1. Then*

$$\psi^i = \frac{1}{1 - \lambda\tau^i - m_{1,\overline{S}}\tau^i R_1^1} = \frac{1}{1 - \tau^i \frac{\lambda}{\lambda + (1-\lambda)/\eta}}, \text{ for all } i \in \mathbb{I} \cup m.$$

Proof. See Appendix A.2. □

Proposition 2 shows that the relative inconvenience yield ψ^i is influenced by three forces. First, an asset's return has to compensate for the transaction cost incurred by selling the asset. Second, an asset's return has to compensate for the transaction cost being incurred *precisely when* the investor is impatient. Third, how much the transaction cost matters for the relative inconvenience yield depends on the value of convenience, η. A lower value of convenience, also lowers the relative inconvenience yields on all assets.

3.3 Effects of transaction costs

We next show how a change in an asset's transaction cost impacts the relative inconvenience yields across the entire cross-section of assets in the economy.

Proposition 3 (**Transaction costs**). *The relative inconvenience yield, of asset i,* ψ^i*, is increasing in that asset's transaction cost,* τ^i*, i.e.* $\frac{\partial \psi^i}{\partial \tau^i} > 0$*,* $\forall i \in \mathbb{I} \cup m$*. It is also increasing in the transaction cost of any other asset,* τ^k*, i.e.* $\frac{\partial \psi^i}{\partial \tau^k} > 0$*, for* $k \neq i$ *and* $i \in \mathbb{I} \cup m$*. The impact is higher, the higher is* τ^i*, i.e.* $\frac{\partial^2 \psi^i}{\partial {\tau^i}^2} > 0$ *and* $\frac{\partial^2 \psi^i}{\partial \tau^k \partial \tau^i} > 0$*,* $\forall i \in \mathbb{I} \cup m$*.*

Proof. See Appendix A.2. □

Therefore, a change in an asset's transaction cost not only impacts its own relative inconvenience yield but also impacts the entire cross-section of asset returns. This spillover effect is driven by a change in the value of convenience η. An increase in the transaction cost of any asset *increases* the value of convenience, since the transaction cost expenditure of sellers increases, decreasing risk-sharing opportunities. A higher value of convenience affects all assets and raises all relative inconvenience yields, making the most liquid asset relatively more valuable. The more illiquid an asset is in terms of its transaction cost, the more its corresponding relative inconvenience yield is affected.

3.4 Effects of changes in asset supplies

We next show how changes to the composition and overall supply of safe assets affect the the cross-section of asset returns. We first consider a compositional change in the supply of the risk-free assets due to an increase in the supply of one asset and an equivalent decrease in the supply of another asset.

Proposition 4 (**Purifying/polluting effects**). *Given a compositional change where* $dQ^l = -dQ^k > 0$ *for two risk-free assets* k *and* l*, the relative inconvenience yield* ψ^i *for any asset* $i \in \mathbb{I} \cup m$ *decreases strictly in response to this change, iff* $\tau^l < \tau^k$*.*

Proof. See Appendix. □

Substituting more liquid for less liquid assets one-for-one can be interpreted as a "purification" of the pool of safe assets by altering its composition. This decreases the value of convenience and compresses relative inconvenience yields. We call this the "purifying/polluting effect" of safe asset supply.

Next we show the asset pricing implications of changes in the overall supply of risk-free assets, keeping the composition of risk-free assets fixed. In the Proposition below we first assume that there is no risky asset in the economy, $Q^m = 0$. In a second step we use the log approximation from Section 2.3 to also discuss the case with $Q^m > 0$. In that case the supply of risk-free assets also impacts the equilibrium risk premium.

Proposition 5 (**Total safe asset supply effect**). *Consider a change in* Q^f*, keeping the composition of risk-free assets fixed.*

- *If* $Q^m = 0$*, then* $\frac{\partial \psi^i}{\partial Q^f} = 0$ *for all* $i \in \mathbb{I} \cup m$*.*
- *If* $Q^m > 0$ *and given the log-approximation of the investor's portfolio problem and the risk premium, then* $\frac{\partial \psi^i}{\partial Q^f} > 0$*, for all* $i \in \mathbb{I} \cup m$*, iff* $(\tau^m - \widetilde{\tau})\left(\mu - \gamma\sigma^2 - 1\right) > 0$*.*

Proof. See Appendix. □

If there are no risky assets, an increase in the aggregate supply of safe assets, holding their composition fixed, does not affect inconvenience yields. This is a direct consequence of having log inter-temporal preferences, and an elasticity of intertemporal substitution (EIS) of unity. In that case, income and substitution effects from intertemporal price changes cancel out and the investor saves a constant share of her wealth. In equilibrium, that leads to an equilibrium value of the risk-free assets that is independent of asset supply. Consequently, the value of convenience remains unchanged and so do relative inconvenience yields.

This conclusion changes once risky assets are present. In that case, there are relative revaluation effects between the risky asset and the risk-free assets. Let $\tau^m > \widetilde{\tau}$, so risk-free assets are on average more liquid than the risky asset, which we view as the empirically-relevant case.

Whenever a higher supply of risk-free assets lowers their value relative to risky assets, so that in equilibrium investors' portfolio weight w would tilt towards risky assets, then the transaction cost expenditure of impatient investors would increase and, by Eq. (11), so would the value of convenience. Consequently, the cross-section of relative inconvenience yields would also widen. This type of revaluation takes place if the expected payoff of risky assets is sufficiently large relative to the payoff variance and investors' risk aversion, as given by the condition $\mu - \gamma\sigma^2 > 1$.

Propositions 4 and 5 can be used to characterize the effect of a change in the supply of *a single* risk-free asset on relative inconvenience yields. That effect is the combination of an increase in the overall supply of risk-free assets and a number of compositional changes, since an increase in the supply of a single risk-free asset increases its relative weight in the pool of risk-free assets and decreases the relative weight of all other assets. For example, suppose that $Q^m = 0$ and there are two risk-free assets. In the simple case without risky assets, only the compositional channel operates: convenience yields rise if the injected asset has higher transaction costs compared to the average (a polluting effect); otherwise, they fall. In the general case with risky assets, both channels are active. Convenience yields increase under the same conditions as in Proposition 5, and, in addition, the injected asset must have transaction costs above the average. Otherwise it is unclear which channel dominates.

3.5 Effects of payoff risk

Our model makes predictions of how changes in aggregate payoff risk during crisis and "flight to safety" episodes impact the cross-section of safe asset returns. The following result characterizes these effects through the lens of our model.

Proposition 6 ("Flight to safety" revaluation effects). *Given the log-approximation of the investor's portfolio problem and the log risk premium, then, for all* $i \in \mathbb{I} \cup m$, $\frac{\partial \psi^i}{\partial \sigma^2} < 0$, *iff* $\tau^m > \widetilde{\tau}$, *and* $\frac{\partial \psi^i}{\partial \gamma} < 0$, *iff* $\tau^m > \widetilde{\tau}$.

Proof. See Appendix. □

An increase in the payoff variance σ^2 or the risk aversion parameter γ increases the price of the risk-free assets and decreases the price of the risky asset. This tilts the equilibrium portfolio weight away from the risky asset and toward the risk-free assets. If safe assets are on average more liquid than the risky asset – which we view as the empirically-relevant case – then an increase in aggregate risk or risk aversion implies that this revaluation effect decreases the total transaction cost expenditure. As a consequence, by Eq. (11), the value of convenience decreases, and so do the relative inconvenience yields for the whole cross-section of safe assets.

4 Welfare and optimal policy

In this section we show that the value of convenience is a measure of welfare in this economy, since it summarizes the degree of liquidity risk-sharing among agents that is achieved in equilibrium. If the value of convenience is greater than one, the decentralized equilibrium does not reach the first-best level of liquidity risk-sharing. The reason for this sub-optimality is a pecuniary externality, since asset prices enter the transaction cost expenditure of asset sellers, which agents fail to internalize when forming their portfolios. In light of this result, we discuss welfare-improving policies.

4.1 The value of convenience as a welfare measure

As already alluded to in Section 3.1, the value of convenience is a welfare measure for this economy, with higher values of η implying lower aggregate welfare. To show this formally, we characterize the welfare maximization problem of a utilitarian social planner that maximizes the *ex ante* ($t = 0$) expected utility of an investor.[12] Specifically, the planner maximizes aggregate welfare, $W = E\left[U_0\right]$, by solving

$$\max_{c_{1,\overline{S}},c_{1,\underline{S}}} \log(Y_0) + \lambda 2\log(c_{1,\overline{S}}) + (1-\lambda)\left[\log(c_{1,\underline{S}}) + \log\left(\left(E\left[\left(Q^f + \varphi_2^m(z)Q^m\right)^{1-\gamma}\right]\right)^{\frac{1}{(1-\gamma)}}\right)\right]$$

$$\text{s.t. } \lambda c_{1,\overline{S}} + (1-\lambda)c_{1,\underline{S}} = Y_1,$$

where we have substituted for the $t = 0$ and $t = 2$ endowments. The optimal period $t = 1$ consumption allocation equalizes the marginal utilities of the patient and impatient investors, and is thus given by $c_{1,\overline{S}} = 2\frac{Y_1}{(1+\lambda)}$, and $c_{1,\underline{S}} = \frac{Y_1}{(1+\lambda)}$. Therefore, from Eq. (9), we have that $1 = \frac{1}{c_{1,\underline{S}}}/\frac{2}{c_{1,\overline{S}}} = \eta$. Moreover, we can express welfare W as a function of η, which is uniquely maximized at $\eta = 1$ (full liquidity risk-sharing), with W monotonically decreasing in η, for $\eta \geq 1$.

Proposition 7 (Welfare). *For $\eta \geq 1$ welfare W is monotone decreasing in the value of convenience, η, $\frac{\partial W}{\partial \eta} < 0$, and W is maximized for $\eta = 1$ (full liquidity risk sharing).*

Proof. See Appendix. □

Given the decentralized equilibrium value of η in this economy (Cf. Eq (10)), it follows directly that with positive transaction costs, $\eta > 1$ and the competitive equilibrium does not achieve the first-best level of risk-sharing and welfare. The reason for this inefficiency is a pecuniary externality, which we discuss next.

[12] Since the planner cannot shift resources across periods it must be that $c_0 = Y_0$.

4.2 Pecuniary externality

Consider a simplified version of our model economy with no payoff risk (no risky asset) and only two risk-free assets. The first asset has zero transaction costs, while the second asset has a transaction cost given by τ. Moreover, suppose that both assets have an equal supply of $Q^f/2$.

Consider the period $t = 1$ portfolio choice of a patient investor. The asset holdings, $X^i_{2,\underline{S}}$, for any of the safe assets satisfy the Euler equations[13]

$$\frac{1}{c_{2,\underline{S}}} = \frac{P^i_1}{c_{1,\underline{S}}}, \quad i \in \{1, 2\}, \tag{13}$$

together with the period $t = 1$ and $t = 2$ budget constraints

$$c_{1,\underline{S}} + \sum_{i\in\{1,2\}} P^i_1 X^i_{2,\underline{S}} = Y_1 + \sum_{i\in\{1,2\}} P^i_1 X^i_1,$$
$$c_{2,\underline{S}} = \sum_{i\in\{1,2\}} X^i_{2,\underline{S}}.$$

The Euler equations and market clearing imply that, in equilibrium, period $t = 1$ asset holdings $X^i_{2,\underline{S}}$ and asset prices P^i_1 are such that $P^1_1 = P^2_1 = P^f_1$. Market clearing, in addition, implies that $X^i_{2,\underline{S}} = Q^f/2\,(1 - \lambda)$, which from the period budget constraints pins down period 1 and 2 equilibrium consumption and the common price $P^f_1 = c_{1,\underline{S}}/c_{2,\underline{S}} = (1 - \lambda)\,Y_1/\left[(1 + \lambda)\,Q^f\right]$. Finally, from Proposition 1, the value of convenience η is $\eta = \frac{2}{1+\lambda}Y_1/\left(\frac{2}{1+\lambda}Y_1 - \Omega\right)$, where the transaction cost expenditure is $\Omega = \tau P^f_1 Q^f/2$.

Suppose that we distort the asset demand schedules of the patient investors away from the privately optimal levels given by the above Euler equations and the period budget constraints (possibly also distorting period 1 consumption). Specifically, suppose that we distort $X^1_{2,\underline{S}}$ up and $X^2_{2,\underline{S}}$ down, as if asset 1 has a period 2 payoff of $1 + \rho > 1$ and asset 2 has a period 2 payoff of $1 - \rho < 1$, for some $\rho > 0$. Put differently, the distorted asset demand schedules (and possibly distorted period 1 consumption) are such that they are at an interior value if the following distorted Euler equations hold

$$\begin{aligned} \frac{1+\rho}{c_{2,\underline{S}}} &= \frac{P^1_1}{c_{1,\underline{S}}}, \\ \frac{1-\rho}{c_{2,\underline{S}}} &= \frac{P^2_1}{c_{1,\underline{S}}}. \end{aligned} \tag{14}$$

Essentially, for small values of ρ, such that the response of period 1 consumption is second order, what this demand distortion does is to shift out the demand curve for asset 1 and shift

[13] The derivation of this equation follows the same steps as the more general case discussed in the next section.

in the demand curve for asset 2.

How do these distorted demand schedules impact equilibrium asset prices? It will no longer be the case that $P_1^1 = P_1^2$. Since market clearing implies that equilibrium asset prices must be such that the investors hold both assets in equilibrium, it follows that P_1^1 and P_1^2 must satisfy the modified Euler equations

$$\frac{1}{c_{2,\underline{S}}} = \frac{\overline{P}_1^i}{c_{1,\underline{S}}}, \quad i \in \{1,2\}, \tag{15}$$

where $\overline{P}_1^1 = P_1^1/(1+\rho)$ and $\overline{P}_1^2 = P_1^2/(1-\rho)$. Therefore, $\overline{P}_1^1 = \overline{P}_1^2 = \overline{P}_1^f$, where from market clearing and the modified Euler equation, $\overline{P}_1^f = (1-\lambda)Y_1/\left[(1+\lambda)Q^f\right]$. Note that $\overline{P}_1^f$ equals P_1^f in the undistorted economy. Intuitively, the symmetric distortion and equal asset supply imply that only the relative prices of the two assets change, while the aggregate value of the assets stays the same.

These equilibrium asset prices give a value of convenience of $\tilde{\eta} = \frac{2}{1+\lambda}Y_1/\left(\frac{2}{1+\lambda}Y_1 - \tilde{\Omega}\right)$, where the transaction cost expenditure is $\tilde{\Omega} = P_1^2 \tau Q^2 = (1-\rho)P_1^f \tau Q^f/2 < \Omega$. Therefore, $\tilde{\eta} < \eta$, and so, welfare in the economy with distorted asset demand is higher than welfare in the undistorted economy.

The reason for why the distortion in investors' asset demand away from their privately optimal demand improves welfare is that the competitive equilibrium level of liquidity risk-sharing is suboptimal. There is a *pecuniary externality* due to asset prices entering the transaction cost bill. Patient agents, who purchase the assets in period $t = 1$ and act as price takers, fail to internalize the impact of their portfolio choices on the transaction costs the selling impatient agents pay. Since patient agents are not directly affected by the transaction costs, they treat all assets as equally valuable. But higher transaction cost expenditures due to higher asset values of the more illiquid assets increase the value of convenience η and decrease welfare. From a social optimum perspective, the more liquid asset is more valuable as it carries a lower transaction cost. Therefore, the distortion in asset demand brings closer together the private and social value of the two assets. As a result, the transaction cost bill decreases and the aggregate welfare in the economy increases. In light of this pecuniary externality, we next discuss possible welfare-improving policies.

4.3 Welfare-improving policies

Again, we assume there is no risky asset but extend our model, such that a risk-free asset i pays a final payoff of $\delta^i > 0$ instead of 1.[14]

[14] In addition, in an extension of our model in the Appendix, we allow for proportional buyer transaction costs or subsidies, τ_i^b (possibly zero), incurred by buyers at $t = 1$. There we show that a suitable set of buyer subsidies

4.3.1 Asset payoff management

Building on the two-asset example above, we show that by increasing the payoff on the most liquid risk-free asset, δ^1, the policy maker can decrease the equilibrium value of convenience, η, and increase welfare. We assume that the higher payoff is financed with a proportional tax on the period $t = 2$ payoffs of all other safe assets. Therefore, all other safe assets have a payoff of $\delta < 1$, which satisfies revenue neutrality $\sum_i \delta^i Q^i = \delta^1 Q^1 + \delta \sum_{j \neq 1} Q^j = Q^f$.

Proposition 8 (**Payoff management**). *Consider the revenue-neutral set of payoff taxes and subsidy described above. Then* $\frac{\partial \eta}{\partial \delta^1} < 0$.

Proof. See Appendix. □

To provide additional intuition for this result, note first that in the model, the period $t = 1$ equilibrium price of asset i is given by

$$P_1^i = \delta^i \frac{1 - \lambda}{1 + \lambda} \frac{Y_1}{Q^f}. \tag{16}$$

This equation arises from the fact that in equilibrium all safe assets have an equal payoff-adjusted $t = 1$ price. Consumption smoothing by the marginal buyer, combined with market clearing pins down that payoff-adjusted common price. A higher period $t = 2$ payoff, δ^i, increases the relative price of asset i, and hence, the equilibrium value of that asset. This valuation effect lowers the transaction cost expenditure of impatient sellers, similar to the compositional changes from Proposition 4 and the valuation effect from Proposition 6, where higher payoff risk decreases the equilibrium value of the relatively illiquid risky asset and increases the equilibrium value of the relatively more liquid risk-free assets. Therefore, an increase in the effective payoffs of more liquid safe assets, for example, via preferential tax treatment of such assets, can improve liquidity risk-sharing and decrease inconvenience yields in the economy.

4.3.2 Asset supply management

An alternative set of policies involves managing the *quantities* and composition of the different risk-free assets. Following Proposition 4, a policy that replaces relatively less liquid with relatively more liquid assets would reduce the equilibrium value of convenience η. Such a policy can be broadly interpreted as conventional central bank liquidity provision or an unconventional balance sheet policy, such as Operation Twist (Greenwood and Vayanos, 2010) or Quantitative Easing (QE) programs (Krishnamurthy and Vissing-Jorgensen, 2011). An alternative interpretation for such a policy is the Treasury managing the maturity structure of government debt by

can restore full efficiency.

issuing relatively more shorter maturity government debt. Indeed, as shown by Bi et al. (Year), Treasury maturity extension shocks that keep the total supply of Treasury debt fixed tend to increase the AAA-Treasury spread, a common measure of convenience yields.

Since both policies involve substituting more liquid assets (i.e. central bank reserves or Treasury bills) for less liquid assets, according to our simple theory, such interventions would decrease relative inconvenience yields and the value of convenience in the economy. The U.S. Treasury Buyback Program provides a recent, concrete example of a government debt policy designed to withdraw illiquid assets from the market. By conducting buybacks, the Treasury allows primary dealers to return less liquid off-the-run Treasuries. This alleviates balance sheet strains and fosters liquidity in the secondary market (Zhou, 2025).

Nevertheless, these asset supply management results only pertain to liquidity effects and may easily be overturned in a richer model. For example, if there are other reasons why investors demand safety in addition to liquidity demand, for example, demand for longer duration assets (e.g. Vayanos and Vila (2021)) or regulatory reasons (Corell et al., 2024). In that case, a central bank that buys longer maturity Treasury notes and bonds and replaces them with very short maturity, highly liquid central bank reserves or a Treasury that issues more Treasury bills and less Treasury notes and bonds may actually end up worsening welfare for some investors in the economy by exacerbating the scarcity of long-duration assets.[15]

5 Empirical evidence

In this section we use data from the US Treasury market to document a set of new stylized facts that are consistent with and which we interpret through the lens of our theoretical model. We use weekly data on Treasury yields and overnight indexed swap (OIS) rates for the period 2010-2023. In different empirical tests we additionally use data on total outstanding US government debt, the MOVE Index of implied bond volatility in the US Treasury market (Mallick et al., 2017), as well as the VIX Index.[16]

We measure the relative convenience yield between two Treasuries as a *relative spread.* First, we compute the spreads or basis of maturity-matched Treasuries and Overnight Indexed Swap (OIS) at different maturities. Second, we take the difference between a specific Treasury-OIS spread and the 3-months Treasury-OIS spread (i.e. a double difference). Formally, for a Treasury-OIS spread of maturity M we denote this relative spread as Relative Spread$_{M-3m}$. As discussed in the related literature, the Treasury-OIS spread is widely used as a proxy for

[15] In addition, the discussion above abstracts completely from the costs associated with the central bank assuming payoff risk or the Treasury getting more exposed to rollover risk.

[16] We list the various data sources in the Appendix, while Table B.2 presents summary statistics.

the convenience yield on Treasuries.[17] From this perspective, the double-differenced relative spread naturally provides a measure of the *relative* convenience yield on Treasuries of different maturities.

The advantage of using the Treasury-OIS spread is that we effectively transform Treasuries of different maturities to instruments with similar duration characteristics – approximately one day. This occurs because the OIS component exchanges fixed rates for floating overnight rates, effectively eliminating the inherent price risk from interest rate changes in longer-dated Treasuries. With this similar duration profile across tenors, our Treasury-OIS relative spread measure compares safe assets with the same interest rate risk exposure.

However, the longer the maturity of the Treasury component, the higher is the probability that the position must be liquidated before it matures with additional costs of reversing the position in the OIS. The liquidation of a Treasury bond combined with an OIS position entails significantly higher transaction costs compared to simply rolling over short-term Treasury securities. Unwinding such structured positions involves various types of expenses: bid-ask spreads on the OIS (that typically widen during market volatility), potential termination fees when closing the swap prior to maturity, counterparty valuation adjustments that may not favor the liquidating party, and execution complexity requiring specialized expertise. Additionally, the mark-to-market calculation for the OIS component introduces pricing opacity that can further disadvantage the position holder. In contrast, rolling short-term Treasury securities benefits from highly liquid markets with narrow bid-ask spreads, transparent pricing mechanisms, minimal execution complexity, and lower operational costs. Despite both approaches achieving similar duration profiles, the transaction cost differential remains substantial.

5.1 Stylized facts

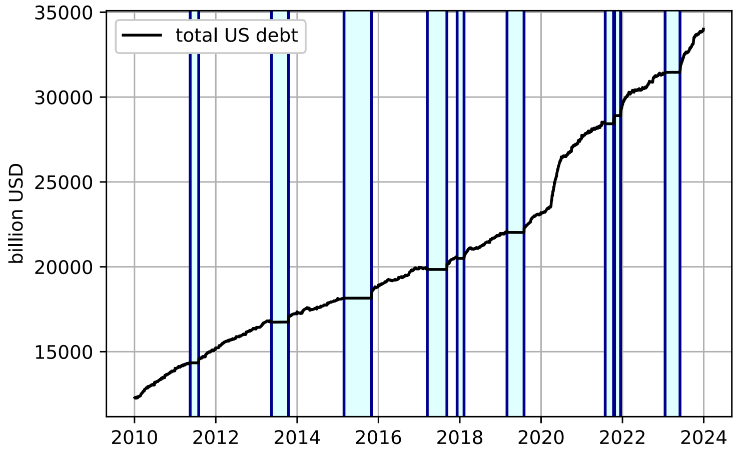
Fact 1: Treasury-OIS relative spread term structure. Figure 1 presents the Treasury-OIS relative spread for several maturities, ranging from 1 to 10 years. There is a striking term structure in the Treasury-OIS relative spread – a monotone effect of Treasury maturity. The effects are also sizable, on the order of around 30 to 40 basis points on average. In addition, whenever the relative spread increases, it increases more for longer maturities. To the extent that longer-maturity safe assets have higher transaction costs (either direct or effective transaction costs – see Section 2.4), this pattern is directly consistent with our theory (Cf. Proposition 2).

Fact 2: Treasury supply effects and the Treasury-OIS relative spread. The second stylized fact is that decreases in the supply of Treasuries tend to comove negatively with the

[17] Figure B.1 in the Appendix plots the Treasury-OIS spreads for different maturities.

relative inconvenience yield. To illustrate this, we use debt ceiling dates to generate variation in the supply of Treasuries.[18] We identify all periods, in which the daily total outstanding government debt did not grow over a period of at least five weeks (i.e., debt ceilings were in place).[19] Figure 2 plots the evolution of total outstanding US government debt, together with the identified "debt ceiling" episodes. In Table 1, we show that Treasury growth is about 0.17 percentage points higher outside of debt ceiling episodes (column 1).

Figure 2: Outstanding government debt and debt ceiling episodes



Notes: This figure shows the daily total outstanding US government debt. All episodes where the debt growth was constant for at least five weeks are marked. Source: Bloomberg and authors calculations.

During debt ceiling episodes, relative spreads decrease by 1.5 basis points on average across maturities (column 2).[20] Through the lens of our model, a decrease in the total supply of safe assets can induce *a decrease* in relative inconvenience yields for all safe assets if risky assets are relatively more illiquid than risk-free assets and their expected payoff is sufficiently high (Cf. Proposition 5).[21]

Fact 3: The MOVE and VIX indices comove with the Treasury-OIS relative spread. We regress the Treasury-OIS relative spread for different maturities on the MOVE and VIX

[18] See Cashin et al. (2017) for a study of how debt ceilings affect Treasury yields.

[19] No growth is defined if the total outstanding debt did not change by more than 10 billion USD on a daily basis.

[20] The coefficient estimates from Table 1 imply that a one percentage point increase in the growth of outstanding government debt, instrumented by the debt ceiling dates, is associated with an increase in relative spreads of 7 basis points on average across maturities.

[21] There are a number of effects in place during debt ceiling periods. For example, another effect is that maturing short term debt might not be replaced. This effect is small in magnitude, however. The overall share of short term debt (measured by the outstanding amount of bills to total outstanding debt) decreases on average by half a percentage point during debt ceiling periods. Figure B.2 in the Appendix shows that total bills outstanding are a small fraction of total debt and grow comparatively slowly.

Table 1: Impact of supply fluctuations (measured by debt ceilings)

	Debt growth	Relative Spread$_{M-3m}$
Debt ceiling dates	-0.1727***	-1.5468***
	(0.012)	(0.405)
N	3485	16708
Adj. R^2	0.071	0.476

Notes: The outcome variable in column (1) is debt growth. In column (2) the outcome variable consists of the relative spread Relative Spread$_{M-3m}$ of maturity $M \in \{1y, 2y, 5y, 7y, 10y\}$. The explanatory variable is in both columns an indicator for any weeks on which daily debt growth was constant for at least five weeks. We include a constant in the first regression and in addition maturity fixed effects in the second. The time horizon is 2010 to mid-May 2023. The frequency is weekly for the first and daily for the second regression. Standard errors are heteroscedasticity robust. *** indicates significance at the 1% level. ** indicates significance at the 5% level. * indicates significance at the 10% level.

indices. To account for effects of the level of interest rates on convenience yields (Nagel (2016), Diamond and Van Tassel (2021)), in some specifications we also control for the level of interest rates by including the effective federal funds rate. We also examine heterogeneous effects of the MOVE and VIX for shorter (1 and 2 years) vs. longer maturities. Table 2 reports the estimated co-movements.

The Treasury-OIS relative spread correlates positively with the MOVE Index and negatively with the VIX Index. A 1 percentage point higher value of the MOVE Index (VIX Index) is associated with an increase (decrease) in the average relative spread of around 10 (8-9) basis points (columns 1 and 2). The effects are, however, much weaker for shorter maturity Treasuries (columns 3-4).

Table 2: Impact of the MOVE Index and the VIX Index

	(1)	(2)	(3)	(4)
ln(MOVE)	8.9292***	8.0620***	14.9371***	14.0441***
	(1.491)	(1.384)	(1.914)	(1.833)
$D_{\{1y\ or\ 2y\}}\times$ln(MOVE)			-14.7981***	-14.6978***
			(1.615)	(1.629)
ln(VIX)	-8.2040***	-7.8005***	-13.0780***	-12.6975***
	(1.255)	(1.282)	(1.708)	(1.734)
$D_{\{1y\ or\ 2y\}}\times$ ln(VIX)			11.9644***	12.0007***
			(1.347)	(1.346)
Fed Funds Rate		0.7743*		0.7603*
		(0.428)		(0.426)
N	3368	3368	3368	3368
Adj. R^2	0.503	0.505	0.520	0.522

Notes: The outcome variable consists of the relative spread Relative Spread$_{M-3m}$ of maturity $M \in \{1y, 2y, 5y, 7y, 10y\}$. The explanatory variables are the MOVE Index and the VIX Index. In Columns (2) and (4), we additionally control for the Effective Federal Funds Rate. $D_{\{1y\ or\ 2y\}}$ is an indicator for the relative spread at 1 or 2 years. In all regressions, we include maturity fixed effects. The frequency is weekly and the time period is 2010 to mid-May 2023. Standard errors are heteroscedasticity and autocorrelation robust (HAC) using 6 lags and without small sample correction. *** indicates significance at the 1% level. ** indicates significance at the 5% level. * indicates significance at the 10% level.

We interpret these comovements as follows. First, the MOVE Index measures the yield volatility of Treasuries, which is closely correlated with illiquidity and transaction costs in the Treasury market (Duffie et al., 2023).[22] Higher values of the MOVE Index can thus be interpreted as higher transaction costs. Therefore, the comovements of the Treasury-OIS relative spread with the MOVE Index are consistent with Proposition 3, namely that an increase in the transaction costs of *any* of the assets increases the relative inconvenience yields for all safe assets. Moreover, the effect is stronger for assets with higher transaction costs, consistent with the estimated interaction effect from Table 2. Regarding the comovements with the VIX Index, since the VIX measures risk sentiment in the economy, the estimated comovements are consistent with our Proposition 6, which shows that an increase in payoff risk or risk aversion

[22]In our model we interpret transaction costs broader than bid-ask spreads. Therefore we consider the MOVE Index as an appropriate proxy measure for these illiquidity costs.

reduces the relative inconvenience yields for all safe assets.

6 Conclusion

The interaction of liquidity self-insurance and transaction cost heterogeneity has important asset pricing and welfare implications. Central to that interaction is the value of convenience, an equilibrium pricing factor, which is also a summary statistic for agents' welfare. By shedding light on this interaction and the inefficiencies that arise in the competitive equilibrium, our theoretical framework puts in focus the importance of policies that promote and facilitate market liquidity and proper market functioning (Duffie, 2023).

Beyond these structural policies, our analysis also points to side-effects of central bank balance sheet policies, via the level of central bank reserves in the system and the asset composition of the central bank's balance sheet. Our theoretical framework suggests that the long-run volume of central bank reserves in the system may affect aggregate welfare via its influence on liquidity conditions in the economy. It also hints at potentially important aggregate liquidity effects of the maturity structure as well as the taxation regime of government debt. Nevertheless, our simple theory includes only one channel through which these policy tools impact aggregate welfare. Understanding the full set of complex trade-offs and the resulting optimal policy mix is an important avenue for future research on this topic.

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A Theory Appendix

A.1 Additional results

A.1.1 Buyer transaction subsidies

A suitable set of buyer subsidies for different assets can restore full efficiency in the competitive equilibrium. We assume that the expenditure is financed with period $t = 1$ lump-sum taxes. Also, to streamline the exposition, we assume that the transaction costs are sufficiently small, so that it is feasible to implement the full set of buyer subsidies that can restore optimality. Specifically, we can show the following result.

Proposition 9 (**Buyer subsidization**). *Suppose that* $\tau_b^i = -\tau^i$, $\forall i$. *Then the economy achieves full liquidity risk-sharing, with* $\eta = 1$ *in equilibrium.*

Proof. See Appendix. □

The intuition for this result is as follows. Given perfectly competitive asset markets at $t = 1$ and perfectly elastic asset demands by buyers, any subsidy that a buyer receives when purchasing a specific asset is fully passed through into the $t = 1$ asset price for that asset. Therefore, the buyer does not benefit from the subsidy in equilibrium, as she only bids up the asset price. This increase in the asset price, however, compensates the seller for incurring the transaction cost when selling that asset. By setting the buyer subsidy appropriately, one can manipulate the equilibrium asset prices in a way that negates the effect of the seller transaction costs, thus achieving the first-best level of liquidity risk-sharing in the competitive equilibrium.

A.2 Omitted proofs

A.2.1 Proof of Lemma 1

Proof. We describe and solve the optimization problems in the second and third period.

$t = 1$ **problem of patient investors** For the problem of the patient investor in period 1, we split the problem into a portfolio choice problem and a consumption-saving problem. Specifically, we conjecture that in equilibrium she only increases her position in each asset. We then split her problem into two: a consumption-saving decision (problem 1) given by

$$V_{1,\underline{S}} = \max_{c_{1,\underline{S}}, A_{2,\underline{S}}} \log(c_{1,\underline{S}}) + \log(V_2(A_{2,\underline{S}}))$$

$$\text{s.t.}$$

$$c_{1,\underline{S}} + A_{2,\underline{S}} = Y_1 + A_1,$$

where $A_{2,\underline{S}}$ denotes savings into $t = 2$, and a portfolio choice problem (problem 2), given by

$$V_2(A_{2,\underline{S}}) = \max_{X^f_{2,\underline{S}}, X^m_{2,\underline{S}}} \left(E\left[c_{2,\underline{S}}(z)^{(1-\gamma)} \right] \right)^{1/(1-\gamma)}$$
$$\text{s.t.}$$
$$P^f_1 X^f_{2,\underline{S}} + P^m_1 X^m_{2,\underline{S}} = A_{2,\underline{S}}$$
$$c_{2,\underline{S}}(z) = X^f_{2,\underline{S}} + \varphi^m(z) X^m_{2,\underline{S}}, \ \forall z.$$

To write down the portfolio choice problem we used $P^f_1 = P^i_1$. We demonstrate in Lemma 3 below.

Note that the solution to this problem can be fully summarized by the portfolio share invested in the risky asset, namely $w \equiv \frac{P^m_1 X^m_{2,\underline{S}}}{A_{2,\underline{S}}}$. In equilibrium, that portfolio share will satisfy $0 \leq w \leq 1$. Therefore, the portfolio problem can equivalently be written as

$$V_2(A_{2,\underline{S}}) = \max_{w} \left(E\left[c_{2,\underline{S}}(z)^{(1-\gamma)} \right] \right)^{1/(1-\gamma)}$$
$$\text{s.t.}$$
$$c_{2,\underline{S}}(z) = \left[(1-w)\frac{1}{P^f_1} + w\frac{\varphi^m(z)}{P^m_1} \right] A_{2,\underline{S}}, \ \forall z.$$

One can then compute the asset allocations as $P^f_1 X^f_{2,\underline{S}} = (1-w) A_{2,\underline{S}}$, and $P^m_1 X^m_{2,\underline{S}} = w A_{2,\underline{S}}$. Given homothetic preferences, Lemma 4 below shows that that the value function $V_2(A_{2,\underline{S}})$ is linear in savings, i.e.

$$V_2(A_{2,\underline{S}}) = R^{CE}_2 A_{2,\underline{S}}.$$

and the optimal value of w satisfies

$$E\left[\left[\frac{1}{P^f_1} + \left(\frac{\varphi^m(z)}{P^m_1} - \frac{1}{P^f_1} \right) w \right]^{-\gamma} \left(\frac{\varphi^m(z)}{P^m_1} - \frac{1}{P^f_1} \right) \right] = 0. \tag{A.1}$$

The period $t = 1$ consumption-saving problem becomes

$$V_{1,\underline{S}} = \max_{c_{1,\underline{S}}, A_{2,\underline{S}}} \log(c_{1,\underline{S}}) + \log(R^{CE}_2 A_{2,\underline{S}})$$
$$\text{s.t.}$$
$$c_{1,\underline{S}} + A_{2,\underline{S}} = Y_1 + A_1.$$

With log utility, we then have $A_{2,\underline{S}} = \frac{Y_1 + A_1}{2}$ and $c_{1,\underline{S}} = \frac{Y_1 + A_1}{2}$. We insert the optimal values into the period $t = 1$ utility function $U_{1,\underline{S}}$ and derive the period $t = 1$ value function, $V_{1,\underline{S}}$, which is given by $V_{1,\underline{S}} = \log R^{CE}_2 \left(\frac{Y_1 + A_1}{2} \right)^2$.

$t = 1$ **problem of impatient investors** The problem of the impatient investor in period 1 is trivial as she sells all her asset holdings and consumes all available resources at $t = 1$. Therefore

$$X^i_{2,\overline{S}} = 0 \; \forall i$$
$$X^m_{2,\overline{S}} = 0$$

and we have $c_{1,\overline{S}} = Y_1 + A_1 - \sum_i \tau^i_1 P^f_1 Q^i - \tau^m P^m_1 Q^m$, where $A_1 \equiv \sum_i P^f_1 Q^i + P^m_1 Q^m$ denotes the investor's financial wealth at $t = 1$.

Market clearing at $t = 1$ At $t = 1$ the patient and impatient investors trade assets with one another. Taking into account the respective mass of each type of investor and their asset holding decisions, market clearing conditions are given by

$$(1-\lambda)X^i_{2,\underline{S}} = Q^i, \; \forall i \in \mathbb{I}$$
$$(1-\lambda)X^m_{2,\underline{S}} = Q^m.$$

Therefore, patient investors hold all the assets at the end of $t = 1$. From these market clearing conditions we can derive the period 1 prices, as well as equilibrium value of period $t = 1$ financial wealth and consumption. Using the market clearing condition $X^m_{2,\underline{S}} = \frac{1}{(1-\lambda)} Q^m$, $A_{2,\underline{S}} = \frac{Y_1 + A_1}{2}$ from the consumption-saving decision and the optimal portfolio weight w on the risky asset, we end up with the following equation for the risky asset

$$\frac{1}{(1-\lambda)} P^m_1 Q^m = w \frac{Y_1 + A_1}{2}. \tag{A.2}$$

and

$$\frac{1}{(1-\lambda)} P^f_1 Q^f = (1-w) \frac{Y_1 + A_1}{2} \tag{A.3}$$

for the risk-free assets. Summing these two equations yields

$$\frac{1}{(1-\lambda)} P^m_1 Q^m + \frac{1}{(1-\lambda)} P^f_1 Q^f = \frac{A_1}{1-\lambda} = \frac{Y_1 + A_1}{2},$$

which further yields the equilibrium value of period $t = 1$ asset holdings of

$$A_1 = \frac{(1-\lambda)}{(1+\lambda)} Y_1. \tag{A.4}$$

Substituting for A_1 into Eqs. (A.5) and (A.6), we arrive at the equilibrium pricing equations

$$P^m_1 Q^m = w \frac{(1-\lambda)}{(1+\lambda)} Y_1, \tag{A.5}$$

and

$$P^f_1 Q^f = (1-w) \frac{(1-\lambda)}{(1+\lambda)} Y_1. \tag{A.6}$$

Lastly, the budget constraints in combination with the optimal asset holdings yield the consumption choices in all periods:

$$c_{1,\overline{S}} = \frac{2}{1+\lambda} Y_1 - \sum_i \tau^i P_1^f Q^i - \tau^m P_1^m Q^m$$
$$c_{1,\underline{S}} = \frac{1}{1+\lambda} Y_1.$$

□

A.2.2 Proof of Lemma 2

Proof. We describe and solve the first period optimization problem.

$t = 0$ problem of investors The period $t = 0$ problem of the representative investor is given, recursively, by

$$V_0 = \max_{c_0, \{X_1^i\}_{i \in \mathbb{I}}, X_1^m} \log(c_0) + \lambda V_{1,\overline{S}} + (1-\lambda) V_{1,\underline{S}}$$
$$\text{s.t.}$$
$$c_0 = Y_0 + \sum_{i \in \mathbb{I}} P_0^i (Q^i - X_1^i) + P_0^m (Q^m - X_1^m),$$

where $V_{1,\overline{S}} = 2 \log(c_{1,\overline{S}})$, with $c_{1,\overline{S}} = Y_1 + A_1 - \sum_{i \in \mathbb{I}} \tau^i P_1^i X_1^i - \tau^m P_1^m X_1^m$ and $V_{1,\underline{S}} = \log R_2^{CE} \left(\frac{Y_1 + A_1}{2}\right)^2$. The Euler equation for holdings of asset $i \in \mathbb{I} \cup \{m\}$ is

$$\frac{\partial U_0}{\partial c_0} = E_s \left[\frac{\partial V_{1,s}}{\partial A_1} \frac{P_1^i}{P_0^i} (1 - \mathcal{I}_{s=\overline{S}} \tau^i) \right],$$

or

$$\frac{\partial U_0}{\partial c_0} = \lambda \frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}} \frac{P_1^i}{P_0^i} (1 - \tau^i) + (1-\lambda) \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \frac{P_1^i}{P_0^i},$$

where we have used the fact that at $t = 1$, by the Envelope theorem for the agent's $t = 1$ problem, $\frac{\partial V_{1,s}}{\partial A_1} = \frac{\partial U_{1,s}}{\partial c_{1,s}}$. Therefore, at the optimum, the marginal utility of consumption today equals the expected marginal utility of consumption tomorrow, where the expectation is taken over the realisation of the idiosyncratic investor's state s and the transaction cost incurred in the impatient state is taken into account.

Market clearing and consumption at $t = 0$ Market clearing at $t = 0$ is given by

$$X_1^i = Q^i, \ \forall i$$
$$X_1^m = Q^m.$$

Substituting into the period budget constraint, we in turn have $c_0 = Y_0$. □

Additionally, we show the following two Lemmas, which constitute parts of the proof of Lemma 1.

Lemma 3 (**Second period prices**). $P_1^i = P_1^f, \forall i \in \mathbb{I}$.

Proof. The Lagrangian of the portfolio choice problem of the patient investor in period 1 is given by

$$\mathcal{L} = \left(\int_z \left[(R_2^f X_{2,\underline{S}}^f + R_2^m(z) X_{2,\underline{S}}^m)^{(1-\gamma)} \right] d(z) \right)^{\frac{1}{(1-\gamma)}} + \mu \left[A_{2,\underline{S}} - \sum_i P_1^i X_{2,\underline{S}}^i - P_1^m X_{2,\underline{S}}^m \right].$$

The first order conditions with respect to the risk-free assets (given an interior solution) are:

$$\frac{1}{(1-\gamma)} \left(\int_z \left[(R_2^f X_{2,\underline{S}}^f + R_2^m(z) X_{2,\underline{S}}^m)^{(1-\gamma)} \right] dF(z) \right)^{\frac{1}{(1-\gamma)}-1} = \frac{\mu P_1^i}{(1-\gamma) R_2^f \left[\int_z (R_2^f X_{2,\underline{S}}^f + R_2^m(z) X_{2,\underline{S}}^m)^{(-\gamma)} dF(z) \right]}$$

for all $i \in \mathbb{I}$. It follows that P_1^i is the same for all $i \in \mathbb{I}$. We denote this price by P_1^f. □

Lemma 4 (Third period utility). $V_2(A_{2,\underline{S}}) = R_2^{CE} A_{2,\underline{S}}$, *and* w *satisfies*

$$E\left[\left[\frac{1}{P_1^f} + \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) w \right]^{-\gamma} \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) \right] = 0. \quad \text{(A.7)}$$

Proof. Using $P_i^1 = P_f^1$, for all i, the portfolio choice problem of the patient investor in period 1 can be rewritten as

$$\max_w \left[\int_z \left[\frac{1}{P_1^f} A_{2,\underline{S}} + \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) w A_{2,\underline{S}} \right]^{(1-\gamma)} dF(z) \right]^{\frac{1}{(1-\gamma)}}.$$

The first order condition for w is given by

$$[V_2(A_{2,\underline{S}})]^{\gamma} \int_z \left[\frac{1}{P_1^f} A_{2,\underline{S}} + \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) w A_{2,\underline{S}} \right]^{-\gamma} \left[\left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) A_{2,\underline{S}} \right] dF(z) = 0.$$

We can re-write as

$$[V_2(A_{2,\underline{S}})]^{\gamma} A_{2,\underline{S}}^{1-\gamma} \int_z \left[\frac{1}{P_1^f} + \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) w \right]^{-\gamma} \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) dF(z) = 0.$$

Note that this condition holds for any value of $A_{2,\underline{S}}$. Therefore, the first-order condition for w simplifies to

$$\int_z \left[\frac{1}{P_1^f} + \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) w \right]^{-\gamma} \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) dF(z) = 0, \quad \text{(A.8)}$$

so that the optimal choice of w is independent of the value of $A_{2,\underline{S}}$. We can, therefore, write

$$V_2(A_{2,\underline{S}}) = R_2^{CE} A_{2,\underline{S}},$$

where

$$R_2^{CE} \equiv \left[\int_z \left[\frac{1}{P_1^f} + \left(\frac{\varphi^m(z)}{P_1^m} - \frac{1}{P_1^f} \right) w \right]^{(1-\gamma)} dF(z) \right]^{\frac{1}{(1-\gamma)}},$$

where w solves the first-order condition, Eq. (A.8).

□

Proof of Proposition 2

Proof. We know that the first order conditions of the period $t = 0$ problem for all assets are

$$\frac{1}{c_0} = E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} \frac{P_1^f (1 - \mathcal{I}_{s=\overline{S}} \tau^i)}{P_0^i} \right)$$

for all $i \in \mathbb{I} \cup m$. Let us define $R_{1,s}^i \equiv \frac{P_1^f (1-\mathcal{I}_{s=\overline{S}} \tau^i)}{P_0^i}$, $R_1^1 \equiv \frac{P_1^f}{P_0^1}$, and $m_{1,s} \equiv \frac{\frac{\partial U_{1,s}}{\partial c_{1,s}}}{\frac{\partial U_0}{\partial c_0}}$. Then $\frac{\partial U_0}{\partial c_0} = E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} R_{1,s}^i \right)$ for all i. This means that also $\frac{\partial U_0}{\partial c_0} = R_1^1 E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} \right)$ must hold. We rearrange the general formula as follows:

$$\begin{aligned}
0 &= E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} R_{1,s}^i - \frac{\partial U_0}{\partial c_0} \right) \\
0 &= E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} \left(R_{1,s}^i - R_1^1 \right) \right) \\
0 &= E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} \right) E_s \left(R_{1,s}^i - R_1^1 \right) + COV \left(\frac{\partial U_{1,s}}{\partial c_{1,s}}, \left(R_{1,s}^i - R_1^1 \right) \right) \\
-\frac{COV \left(\frac{\partial U_{1,s}}{\partial c_{1,s}}, \left(R_{1,s}^i - R_1^1 \right) \right)}{E_s \left(\frac{\partial U_{1,s}}{\partial c_{1,s}} \right)} &= E_s \left(R_{1,s}^i \right) - R_1^1 \\
-COV \left(m_{1,s}, \left(R_{1,s}^i - R_1^1 \right) \right) R_1^1 &= E_s \left(R_{1,s}^i \right) - R_1^1 \\
-COV \left(m_{1,s}, R_{1,s}^i \right) &= \frac{E_s \left(R_{1,s}^i \right)}{R_1^1} - 1.
\end{aligned}$$

We define the relative inconvenience yield $\psi^i \equiv \frac{R_1^i}{R_1^1}$ with $R_1^i \equiv \frac{P_1^f}{P_0^i}$. We insert for $R_{1,s}^i$ and rearrange:

$$\begin{aligned}
\frac{E_s\left(\frac{P_1^f(1-\mathcal{I}_{s=\overline{S}}\tau^i)}{P_0^i}\right)}{R_1^1} - 1 &= -COV\left(m_{1,s}, \frac{P_1^f(1-\mathcal{I}_{s=\overline{S}}\tau^i)}{P_0^i}\right) \\
\frac{\frac{P_1^f}{P_0^i}}{R_1^1}(1-\lambda\tau^i) - 1 &= -\frac{\frac{P_1^f}{P_0^i}}{R_1^1} COV\left(m_{1,s}, (1-\mathcal{I}_{s=\overline{S}}\tau^i)\right) R_1^1 \\
\psi^i &= \frac{1}{(1-\lambda\tau^i) - COV\left(m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^i\right) R_1^1}.
\end{aligned}$$

We then rearrange the covariance term:

$$\begin{aligned}
COV\left(m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^i\right) &= \lambda\tau^i m_{1,\overline{S}} - E_s\left(m_{1,s}\right)\lambda\tau^i \\
COV\left(m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^i\right) &= \lambda\tau^i\left(m_{1,\overline{S}} - \frac{1}{R_1^1}\right).
\end{aligned}$$

We insert it into our main term of interest:

$$\begin{aligned}
\psi^i &= \frac{1}{1-\lambda\tau^i m_{1,\overline{S}} R_1^1} \\
\psi^i &= \frac{1}{1-\lambda\tau^i m_{1,\overline{S}} \frac{1}{E_s(m_{1,s})}} \\
\psi^i &= \frac{1}{1-\tau^i \frac{\lambda m_{1,\overline{S}}}{\lambda m_{1,\overline{S}} + (1-\lambda) m_{1,\underline{S}}}}.
\end{aligned}$$

Therefore the relative inconvenience yield is given by

$$\psi^i = \frac{1}{1-\tau^i \frac{\lambda}{\lambda+(1-\lambda)/\eta}}$$

where $\eta \equiv \frac{m_{1,\overline{S}}}{m_{1,\underline{S}}}$ is the relative kernel, which we later call the value of convenience. □

Proof of Proposition 3

Proof. We analyse the impact of the fees on the relative inconvenience yields. The derivatives are given by

$$\frac{\partial\psi^i}{\partial\tau^i} = -\left(\psi^i\right)^2 \frac{\lambda}{\lambda+(1-\lambda)/\eta}\left(\frac{(1-\lambda)\tau^i}{\lambda+(1-\lambda)/\eta}\frac{\partial(1/\eta)}{\partial\tau^i} - 1\right) > 0$$

and

$$\frac{\partial\psi^i}{\partial\tau^k} = \frac{\partial\psi^i}{\partial(1/\eta)}\frac{\partial(1/\eta)}{\partial\tau^k} > 0 \text{ for all } i \neq k,$$

where $\frac{\partial \psi^i}{\partial(1/\eta)} = -\left(\psi^i\right)^2 \frac{\tau^i\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2} < 0$, $\frac{\partial(1/\eta)}{\partial\tau^i} = \frac{-P_1^i Q^i}{\frac{2}{1+\lambda}Y_1} < 0$, and $\frac{\partial(1/\eta)}{\partial\tau^k} = \frac{-P_1^k Q^k}{\frac{2}{1+\lambda}Y_1} < 0$ and i and $k \in \mathbb{I} \cup m$. In addition it follows that

$$\begin{aligned}
\frac{\partial^2\psi^i}{\partial{\tau^i}^2} =& -2\psi^i \frac{\partial\psi^i}{\partial\tau^i}\frac{\lambda}{\lambda+(1-\lambda)/\eta}\left(\frac{(1-\lambda)\tau^i}{\lambda+(1-\lambda)/\eta}\frac{\partial(1/\eta)}{\partial\tau^i} - 1\right) \\
& -\left(\psi^i\right)^2 \frac{\lambda}{\lambda+(1-\lambda)/\eta}\left(\frac{(1-\lambda)}{\lambda+(1-\lambda)/\eta}\frac{\partial(1/\eta)}{\partial\tau^i} - 1\right) > 0 \\
\frac{\partial^2\psi^i}{\partial\tau^k\partial\tau^i} =& -\left(2\psi^j\frac{\partial\psi^i}{\partial\tau^i}\frac{\tau^i\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2} + \left(\psi^i\right)^2\frac{\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2}\right)\frac{\partial(1/\eta)}{\partial\tau^k} > 0.
\end{aligned}$$

□

Proof of Propositon 4

Proof. We study the impact on the relative inconvenience yields due to a compositional change in the supply of the risk-free assets. Suppose $dQ^l = -dQ^k$ for any l and k in $\in \mathbb{I}$. Then

$$\frac{d\psi^i}{dQ^l} = \frac{\partial\psi^i}{\partial(1/\eta)}\frac{d(1/\eta)}{dQ^l}$$

where $\frac{\partial \psi^i}{\partial(1/\eta)} = -\left(\psi^i\right)^2 \frac{\tau^i\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2} < 0$, $\frac{d(1/\eta)}{dQ^l} = \frac{\partial(1/\eta)}{\partial Q^l} + \frac{\partial(1/\eta)}{\partial Q^k}\frac{dQ^k}{dQ^l} = \left(\tau^k - \tau^l\right)\frac{1}{\frac{2}{1+\lambda}Y_1}$ for all $i \in \mathbb{I} \cup m$. □

Proof of Proposition 5

Proof. We analyse the effect of increasing Q^f for two cases: The case where there is no risky asset in the economy, i.e. $Q^m = 0$ and where there is, i.e. $Q^m > 0$. Specifically we look at the comparative static

$$\frac{\partial\psi^i}{\partial Q^f} = \frac{\partial\psi^i}{\partial(1/\eta)}\frac{\partial(1/\eta)}{\partial Q^f}$$

where $\frac{\partial \psi^i}{\partial(1/\eta)} = -\left(\psi^i\right)^2 \frac{\tau^i\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2} < 0$ for all $i \in \mathbb{I} \cup m$.

If $Q^m = 0$, then by Eq. (12), the value of convenience η is given by $\eta = \frac{1}{1-(1-\lambda)\widetilde{\tau}/2}$ and so $\frac{\partial(1/\eta)}{\partial Q^f} = \frac{(1-\lambda)}{2}\frac{1}{Q^f}\left(1 - \frac{\partial Q^i}{\partial Q^f}\frac{Q^f}{Q^i}\right)\widetilde{\tau} = 0$. This shows the first part of the proposition.

Next, consider the case of $Q^m > 0$. In this case, by equation (11), we have

$$\frac{\partial(1/\eta)}{\partial Q^f} = -\frac{1-\lambda}{2}(\tau^m - \widetilde{\tau})\frac{1}{\gamma\sigma^2}\frac{\partial w}{\partial Q^f} = -\frac{1-\lambda}{2}(\tau^m - \widetilde{\tau})\frac{1}{\gamma\sigma^2}\frac{\partial\pi}{\partial Q^f}.$$

To find the derivative of the log risk premium with respect to Q^f, given the log-approximation from Section 2.3, we can find a closed form solution for the log risk premium, as we show in the following Lemma:

Lemma 5 (**Log risk premium**). *Under the log approximation of the investor's portfolio problem and log risk premium the period $t = 1$ log risk premium is given by*

$$\pi = \frac{\mu - 1 - q + \sqrt{(\mu-1-q)^2 + 4\gamma\sigma^2 q}}{2},$$

where $q = Q^m/Q^f$.

Proof. First, from the approximation from Section 2.3, the log risk premium on the risky asset is approximately

$$\pi \approx \log E[\varphi^m(z)] + \frac{P_1^f - P_1^m}{P_1^m} = \mu - 1 + \frac{P_1^f}{P_1^m}.$$

Combining this equation for the log risk premium with the equations period $t = 1$ equilibrium asset prices from Lemma 1 and the equation for the optimal portfolio share w from Eq. (6), we find that

$$\pi = \mu - 1 + \left(\frac{\gamma\sigma^2}{\pi} - 1\right)\frac{Q^m}{Q^f}.$$

Therefore, we have a quadratic equation for the log risk premium of the form

$$\pi^2 - (\mu - 1 - q)\,\pi - \gamma\sigma^2 q = 0,$$

where $q = Q^m/Q^f$ is the ratio of the supply of risky to risk-free assets. Notice that since the third term of the quadratic equation is negative, it follows that the equation has one positive and one negative root. Only the positive root is relevant in our case. Therefore,

$$\pi = \frac{\mu - 1 - q + \sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q}}{2}.$$

□

Differentiating with respect to q, we find that

$$\frac{\partial \pi}{\partial q} = \frac{1}{2}\left(-1 + \frac{q - \mu - 1 + 2\gamma\sigma^2}{\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q}}\right).$$

Consider the sign of the term in parentheses and notice that

$$(\mu - 1 - q)^2 + 4\gamma\sigma^2 q - \left(q - \mu - 1 + 2\gamma\sigma^2\right)^2 = 4\gamma\sigma^2\left(\mu - 1 - \gamma\sigma^2\right).$$

Therefore, if $\mu - 1 > \gamma\sigma^2$, then

$$(\mu - 1 - q)^2 + 4\gamma\sigma^2 q > \left(q - \mu - 1 + 2\gamma\sigma^2\right)^2,$$

and so the sign of the term in parenthesis is negative, or $\partial\pi/\partial q < 0$. Similarly, if $\mu - 1 \leq \gamma\sigma^2$, we can conclude that $\partial\pi/\partial q \geq 0$. Since $\partial q/\partial Q^f < 0$, it follows that $\partial\pi/\partial Q^f > 0$, iff $\mu - 1 > \gamma\sigma^2$.

Therefore, $\partial(1/\eta)/\partial Q^f < 0$, iff $(\tau^m - \widetilde{\tau})\frac{\partial\pi}{\partial Q^f} > 0$, or using the condition for the sign of $\partial\pi/\partial Q^f > 0$, $\partial(1/\eta)/\partial Q^f < 0$, iff $(\tau^m - \widetilde{\tau})\left(\mu - \gamma\sigma^2 - 1\right) > 0$. The second part of the proposition follows directly from combining this partial derivative with the comparative static for ψ^i with respect to η above. □

Proof of Proposition 6

Proof. Consider the derivative $\partial\psi^i/\partial\sigma^2$, for any $i \in \mathbb{I} \cup m$. We have that

$$\frac{\partial\psi^i}{\partial\sigma^2} = \frac{\partial\psi^i}{\partial(1/\eta)}\frac{\partial(1/\eta)}{\partial\sigma^2},$$

where $\frac{\partial\psi^i}{\partial(1/\eta)} = -\left(\psi^i\right)^2 \frac{\tau^i\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2} < 0$. Furthermore, by Eq. (11), we have that

$$\frac{\partial(1/\eta)}{\partial\sigma^2} = -\frac{1-\lambda}{2}(\tau^m - \widetilde{\tau})\frac{\partial w}{\partial\sigma^2} = \frac{1-\lambda}{2}(\tau^m - \widetilde{\tau})\frac{1}{\gamma\sigma^2}\left(\frac{\pi}{\sigma^2} - \frac{\partial\pi}{\partial\sigma^2}\right).$$

Note first that

$$\frac{\partial\pi}{\partial\sigma^2} = \frac{\gamma q}{\sqrt{(\mu-1-q)^2 + 4\gamma\sigma^2 q}} > 0$$

Next, note that

$$\frac{\pi}{\sigma^2} - \frac{\partial\pi}{\partial\sigma^2} = \frac{(\mu-1-q)\sqrt{(\mu-1-q)^2 + 4\gamma\sigma^2 q} + (\mu-1-q)^2 + 2\gamma\sigma^2 q}{2}$$

Suppose that $\mu - 1 - q \geq 0$. In that case it clearly follows that the right-hand side is positive. Suppose instead that $\mu - 1 - q \leq 0$. In that case, note that

$$\begin{aligned}
(\mu-1-q)\sqrt{(\mu-1-q)^2 + 4\gamma\sigma^2 q} + (\mu-1-q)^2 + 2\gamma\sigma^2 q &> \\
-\frac{(\mu-1-q)^2 + \left(\sqrt{(\mu-1-q)^2 + 4\gamma\sigma^2 q}\right)^2}{2} + (\mu-1-q)^2 + 2\gamma\sigma^2 q &= \\
\frac{(\mu-1-q)^2 - \left(\sqrt{(\mu-1-q)^2 + 4\gamma\sigma^2 q}\right)^2}{2} + 2\gamma\sigma^2 q &= \\
-2\gamma\sigma^2 q + 2\gamma\sigma^2 q &= 0.
\end{aligned}$$

Therefore, we have that

$$\frac{\pi}{\sigma^2} - \frac{\partial\pi}{\partial\sigma^2} > 0,$$

and so the sign of $\frac{\partial(1/\eta)}{\partial\sigma^2}$ depends on the sign of $\tau^m - \widetilde{\tau}$.

The derivation of the comparative statics with respect to risk aversion γ is analogous. □

Proof of Proposition 7

Proof. We can express $c_{1,\underline{S}}$ as a function of η: $c_{1,\underline{S}} = \frac{Y_1}{2\lambda\eta+(1-\lambda)}$. We insert it in the constraints and the constraints into the objective function of the social planner. This implies:

$$\begin{aligned}
W \equiv \log(Y_0) + 2\lambda\log\left(\frac{1}{\lambda}\right) + 2\lambda\log\left(Y_1 - (1-\lambda)\frac{Y_1}{2\lambda\eta + (1-\lambda)}\right) \\
+ (1-\lambda)\log\left(\frac{Y_1}{2\lambda\eta + (1-\lambda)}\right) + (1-\lambda)\log\left(\left(E\left[\left(Q^f + \varphi_2^m(z)Q^m\right)^{1-\gamma}\right]\right)^{1/(1-\gamma)}\right).
\end{aligned}$$

We want to analyze the impact of η on W. Taking the respective first-order condition we have

$$\frac{\partial W}{\partial \eta} = \left(\frac{1}{\eta}(1-\lambda) - (1-\lambda)\right)\frac{2\lambda}{2\lambda\eta + (1-\lambda)}.$$

It follows that $\frac{\partial W}{\partial \eta} = 0$ iff $\eta = 1$. We observe that for $\eta \geq 1$[1] welfare is monotone decreasing in the value of convenience, i.e. $\frac{\partial W}{\partial \eta} < 0$. $\square$

Proof of Proposition 8

Proof. We first modify Lemmas 1 and 2 to allow for the more general period $t = 2$ asset payoffs as well as buyer transaction costs/subsidies. The maximization problem of the impatient investor in period $t = 1$ is trivial – it is optimal for her to sell all her asset holdings and consume. Therefore, for that investor we have

$$V_{1,\overline{S}} = 2\log(c_{1,\overline{S}}),$$

where $c_{1,\overline{S}} = Y_1 + \sum_{i\in\mathbb{I}}(1-\tau^i)P_1^iX_1^i$, with $X_1^i = Q^i$ in equilibrium.

The maximization problem of the patient investor, who buys all assets in equilibrium at $t = 1$, in the absence of $t = 2$ payoff risk, is now given by

$$V_{1,\underline{S}} = \max_{c_{1,\underline{S}},c_{2,\underline{S}},\left\{X^i_{2,\underline{S}}\right\}_i} \log(c_{1,\underline{S}}) + \log(c_{2,\underline{S}})$$

$$\text{s.t.}$$

$$c_{1,\underline{S}} + \sum_{i\in\mathbb{I}} P_1^i X_{2,\underline{S}}^i = Y_1 + \sum_{i\in\mathbb{I}} P_1^i X_1^i - \sum_{i\in\mathbb{I}} \tau_b^i P_1^i (X_{2,\underline{S}}^i - X_1^i),$$

$$c_{2,\underline{S}} = \sum_{i\in\mathbb{I}} \delta^i X_{2,\underline{S}}^i.$$

Assuming an interior solution for each asset i (which will be the case in equilibrium), the Euler equation for asset i is given by

$$\frac{1}{c_{2,\underline{S}}} = \frac{\overline{P}_1^i}{c_{1,\underline{S}}}, \tag{A.9}$$

where $\overline{P}_1^i \equiv \frac{(1+\tau_b^i)}{\delta^i}P_1^i$. Since these first-order conditions have to hold for each asset i in equilibrium, this implies that $\overline{P}_1^i$ must be equalized across assets i. We therefore define $\overline{P}_1^f \equiv \overline{P}_1^i$. Since $X_1^i = Q^i$ in equilibrium, market clearing implies that $X_{2,\underline{S}}^i = \frac{1}{(1-\lambda)}Q^i$, which we insert into the budget constraints. This yields

$$c_{1,\underline{S}} = Y_1 - \frac{\lambda}{(1-\lambda)}\sum_{i\in\mathbb{I}} \overline{P}_1^i \overline{Q}^i,$$

$$c_{2,\underline{S}} = \frac{1}{(1-\lambda)}\sum_{i\in\mathbb{I}} \overline{Q}^i,$$

where

$$\overline{Q}^i \equiv \delta^i Q^i$$

[1] It is possible for the social planner to set $\eta < 1$ but in our model this is not a possible equilibrium outcome.

can be thought of as a payoff-adjusted effective supply for asset i.

Substituting for the period 1 and 2 equilibrium consumption in the Euler equation, Eq. (A.9), and solving for the common adjusted price $\overline{P}_1^f$, we obtain

$$\overline{P}_1^f = \frac{1-\lambda}{1+\lambda} \frac{Y_1}{\sum_{i\in\mathbb{I}} \overline{Q}^i}. \tag{A.10}$$

Next, we characterize the $t = 0$ maximization problem of an investor:

$$\begin{aligned} V_0 = & \max_{\{X_1^i\}_{i\in\mathbb{I}}} \log(c_0) + \lambda V_{1,\overline{S}} + (1-\lambda) V_{1,\underline{S}} \\ & \text{s.t. } c_0 = Y_0 + \sum_{i=1}^{N} P_0^i (Q^i - X_1^i). \end{aligned}$$

Taking first-order conditions, and using the Envelope theorem for the period $t = 1$ marginal values, we obtain the period $t = 0$ Euler equations

$$\frac{\partial U_0}{\partial c_0} = \lambda \frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}} \frac{P_1^i}{P_0^i}(1-\tau^i) + (1-\lambda) \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \frac{P_1^i}{P_0^i}(1+\tau_b^i).$$

Next, we derive the equilibrium value of convenience, η. Recall that $\eta = \frac{m_{1,\overline{S}}}{m_{1,\underline{S}}} = \frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}} / \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}}$. We know that

$$\begin{aligned} \frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}} &= \frac{2}{c_{1,\overline{S}}}, \\ &= \frac{2}{Y_1 + \sum\limits_{i\in\mathbb{I}} (1-\tau^i) P_1^i X_1^i}, \\ &= \frac{2}{Y_1 + \sum\limits_{i\in\mathbb{I}} \frac{(1-\tau^i)}{(1+\tau_b^i)} \overline{P}_1^f \overline{Q}^i}, \\ &= \frac{2}{Y_1 + \left[\sum\limits_{i\in\mathbb{I}} \frac{(1-\tau^i)}{(1+\tau_b^i)} \overline{Q}^i \right] \frac{(1-\lambda)}{(1+\lambda)} \frac{Y_1}{\sum\limits_{i\in\mathbb{I}} \overline{Q}^i}} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} &= \frac{1}{c_{1,\underline{S}}}, \\ &= \frac{1}{Y_1 - \frac{\lambda}{(1-\lambda)} \sum\limits_{i\in\mathbb{I}} \overline{P}_1^i \overline{Q}^i}, \\ &= \frac{(1+\lambda)}{Y_1}. \end{aligned}$$

Therefore

$$\eta = \frac{2}{1 + \lambda + (1-\lambda) \sum\limits_{i\in\mathbb{I}} \frac{\overline{Q}^i}{\sum_{i\in\mathbb{I}} \overline{Q}^i} \frac{(1-\tau^i)}{(1+\tau_b^i)}}, \tag{A.11}$$

which we can further simplify to

$$\eta = \frac{1}{1 - \frac{(1-\lambda)}{2}\sum_{i\in\mathbb{I}} \frac{\overline{Q}^i}{\sum_{i\in\mathbb{I}} \overline{Q}^i} \frac{\tau^i+\tau_b^i}{(1+\tau_b^i)}}. \tag{A.12}$$

Recall that $\overline{Q}^i = \delta^i Q^i, \ \forall i$, so that Eq. (A.12) simplifies to equation (12) for $\tau_b^i = 0$ and $\delta^i = 1, \forall i$.

Suppose that $\tau_b^i = 0, \forall i$. In that case, Eq. (A.12) simplifies to

$$\eta = \frac{1}{1 - \frac{(1-\lambda)}{2}\sum_{i\in\mathbb{I}} \frac{\overline{Q}^i}{\sum_{i\in\mathbb{I}} \overline{Q}^i} \tau^i}. \tag{A.13}$$

Note that this expression is identical to Eq. (12), apart from the different weighting of the transaction costs, τ^i. Therefore, by increasing δ^1, the payoff on the most liquid asset, and at the same time decreasing the payoffs δ of the other assets, to ensure budget balance, one can decrease the value of η. This gives us the result that $\frac{\partial \eta}{\partial \delta^1} < 0$. □

Proof of Proposition 9

Proof. Inspecting, Eq. (A.12), we see that if $\tau_b^i = -\tau^i$, $\forall i$, then $\eta = 1$. □

Relating transaction costs and maturity – microfoundations

Summary In this section we provide two micro-founded examples for higher transaction costs on longer maturity assets. For both examples we adapt our framework by assuming, for simplicity, that there are only two risk-free assets. We further modify the framework slightly by assuming that one asset matures in period $t = 1$, while the other matures in $t = 2$, which introduces a term structure into our model.

In the first example we additionally assume that if the two-period asset has to be resold at $t = 1$, the investor has to pay a fee, which can be thought of as a market access fee. Our results show that the long maturity asset is less convenient and therefore may be associated with higher transaction costs. Unlike the short maturity asset, which pays off precisely when an investor has liquidity needs, the long maturity asset has to be liquidated, while incurring a cost to do so.

In the second example we do not assume that agents incur any transaction costs when selling assets. In contrast, we focus on the impact of aggregate uncertainty on the convenience of assets with different maturities. As a concrete example (which we later generalise) we assume that the endowment in $t = 1$ is stochastic. There is a news shock about the aggregate endowment realised in $t = 1$, so that at $t = 0$ agents do not know if their endowment will be high or low in the second period. Our results reveal that the price, and hence the return, of the long maturity asset in period $t = 1$ covaries negatively with the marginal utility of consumption. In comparison, the short maturity asset matures in $t = 1$ and therefore its payoff is not correlated with endowment uncertainty. The negative covariance between the future return and future marginal utility (and therefore also with the aggregate value of convenience) is another reason why the long maturity asset is less convenient to be held at $t = 0$ and trades at a discount relative to the short maturity asset.

We also generalise these examples, by deriving a general expression for the relative inconvenience yield within our specialised set-up with two assets with different maturities. We assume both market fees and aggregate uncertainty, thus combining the two examples. Compared to the second example we further assume that the aggregate uncertainty can be with respect to any exogenous variable, for example payoffs. Additionally, we assume an isoelastic utility function instead of a log utility function to generalise further. We also assume that the third period is discounted with a discount factor which is possibly smaller than one (to allow for aggregate uncertainty on the discount factor). We show that the expression for the relative inconvenience yield nests our two examples from beforehand, and that the drivers are the same. With respect to the influence of aggregate uncertainty it depends on the specific kind of uncertainty if the long or the short maturity asset are more convenient.

Preliminaries We use our original model and make the following adjustment to explicitly account for short and long maturity assets: We assume (for simplicity) that there are only two risk-free assets with payoff 1 in the economy. One matures after one period and the other after two periods. We denote the short maturity asset by l ("low") and the long maturity asset by h ("high"). We will give two examples of why the long maturity asset is less convenient relative to the short maturity asset.

Example 1

First we assume that if an asset is sold before maturing, the agents incur a fee. As this can only happen for the long maturity asset, the fee is denoted by τ^h. We start by describing the optimal behaviour in period 1. As in our standard model, the liquidity constrained agent will liquidate all assets in period 1 and therefore $X^l_{2,\overline{S}} = X^h_{2,\overline{S}} = 0$. The other kind of agent solves $\max_{X^h_{2,\underline{S}}} log(Y_1 + A_1 - P^h_1 X^h_{2,\underline{S}}) + log(X^h_{2,\underline{S}})$, where $A_1 \equiv X^l_1 + P^h_1 X^h_1$. The first order condition is given by $\frac{P^h_1}{Y_1+A_1-P^h_1 X^h_{2,\underline{S}}} = \frac{1}{X^h_{2,\underline{S}}}$, or $P^h_1 X^h_{2,\underline{S}} = \frac{Y_1+A_1}{2}$. Market clearing implies $\lambda(X^h_{2,\overline{S}} - X^h_1) + (1-\lambda)(X^h_{2,\underline{S}} - X^h_1) = 0$, or $X^h_{2,\underline{S}} = \frac{X^h_1}{(1-\lambda)}$. In period 0 the agents solve the following maximisation problem:

$$\max_{c_0, X^l_1, X^h_1} log(c_0) + \lambda\chi V_{1,\overline{S}} + (1-\lambda)V_{1,\underline{S}}$$

$$\text{s.t.}$$

$$Y_0 = c_0 + P^l_0(Q^l - X^l_1) + P^h_0(Q^h - X^h_1)$$

where $V_{1,\overline{S}} = log(Y_1 + A_1 - \tau^h P^h_1 X^h_1)$ and $V_{1,\underline{S}} = log\left(Y_1 + A_1 - \frac{P^h_1 X^h_1}{1-\lambda}\right) + log\left(\frac{X^h_1}{1-\lambda}\right)$, or $V_{1,\underline{S}} = log\left[Y_1 + A_1 - \frac{Y_1+A_1}{2}\right] + log\left[\frac{Y_1+A_1}{2}\right]$. The first order conditions are given by

$$\frac{1}{Y_0 + P^l_0(Q^l - X^l_1) + P^h_0(Q^h - X^h_1)} = \lambda\left(\frac{\chi}{Y_1 + A_1 - \tau^h X^h_1}\frac{1}{P^l_0}\right) + (1-\lambda)\left(\frac{2}{Y_1 + A_1}\frac{1}{P^l_0}\right)$$

and

$$\frac{1}{Y_0 + P_0^l(Q^l - X_1^l) + P_0^h(Q^h - X_1^h)} = \lambda\left(\frac{\chi}{Y_1 + A_1 - \tau^h X_1^h}\frac{P_1^h}{P_0^h}(1-\tau^h)\right) + (1-\lambda)\left(\frac{2}{Y_1 + A_1}\frac{P_1^h}{P_0^h}\right).$$

Market clearing in period 0 implies that $X_1^l = Q^l$ and $X_1^h = Q^h$. From combining the first order condition of the liquidity unconstrained agent in period 1 and market clearing it follows that $P_1^h = \frac{1}{\frac{2}{(1-\lambda)}-1}\frac{Y_1+Q^l}{Q^h}$. From now on we assume that $\chi = 2$ (as in the main body). In addition we define $R_1^h \equiv \frac{P_1^h}{P_0^h}$ and $R_1^l \equiv \frac{1}{P_0^l}$. Note that $\frac{\partial V_{1,s}}{\partial A_1} = \frac{\partial U_{1,s}}{\partial c_{1,s}}$ and rewriting the first of the two first order conditions implies $\frac{1}{R_1^l} = E_s\left(\frac{\frac{\partial U_{1,s}}{\partial c_{1,s}}}{\frac{\partial U_1}{\partial c_0}}\right)$. As a next step we combine the first order conditions in period 1. To rearrange we use $E_s\left(\frac{\partial U_{1,s}}{\partial c_{1,s}}\right) \equiv \lambda\left(\frac{\partial U_{1,\overline{S}}}{\partial c_{1,\overline{S}}}\right) + (1-\lambda)\left(\frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}}\right)$. Rearranging yields

$$\begin{aligned}
0 &= \lambda\frac{2}{Y_1 + A_1}\left[R_1^l - R_1^h + R_1^h\tau^h\right] + (1-\lambda)\left[\frac{2}{Y_1 + A_1}(R_1^l - R_1^h)\right] \\
0 &= E_s\left\{\frac{\partial U_{1,s}}{\partial c_{1,s}}\left[1 - \frac{R_1^h}{R_1^l}(1 + \mathcal{I}_{s=\overline{S}}\tau^h)\right]\right\} \\
0 &= COV\left[\frac{\partial U_{1,s}}{\partial c_{1,s}}, \mathcal{I}_{s=\overline{S}}\tau^h\right]\frac{R_1^h}{R_1^l} + E_s\left[\frac{\partial U_{1,s}}{\partial c_{1,s}}\right]\left[1 - \frac{R_1^h}{R_1^l}(1 - \lambda\tau^h)\right] \\
0 &= COV\left[m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^h\right]\frac{R_1^h}{R_1^l} + E_s\left[m_{1,s}\right]\left[1 - \frac{R_1^h}{R_1^l}(1 - \lambda\tau^h)\right] \\
0 &= \frac{1}{R_1^l} + \left[COV\left[m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^h\right] - E_s\left[m_{1,s}\right](1 - \lambda\tau^h)\right]\frac{R_1^h}{R_1^l}.
\end{aligned}$$

Further rearranging yields

$$\frac{R_1^h}{R_1^l} = \frac{1}{R_1^l}\frac{1}{\left\{E_s\left[m_{1,s}\right](1-\lambda\tau^h) - COV\left[m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^h\right]\right\}}.$$

We know that $COV\left[m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^h\right] = \lambda\tau^h m_{1,\overline{S}} - E_s\left[m_{1,s}\right]\lambda\tau^h$. Therefore

$$\begin{aligned}
\frac{R_1^h}{R_1^l} &= \frac{1}{1 - \lambda\tau^h - \lambda\tau^h\left[m_{1,\overline{S}} - E_s\left[m_{1,s}\right]\right]R_1^l} > 0 \\
\frac{R_1^h}{R_1^l} &= \frac{1}{1 - \lambda\tau^h - \lambda\tau^h(1-\lambda)(\eta - 1)m_{1,\underline{S}}R_1^l} > 0
\end{aligned}$$

or rewritten

$$\frac{R_1^h}{R_1^l} = \frac{1}{1 - \lambda\tau^h - \lambda\tau^h(1-\lambda)Y_0\left[\frac{2}{Y_1+A_1-\tau^h P_1^h Q^h} - \frac{2}{Y_1+A_1}\right]R_1^l} > 0.$$

The long maturity asset is less convenient. The short maturity asset carries a convenience premium when being compared to the long maturity asset. The reason is that the long maturity asset has a lower probability to mature at the moment the liquidity is needed.

Example 2

Second, we assume that there is an aggregate uncertainty in $t = 1$. We assume that the endowment will be high or low in the first period. In period 0, the value of the endowment, denoted by Y_1^k, is unknown, but the agents know the distribution. We use this explicit example for illustrative purposes and later generalise the microfoundation. In the general version, the uncertainty can be about any exogenous variable.

Analogous to above and adjusted to the current example, the first order conditions are given by

$$\frac{1}{Y_0 + P_0^l(Q^l - X_1^l) + P_0^h(Q^h - X_1^h)} = E\left(\frac{2}{Y_1^k + A_1}\frac{1}{P_0^l}\right)$$

and

$$\frac{1}{Y_0 + P_0^l(Q^l - X_1^l) + P_0^h(Q^h - X_1^h)} = E\left(\frac{2}{Y_1^k + A_1}\frac{P_1^h}{P_0^h}\right).$$

We combine the first order conditions. Note that $\frac{\partial U_{1,\underline{S}}}{\partial A_1} = \frac{\partial U_{1,\overline{S}}}{\partial A_1}$ and $m_{1,\underline{S}} = m_{1,\overline{S}}$. We will use $\frac{1}{R_1^l} = E\left(\frac{\frac{\partial U_{1,\underline{S}}}{\partial A_1}}{\frac{\partial U_1}{\partial c_0}}\right)$. Therefore

$$\begin{aligned}
0 =& E\left[\frac{2}{Y_1^k + A_1}\left(R_1^l - R_1^h\right)\right] \\
0 =& E\left[\frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}}\left(1 - \frac{R_1^h}{R_1^l}\right)\right] \\
0 =& E\left[m_{1,\underline{S}}\left(1 - \frac{R_1^h}{R_1^l}\right)\right] \\
0 =& \frac{1}{R_1^l} - E\left(m_{1,\underline{S}}\frac{R_1^h}{R_1^l}\right) \\
0 =& \frac{1}{R_1^l} - COV\left(\frac{R_1^h}{R_1^l}, m_{1,\underline{S}}\right) - E\left(\frac{R_1^h}{R_1^l}\right)E\left(m_{1,\underline{S}}\right).
\end{aligned}$$

Further rearranging yields

$$\begin{aligned}
E\left(\frac{R_1^h}{R_1^l}\right) =& \frac{\frac{1}{R_1^l} - COV\left(m_{1,\underline{S}}, \frac{R_1^h}{R_1^l}\right)}{E\left(m_{1,\underline{S}}\right)} \\
E\left(\frac{R_1^h}{R_1^l}\right) =& 1 - COV\left(m_{1,\underline{S}}, R_1^h\right) \\
E\left(\frac{R_1^h}{R_1^l}\right) =& 1 - COV\left(\frac{m_{1,\overline{S}}}{\eta}, R_1^h\right) \\
E\left(\frac{R_1^h}{R_1^l}\right) =& 1 - 2\,COV\left(\frac{Y_0}{Y_1^k + A_1}, R_1^h\right).
\end{aligned}$$

From $P_1^h = \frac{1}{\frac{2}{1-\lambda} - 1}\frac{Y_1^k + Q^l}{Q^h}$ it follows that $\frac{\partial P_1^h}{\partial Y_1^k} > 0$. Therefore $COV\left[\frac{Y_0}{Y_1^k + A_1}, R_1^h\right] < 0$. It follows that

$$E\left(\frac{R_1^h}{R_1^l}\right) > 1.$$

The long maturity asset is on average less convenient because in period $t = 1$ the asset has a high price, or return when the marginal utility is low.

Generalisation

Finally, we can derive (following the same steps as in the examples above) a more general function that incorporates both channels, transaction costs and aggregate uncertainty, and further generalises the latter. First, we generalise by using an isoelastic utility function instead of a log utility function. Second, we allow agents to discount the second period with the factor $\beta \leq 1$. Third, we now leave open which variable x^i is affected by an aggregate uncertainty in period 0 that is revealed in period 1. For example, we could introduce the payoff risk as an aggregate risk or an uncertainty in the discount factor β. The aggregate uncertainty in the endowment was just an example.

The result that the covariance and not any volatility implies the effect of aggregate uncertainty on the relative convenience return is more general. We derive a general function for the relative inconvenience yields (which nests the relative inconvenience yields of the above examples):

$$E\left(\frac{R_1^h}{R_1^l}\right) = \frac{1 + COV\left[COV\left[m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^h\right] - E_s\left[m_{1,s}\right](1-\lambda\tau^h), R_1^h\right]}{1 - \lambda\tau^h - E\left\{COV\left[m_{1,s}, \mathcal{I}_{s=\overline{S}}\tau^h\right] R_1^l\right\}},$$

or

$$E\left(\frac{R_1^h}{R_1^l}\right) = \frac{1 + COV\left[\lambda\tau^h m_{1,\overline{S}} - E_s\left[m_{1,s}\right], R_1^h\right]}{1 - \lambda\tau^h - \lambda\tau^h E\left[m_{1,\overline{S}} - E_s\left[m_{1,s}\right]\right] R_1^l}$$

$$E\left(\frac{R_1^h}{R_1^l}\right) = \frac{1 + COV\left[\left[\lambda(\tau^h - 1)\eta - (1-\lambda)\right] m_{1,\underline{S}}, R_1^h\right]}{1 - \lambda\tau^h - \lambda\tau^h E\left[(1-\lambda)(\eta - 1)m_{1,\underline{S}}\right] R_1^l}.$$

B Data Appendix

B.1 Data sources and summary statistics

We obtain government security yields and OIS rates from Bloomberg (see table B.1). We use it to calculate the Treasury-OIS spreads. The maturities we use are 3 months, 1 year, 2 years, 5 years, 7 years and 10 years. The frequency is daily, which we aggregate to weekly. Data on total outstanding debt of the US government and outstanding bills are obtained from Bloomberg (tickers: PUBLDEBT Index and DEBPBILL Index). The frequency is daily for the former and monthly for the latter. The MOVE Index also comes from Bloomberg (ticker: MOVE Index), and the VIX Index is obtained from FRED. The frequency is daily, which we aggregate to weekly. Lastly, we obtain the Effective Federal Funds Rate from FRED. The frequency is daily, which we aggregate to weekly. This following table gives an overview over the Bloomberg tickers of the used data series for the Treasury yields and OIS rates.[1]

Table B.1: Data sources

Data	Maturity	Source	Ticker
Treasury	3m	Bloomberg	USGG3M Index
	1y	Bloomberg	USGG1Y Index
	2y	Bloomberg	USGG2Y Index
	5y	Bloomberg	USGG5Y Index
	7y	Bloomberg	USGG7Y Index
	10y	Bloomberg	USGG10Y Index
OIS	3m	Bloomberg	USSOC Curncy
	1y	Bloomberg	USSO1 Curncy
	2y	Bloomberg	USSO2 Curncy
	5y	Bloomberg	USSO5 Curncy
	7y	Bloomberg	USSO7 Curncy
	10y	Bloomberg	USOSFR10 Curncy

[1] When we change the frequency of the data to weekly for the regressions, we use the last available day during each week. When we adjust the frequency of the time series plotted in the figures, we use the average instead of the last available day, as this gives a more accurate overview and is uncritical to do, as no different time series are matched. The only exception is Figure B.2, where we use the last day because we are comparing two series.

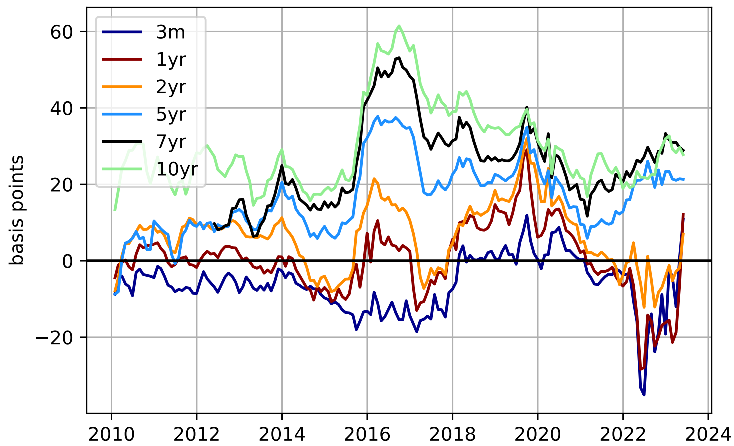
Table B.2: Summary statistics US

Data	Maturity	Mean	Median	St. Dev.
Treasury-OIS	1y	6.65	6.15	6.75
spread in bp	2y	12.83	12.19	8.34
	5y	22.95	19.10	13.10
	7y	32.55	24.54	15.28
	10y	36.26	31.81	14.32
Debt growth in %		0.14	0.05	0.28
MOVE Index		75.05	70.00	23.41
VIX Index		18.51	16.77	7.10
Effective Fed Funds Rate in %		0.76	0.16	1.09

Notes: The time horizon used for the summary statistics is 2010 to mid May 2023. If the frequency is not weekly, we change it to weekly and use the last value of the week.

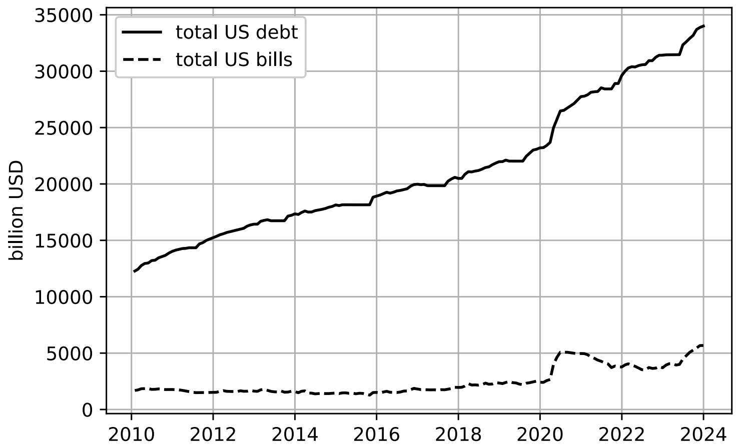
B.2 Additional figures

Figure B.1: Treasury-OIS spread



Notes: This figure shows the US Treasury-OIS spread across different maturities. Source: Bloomberg and authors calculations.

Figure B.2: Total outstanding US government debt and bills



Notes: This figure shows the monthly total outstanding US government debt and total oustanding US government bills. Source: Bloomberg and authors calculations.