

# Bailouts and Financial Fragility

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## Abstract

How does the belief that policy makers will bail out investors in the event of a crisis affect the allocation of resources and the stability of the financial system? I study this question in a model of financial intermediation with limited commitment. When a crisis occurs, the efficient policy response is to use public resources to augment the private consumption of those investors facing losses. The anticipation of such a “bailout” distorts ex ante incentives, leading intermediaries to choose arrangements with excessive illiquidity and thereby increasing financial fragility. Prohibiting bailouts is not necessarily desirable, however: it induces intermediaries to become too *liquid* from a social point of view and may in addition leave the economy *more* susceptible to a crisis. A policy of taxing short-term liabilities, in contrast, can correct the incentive problem while improving financial stability.

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# 1 Introduction

The recent financial crisis has generated a heated debate about the economic effects of public-sector bailouts of private financial institutions. A wide range of policy interventions undertaken in various countries over the past three years can be thought of as “bailouts,” including loans to individual institutions, guarantees of private debt, and direct purchases of certain types of assets. Most observers agree that the anticipation of such bailouts in the event of a crisis distorts the incentives faced by financial institutions and other investors. By insulating these agents from the full consequences of a negative outcome, an anticipated bailout results in a misallocation of resources and encourages risky behavior that may leave the economy more susceptible to a future crisis.

Opinions differ widely, however, on the best way for policy makers to deal with this problem. Some observers argue that policy makers should focus on making credible commitments to not bail out financial institutions in the event of a future crisis. Such a commitment would encourage investors to provision for bad outcomes and, it is claimed, these actions would collectively make the financial system more stable. Others argue that policy makers should focus instead on improving the regulation and supervision of financial institutions and markets. Proponents of this second view believe that it is either infeasible or perhaps even undesirable to limit future policy makers’ actions. They view the distortions caused by the anticipation of future bailouts as inevitable and argue that policy makers must aim to correct the distortions and promote financial stability through improved regulation in normal times.

Given these widely differing views, it is important to investigate the effects of bailouts in formal economic models and to use these models to ask how policy makers can best address the issue. Would it be desirable for policy makers to commit to never bail out financial institutions? Would doing so be an effective way to promote financial stability? Or is it better to allow bailouts to occur and attempt to offset their distortionary effects through regulation?

I address these questions in a model of financial intermediation and fragility based on the classic paper of Diamond and Dybvig [8]. In particular, I study an environment with idiosyncratic liquidity risk and with limited commitment, as in Ennis and Keister [9]. Individuals deposit resources with financial intermediaries, and these resources are invested in a nonstochastic production technology. Intermediaries perform maturity transformation and thereby insure investors against their

individual liquidity risk. This maturity transformation makes intermediaries illiquid and may leave them susceptible to a self-fulfilling run by investors. Fiscal policy is introduced into this framework by adding a public good that is financed by taxing households' endowments. In the event of a crisis, some of this tax revenue may be diverted from production of the public good and instead given as private consumption to investors facing losses in the financial system. The size of this "bailout" payment is chosen to achieve an ex post efficient allocation of the remaining resources in the economy.

I begin the analysis by characterizing a benchmark allocation that represents the efficient distribution of resources in this environment conditional on investors running on the financial system in some states of the world. I show that this allocation always involves a transfer of public resources to private investors in those states. In other words, a bailout is part of the *efficient* allocation of resources in this environment whenever a crisis is possible. The logic behind this result is straightforward and fairly general. In normal times, the policy maker chooses the tax rate and the level of public good provision to equate the marginal social values of public and private consumption. A crisis results in a misallocation of resources, which raises the marginal value of private consumption for some investors. The optimal response to this situation is to decrease public consumption and transfer resources to these investors – a "bailout." The efficient bailout policy thus provides investors with (partial) insurance against the losses associated with a financial crisis.

In a decentralized setting, the anticipation of this type of bailout distorts the ex ante incentives of investors and their intermediaries. As a result, intermediaries choose to perform more maturity transformation, and hence become more illiquid, than in the benchmark allocation. This excessive illiquidity, in turn, implies that the financial system is more *fragile* in the sense that a self-fulfilling run can occur in equilibrium for a strictly larger set of parameter values. The incentive problem created by the anticipated bailout thus has two negative effects in this environment: it both distorts the allocation of resources in normal times and increases the financial system's susceptibility to a crisis.

A policy of committing to no bailouts is not necessarily desirable, however. Such a policy would require intermediaries to completely self-insure against the possibility of a crisis, which would lead them to become more liquid (by performing less maturity transformation) than in the benchmark efficient allocation. Despite this increase in liquidity, the economy would remain more fragile than in the benchmark allocation. A no-bailouts policy would also leave the level of public

good provision inefficiently high if a crisis does occur. If the probability of a crisis is sufficiently small, a no-bailouts commitment is strictly inferior to a discretionary policy regime – it lowers equilibrium welfare without improving financial stability. For higher probabilities of a crisis, a no-bailouts policy may or may not be preferable, depending on parameter values, but it will never achieve the efficient allocation of resources. Interestingly, for some economies that are not fragile in a discretionary regime, a no-bailouts policy would actually *introduce* the possibility of a self-fulfilling run.

The idea that a credible no-bailout commitment can increase the fragility of the financial system may seem surprising at first, but the mechanism behind this result is easy to understand. A bailout policy provides insurance – it lessens the potential loss an investor faces if she does not withdraw her funds and a crisis occurs. Removing this insurance increases each individual’s incentive to withdraw early if she expects others to do so, which makes the financial system more susceptible to a self-fulfilling crisis. This argument is familiar in the context of retail banking: government-sponsored deposit insurance programs can be thought of as a type of “bailout” policy that is explicitly designed to play a stabilizing role. Despite this similarity, discussion of the insurance role of bailouts has been largely absent in the current policy debate.

An optimal policy arrangement in the environment studied here requires permitting bailouts to occur, so that investors benefit from the efficient level of insurance, while offsetting the negative effects on ex ante incentives. One way this can be accomplished is by placing a Pigouvian tax on intermediaries’ short-term liabilities, which can also be interpreted as a tax on the activity of maturity transformation. In the simple environment studied here, the appropriate choice of tax rate will implement the benchmark efficient allocation and will decrease the scope for financial fragility relative to either the discretionary or the no-bailouts regime.

There is a large literature in which versions of the Diamond-Dybvig model are used to address issues related to banking policy and financial fragility. This paper follows Green and Lin [17], Peck and Shell [21], Ennis and Keister [10] and other recent work in specifying an explicit sequential service constraint and allowing intermediaries to offer any contract that is consistent with the information flow generated by that constraint. In particular, intermediaries and the policy maker are able to react as soon as they infer that a run is under way, rather than following a simple rule such as allowing investors to withdraw until all funds are depleted. The paper also focuses on the implications of a lack of commitment power on the part of the banking authorities, as in Mailath

and Mester [18], Acharya and Yorulmazer [1], Ennis and Keister [9] and others.

There is a small but growing literature on the incentive effects of financial-sector bailouts and optimal regulatory policy in the presence of limited commitment. Chari and Kehoe [5] study an environment in which committing to a no-bailout policy would generate the first-best allocation of resources if it were feasible and show how, in the absence of commitment, ex ante regulation of private contracts can be welfare improving. In the environment studied here, in contrast, committing to a no-bailout policy is not first-best optimal because bailout payments provide socially-valuable insurance. This aspect of the model is similar in some respects to Green [16], who also highlights the fact that policies resembling a bailout can be part of a desirable ex ante insurance arrangement. Other related work includes Gale and Vives [14], who study dollarization as a device for limiting a central bank's ability to engage in bailouts, Fahri and Tirole [12], who focus on the strategic complementarities generated by indiscriminate bailouts, Cooper and Kempf [6], who study the redistributive effects of deposit insurance when agents are ex ante heterogeneous, and Niepmann and Schmidt-Eisenlohr [20], who examine the strategic interaction between governments when bailouts have international spillover effects. In contrast to these papers, a primary focus here is on financial fragility, that is, the conditions under which an economy becomes susceptible to a crisis driven by the self-fulfilling beliefs of investors.

## 2 The Model

I begin with a fairly standard version of the Diamond and Dybvig [8] model and augment this basic framework by introducing a public good. This section describes the physical environment and the model of the decentralized economy.

### 2.1 The environment

There are three time periods,  $t = 0, 1, 2$ , and a continuum of investors, indexed by  $i \in [0, 1]$ . Each investor has preferences given by

$$U(c_1, c_2, g; \theta_i) = u(c_1 + \theta_i c_2) + v(g), \quad (1)$$

where  $c_t$  is consumption of the private good in period  $t$  and  $g$  is the level of public good, which is provided in period 1. The functions  $u$  and  $v$  are assumed to be strictly increasing, strictly concave, and to satisfy the usual Inada conditions. In addition, the coefficient of relative risk aversion for the

function  $u$  is assumed to be constant and greater than one. The parameter  $\theta_i$  is a binomial random variable with support  $\Theta = \{0, 1\}$ . If the realized value of  $\theta_i$  is zero, investor  $i$  is *impatient* and only cares about early consumption. An investor's type  $\theta_i$  is revealed to her in period 1 and remains private information. Let  $\omega$  denote a profile of preference types for each investor and let  $\Omega$  denote the set of all such profiles. Let  $\pi$  denote the probability with which each individual investor will be impatient. By a law of large numbers,  $\pi$  is also the fraction of investors in the population who will be impatient.

Each investor is endowed with one unit of the private good in period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into private consumption in the later periods. A unit of the good invested in period 0 yields  $R > 1$  units in period 2, but only one unit in period 1. This investment technology is operated in a central location, where investors can pool resources in an intermediation technology to insure against individual liquidity risk. Investors are isolated from each other in periods 1 and 2 and no trade can occur among them. Upon learning her preference type, each investor chooses either to contact the intermediation technology in period 1 to withdraw funds or to wait and withdraw in period 2. There is also a technology for transforming units of the private good one-for-one into units of the public good. This technology is operated in period 1, using goods that were placed into the investment technology in period 0.

An (ex post) *allocation* in this environment is a pair  $(c, g)$ , where  $c : [0, 1] \rightarrow \mathbb{R}_+^2$  is an assignment of a private consumption level to each investor in each period and  $g \in \mathbb{R}_+$  is a level of public good provision. An allocation is *feasible* if it can be produced from the period-0 endowments using the technologies described above, that is, if

$$\int_0^1 c_1(i) di + \frac{1}{R} \int_0^1 c_2(i) di \leq 1 - g.$$

Let  $\mathbb{A}$  denote the set of feasible allocations. A *state-contingent allocation* is a mapping  $\mathbf{c} : \Omega \rightarrow \mathbb{A}$  from the set of realized preference types to the set of feasible allocations.

Investors who choose to withdraw in period 1 arrive one at a time in a randomly-determined order. As in Wallace [23], [24], these investors must consume immediately upon arrival. This sequential-service constraint implies that the payment made to such an investor can only depend on the information received by the intermediation technology up to that point. In particular, this payment can be contingent on the number of early withdrawals that have taken place *so far*, but not on the *total* number of early withdrawals that will occur because this latter number will not be

known until the end of the period.

Since investors ex ante identical, it is natural to measure ex ante welfare in this economy as the period-0 expected utility of each investor. For ex post measures of welfare, after preference types (and potentially some consumption levels) have been realized, I use an equal-weighted sum of individual utilities to measure welfare. The expression

$$W = \int_0^1 E [u (c_1 (i) , c_2 (i) , g; \theta_i)] di$$

captures both of these notions and is, therefore, used to measure welfare throughout the analysis.

## 2.2 The decentralized economy

In the decentralized economy, the intermediation technology is operated by a large number of competitive intermediaries, each of which aims to maximize the expected utility of its investors. Each intermediary serves a large number of investors and, hence, knows that a fraction  $\pi$  of its investors will be impatient. Because investors' types are private information, the payment an investor receives from her intermediary cannot depend directly on her realized type. Instead, the intermediary allows each investor to choose the period in which she will withdraw. This arrangement, which resembles a variety of demand-deposit contracts used in reality, is well known to be a useful tool for implementing desirable allocations in economies with private information. However, such arrangements may also create the possibility of a "run" on the financial system in which all investors attempt to withdraw early, regardless of their realized preference type.

Intermediaries act to maximize the expected utility of their investors at all times. In reality, there are important agency problems that cause the incentives of financial intermediaries to differ from those of their investors and creditors. I abstract from these agency problems here in order to focus more directly on the distortions in investors' incentives that are created by the anticipation of a bailout in the event of a crisis. As in Ennis and Keister [9], [11], intermediaries cannot commit to future actions. This inability to commit implies that they are unable to use the type of suspension of convertibility plans discussed in Diamond and Dybvig [8] or the type of run-proof contracts studied in Cooper and Ross [7]. Instead, the payment given to each investor who withdraws in period 1 will be a best response given the intermediary's current beliefs.

The public good is provided by a benevolent policy maker who has the ability to tax endowments in period 0. The revenue from this tax is placed into the investment technology and transformed

into period 1 private goods. In period 1, the policy maker can use these private goods to produce units of the public good or, if a crisis is underway, can transfer some of these private goods to the financial intermediaries. I refer to this latter option as a “bailout” payment to the financial system.<sup>1</sup>

## 2.3 Financial crises

In order to allow a run on the financial system to occur with nontrivial probability, I introduce an extrinsic “sunspot” signal on which investors can potentially condition their actions. Let  $S = \{s_1, s_2\}$  be the set of possible sunspot states, with  $\text{prob}[s = s_2] = q \in [0, 1]$ . Investor  $i$  chooses a strategy that assigns a decision to withdraw in either period 1 or period 2 to each possible realization of her preference type  $\theta_i$  and of the sunspot variable

$$y_i : \Theta \times S \rightarrow \{1, 2\}.$$

Neither the intermediaries nor the policy maker observe the realization of the sunspot variable. Instead, they must try to infer the state from the flow of withdrawals. This approach is standard<sup>2</sup> and, combined with the sequential service constraint, implies that some payments must be made to withdrawing investors before the intermediaries or policy maker know whether or not a run is underway. I focus on system-wide financial crises, in which all intermediaries face a potential run in the same sunspot state. Suppose that all investors attempt to withdraw early in state  $s_2$ . Intermediaries and the policy maker know that at least  $\pi$  investors will withdraw in both states and, therefore, as the first  $\pi$  withdrawals take place they are unable to infer anything about the realized sunspot state. If the fraction of early withdrawals goes past  $\pi$ , however, they can immediately infer that the state is  $s_2$  and that a run is underway.

An important element of the model is specifying how intermediaries and the policy maker respond once they discover that a run is underway and how those investors who have not yet been able to withdraw react to this response. In general, this interaction may be quite complex and different patterns of behavior are possible (see Ennis and Keister [11]). To simplify matters, I assume here that once it has discovered a run is underway, an intermediary is able to implement the efficient allocation of its remaining resources among the remaining investors. As part of this

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<sup>1</sup> Notice that this type of bailout policy is entirely consistent with the sequential service constraint, since all taxes are collected before any consumption takes place. I assume the sequential service constraint applies to the policy maker as well as to the intermediaries and, hence, the approach here is not subject to the Wallace [23] critique of Diamond and Dybvig [8]. Other papers have introduced taxation into the Diamond-Dybvig framework in a similar way; see, for example, Freeman [13], Boyd *et al.* [3], and Martin [19]. The goal of fiscal policy in those papers, however, is to fund a deposit insurance system rather than to pursue an independent objective like the provision of a public good.

<sup>2</sup> See, for example, Diamond and Dybvig [8, Section 4], Cooper and Ross [7], and Peck and Shell [21].



allocation, only those remaining investors who are impatient withdraw early; the remaining patient investors wait until period 2 to withdraw.

There are several different ways in which this allocation could come about. It could, for example, be the result of a screening technology that can be used in the event of a run, as in Ennis and Keister [9]. Alternatively, it could be the result of equilibrium behavior in a game played by the intermediary and those investors who anticipate they will be late to arrive at their intermediary in period 1, as in Ennis and Keister [11]. Whatever the mechanism, this approach ensures that none of the results below are driven by some assumed inefficiency in the distribution of resources following a run.<sup>3</sup>

### 3 Efficient Allocations and Bailouts

In this section, I study the efficient allocation of resources under the assumption that only impatient investors withdraw early in state  $s_1$  but all investors attempt to withdraw early in  $s_2$ , so that a financial crisis occurs with probability  $q$ . The question of whether this behavior is consistent with equilibrium under the different policy regimes is taken up in subsequent sections. The objective in this section is simply to determine the efficient way to allocate resources conditional on this behavior and subject to the constraints imposed by the environment.

#### 3.1 The $q$ -efficient allocation

Using the structure of the model, particularly the absence of any intrinsic aggregate uncertainty, the problem of finding the efficient allocation of resources under this scenario can be simplified considerably. First, note that the form of the utility function (1) implies that a planner would want to give consumption to impatient investors only in period 1 and to patient investors only in period 2. Moreover, because investors are risk averse, the planner would like to give the same amount of consumption to all investors of a given type. However, I assume the planner faces the same informational constraints that intermediaries and the policy maker face in the decentralized economy. In particular, the planner correctly anticipates investors' withdrawal strategies as a function of the sunspot state, but is unable to observe the realized state. Instead, it must infer the state from the observed withdrawal behavior of investors.

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<sup>3</sup> The results would not change if, for example, a fraction of the intermediary's remaining assets were lost in the event of a run. Such an inefficiency would only serve to increase the scope for financial fragility under all of the policy regimes studied here.

In the scenario considered here, the fraction of investors who attempt to withdraw early will be  $\pi$  in state  $s_1$  and 1 in state  $s_2$ . As the first  $\pi$  withdrawals are taking place, therefore, no information about the state is revealed to the planner. The efficient policy must give the same consumption level to all of these investors; any feasible allocation in which these investors consume different amounts is strictly dominated by another feasible allocation in which their consumption levels are equalized. Let  $c_E$  denote the payment given to these investors, who withdraw “early.” If withdrawals cease after a fraction  $\pi$  of investors has withdrawn, the planner can infer that the remaining investors are all patient and will withdraw in period 2. The planner will then divide the remaining resources between a common payment  $c_L$  for those investors who withdraw “late” and an amount  $g$  of the public good.

If, on the other hand, the fraction of investors withdrawing in period 1 goes past  $\pi$ , the planner is immediately able to infer that state  $s_2$  has occurred. At this point, the planner is able to implement the efficient continuation allocation among the remaining investors. This allocation gives a common amount of consumption, denoted  $\hat{c}_E$ , to each remaining impatient investor in period 1. Note that  $\hat{c}_E$  will, in general, be different from the consumption level of the first  $\pi$  investors to withdraw,  $c_E$ . Similarly, the planner will give a common amount  $\hat{c}_L$  to each remaining patient investor in period 2. Let  $\hat{g}$  denote the amount of public good provided in this case. Notice the importance of the sequential service constraint here: a fraction  $\pi$  of investors must be served, and will consume, before the planner is able to infer the state and thus determine the appropriate consumption levels.

The problem of finding the efficient allocation of resources given that a run will occur in state  $s_2$  can, therefore, be reduced to choosing the consumption levels  $(c_E, c_L, \hat{c}_E, \hat{c}_L)$  and the levels of public good provision  $(g, \hat{g})$  to solve

$$\begin{aligned} \max \quad & (1 - q) [\pi u(c_E) + (1 - \pi) u(c_L) + v(g)] + \\ & q [\pi u(c_E) + (1 - \pi) [\pi u(\hat{c}_E) + (1 - \pi) u(\hat{c}_L)] + v(\hat{g})] \end{aligned}$$

subject to

$$\pi c_E + (1 - \pi) \frac{c_L}{R} + g \leq 1, \quad (2)$$

$$(1 - \pi) \left( \pi \hat{c}_E + (1 - \pi) \frac{\hat{c}_L}{R} \right) + \hat{g} \leq 1 - \pi c_E, \quad (3)$$

and

$$c_L \geq c_E, \quad \hat{c}_L \geq \hat{c}_E.$$

Expression (2) is the resource constraint that applies in state  $s_1$ , while (3) applies in state  $s_2$ . The final two constraints are incentive compatibility conditions that, in a decentralized economy, ensure withdrawing early is not a dominant strategy. One can show that these latter constraints never bind at the solution. The solution to this problem is called the *q-efficient allocation*.

Letting  $(1 - q)\mu$  and  $q\hat{\mu}$  denote the multipliers on constraints (2) and (3), respectively, the solution to this problem is characterized by the conditions

$$u'(c_E) = (1 - q)\mu + q\hat{\mu} \quad (4)$$

$$Ru'(c_L) = v'(g) = \mu, \quad \text{and} \quad (5)$$

$$u'(\hat{c}_E) = Ru'(\hat{c}_L) = v'(\hat{g}) = \hat{\mu}. \quad (6)$$

The first condition says that the marginal value assigned to resources paid out before the planner knows whether a run is underway should be equal to the expected future marginal value of resources. The other equations can be interpreted as the standard Samuelson condition for the efficient provision of a public good, which equates the sum of individuals' marginal rates of substitution to the marginal rate of transformation, in each of the two states.<sup>4</sup>

Let  $\mathbf{c}^* = (c_E^*, c_L^*, g^*, \hat{c}_E^*, \hat{c}_L^*, \hat{g}^*)$  denote the solution to this problem and let  $(\mu^*, \hat{\mu}^*)$  denote the corresponding values of the (normalized) multipliers. It is straightforward to show that each element of this solution varies continuously with the probability of a crisis  $q$ , and that evaluating  $\mathbf{c}^*$  in the limit as  $q \rightarrow 0$  yields the first-best allocation of resources in this environment.

### 3.2 Illiquidity

For any given allocation, define the *degree of illiquidity* in the financial system to be

$$\rho \equiv \frac{c_E}{1 - g}.$$

Since each investor has the option of withdrawing early,  $c_E$  represents the short term liabilities of the financial system in per-capita terms. The short-run value of intermediaries' assets per capita is equal to the fraction of endowments that are invested to provide private consumption,  $1 - g$ . Hence  $\rho$  represents the ratio of the short-term liabilities of the financial system to the short-run value of its assets. I will say that the financial system is *illiquid* whenever  $\rho > 1$  holds.

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<sup>4</sup> Note that because the *q*-efficient allocation is symmetric and there is a measure 1 of depositors, the sum of all investors' marginal rates of substitution is equal to each individual's marginal rate of substitution.

The following proposition shows that the financial system is illiquid under the  $q$ -efficient allocation of resources for any value of  $q$ . As is standard in Diamond-Dybvig models, this illiquidity is what potentially opens the door to self-fulfilling financial crises. In addition, the proposition shows that the efficient response to an increase in the probability of a crisis is to decrease the degree of illiquidity. Proofs of all propositions are contained in the appendix.

**Proposition 1**  $\rho^* > 1$  holds for all  $q \geq 0$  and  $\rho^*$  is strictly decreasing in  $q$ .

### 3.3 Bailouts

The next proposition establishes a key feature of the  $q$ -efficient allocation: less of the public good is provided in the event of a crisis than in normal times.

**Proposition 2**  $\hat{g}^* < g^*$  holds for all  $q \geq 0$

Recall that  $g^*$  is the quantity of resources initially set aside to provide the public good. If a crisis occurs, some of these resources are instead used to provide private consumption to those investors who have not yet been able to withdraw. The property  $\hat{g}^* < g^*$  can, therefore, be interpreted as a “bailout” of the financial system. In the event of a run, all investors pay a cost in terms of a lower level of the public good in order to augment the private consumption of those agents facing losses on their financial investments.<sup>5</sup>

Proposition 2 shows that this bailout is part of the *efficient* allocation of resources. The logic behind the result is fairly general and seems likely to appear in a wide range of settings. The efficient fiscal plan is designed so that the marginal social value of public consumption will equal the marginal value of the private consumption in normal times. When a crisis occurs, it leads to a misallocation of resources that lowers private consumption for some investors, which raises their marginal value of consumption. The efficient response must, therefore, be to shift some resources away from public consumption and into the private consumption of these investors. Notice that this “bailout” is efficient even from an ex ante point of view; it provides investors with insurance against the losses they may suffer in the event of a crisis.

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<sup>5</sup> Note that total government spending is unaffected by a financial crisis in this model, since all tax revenue is collected in the initial period and the government budget is always balanced. What changes during a crisis is the composition of government spending between public services and transfer payments. In reality, governments typically do cut public services in response to budgetary pressures that arise during a crisis.

### 3.4 Financial fragility

The concept of financial fragility – or the susceptibility of the financial system to a crisis – has been defined in a variety of different ways. In the environment studied here, it is natural to say that the financial system is fragile if a crisis can occur with positive probability in an equilibrium of the decentralized economy.

**Definition:** The financial system of an economy is *fragile* under a given policy regime if there exists an equilibrium in which all investors attempt to withdraw early in state  $s_2$ .

For making comparisons across different policy regimes, I examine the set of economies that fit this definition of fragility under each regime. An economy is characterized by a set of parameter values; let  $e \equiv (R, \pi, u, v, q)$  denote a typical economy. For each policy regime, I ask what subset of economies have an equilibrium in which investors run on the financial system in state  $s_2$ . If this set is strictly larger under some policy regime A than under regime B, I say that A increases the scope for financial fragility relative to B.

Other approaches to defining financial fragility are possible and would likely lead to similar results. One could, for example, impose an equilibrium selection rule such as risk dominance to resolve the multiplicity of equilibrium in the coordination game played by investors. Aggregate uncertainty about, say, the long-run return  $R$  could be introduced so that agents are initially uncertain whether or not a run will occur. Financial fragility could then be measured by the probability of the set of realizations of  $R$  for which the equilibrium selected by this rule involves a run. While this approach would alter the way financial fragility is measured, the comparative statics of fragility would be qualitatively unchanged. A policy change that makes it more attractive for an individual investor to withdraw early if she believes that others are doing so will tend to increase financial fragility in this setting under any reasonable definition.<sup>6</sup>

The definition of fragility can be extended in a natural way to the benchmark allocation studied

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<sup>6</sup> Another alternative would be to attempt to resolve the multiplicity of equilibrium by introducing private information as in the literature on global games pioneered by Carlsson and van Damme [4]. However, this approach places rather strict requirements on the information structure of the model. Papers that have used the global games methodology in Diamond-Dybvig type models have done so by placing arbitrary restrictions on contracts between intermediaries and their investors (see, for example, Rochet and Vives [22] and Goldstein and Pauzner [15]). These restrictions themselves are potential sources of financial fragility, quite separate from the issues related to bailouts under consideration here. The approach taken here captures the effects of changes in the incentives faced by investors in a reasonably clear and transparent way, and does not place any additional restrictions on agents other than those imposed by the physical environment.

above. In the decentralized economy, a patient investor who runs when all other investors are running and is served before the planner discovers that a run is underway receives  $c_E$ . She would instead receive  $\widehat{c}_L$  if she waits until period 2 to withdraw. We can, therefore, identify fragility with a situation in which this investor has an incentive to participate in the run, that is, in which  $c_E \geq \widehat{c}_L$  holds. I will say that the financial system of an economy is *fragile under the  $q$ -efficient allocation* if  $c_E^* \geq \widehat{c}_L^*$  holds.

Let  $\Phi^*$  denote the set of economies  $e$  such that the financial system is fragile under the  $q$ -efficient allocation. Using the first-order conditions (4) – (6), the condition  $c_E^* \geq \widehat{c}_L^*$  can be written as

$$\frac{\mu^*}{\widehat{\mu}^*} \leq \frac{R^{-1} - q}{1 - q}. \quad (7)$$

It is straightforward to show that there exist parameter values such that this condition is satisfied and, hence, the set  $\Phi^*$  is nonempty.

## 4 Equilibrium under Discretion

In this section, I study the allocation of resources that emerges in an equilibrium of the decentralized economy and compare this outcome to the  $q$ -efficient allocation derived above. The equilibrium is constructed by working backward, beginning with the division of resources among the remaining investors in the event of a run.

### 4.1 The post-run allocation and bailout policy

Suppose the realized state is  $s_2$  and a run occurs. Once it discovers that a run has taken place, each intermediary  $j$  efficiently divides whatever resources it has left among its remaining investors. Let  $\psi_j$  denote the amount of resources, per remaining investor, available to intermediary  $j$ . The intermediary sets the consumption levels  $(\widehat{c}_{E,j}, \widehat{c}_{L,j})$  to solve

$$\widehat{V}(\psi_j) \equiv \max \pi u(\widehat{c}_{E,j}) + (1 - \pi) u(\widehat{c}_{L,j}) \quad (8)$$

subject to

$$\begin{aligned} \pi \widehat{c}_{E,j} + (1 - \pi) \frac{\widehat{c}_{L,j}}{R} &\leq \psi_j \quad \text{and} \\ \widehat{c}_{L,j} &\geq \widehat{c}_{E,j}. \end{aligned} \quad (9)$$

The solution to this problem is characterized by the first-order conditions

$$u'(\widehat{c}_{E,j}) = Ru'(\widehat{c}_{L,j}) = \widehat{\mu}_j, \quad (10)$$

where  $\widehat{\mu}_j$  is the multiplier on the resource constraint (9).

The variable  $\psi_j$  represents the intermediary's own remaining funds plus any bailout payment received from the policy maker. Let  $\tau$  denote the fraction of investors' endowments collected in taxes in the initial period, so that  $1 - \tau$  is the size of the deposit made by each investor. Let  $c_{E,j}$  denote the amount received by each of the first  $\pi$  investors to withdraw from intermediary  $j$  and let  $b_j \geq 0$  denote the size of the bailout payment received by the intermediary. Then resources available to intermediary  $j$ , per remaining investor, are given by

$$\psi_j = \frac{1 - \tau - \pi c_{E,j} + b_j}{1 - \pi}. \quad (11)$$

The policy maker divides its revenue  $\tau$  between a level of the public good  $\widehat{g}$  and bailout payments  $b_j$ . These bailout payments are allocated across intermediaries in an ex post efficient manner. Let  $\sigma_j$  denote the fraction of investors in the economy who have deposited with intermediary  $j$ . The problem of choosing the optimal bailout policy can be written as

$$\max_{\{b_j, \widehat{g}\}} \sum_j \sigma_j (1 - \pi) \widehat{V}(\psi_j) + v(\widehat{g}) \quad (12)$$

subject to the relationship (11) and the budget constraint

$$\widehat{g} + \sum_j \sigma_j b_j = \tau.$$

The solution to this problem is characterized by first-order conditions

$$\widehat{V}'(\psi_j) = v'(\widehat{g}) \quad \text{for all } j,$$

which immediately imply

$$\psi_j = \psi_{j'} \quad \text{for all } j \text{ and } j'. \quad (13)$$

In other words, the ex post efficient bailout payments equalize the resources available for private consumption across intermediaries. The incentive problems that will be caused by this bailout policy are clear: an intermediary with fewer remaining resources (because it chose a higher value

of  $c_{E,j}$ ) will receive a larger bailout.<sup>7</sup> The total size of the bailout payments is then given by

$$b \equiv \sum_j \sigma_j b_j = \tau - \hat{g}. \quad (14)$$

## 4.2 The ex ante allocation

The remaining elements of the equilibrium allocation to be determined are the payments given by intermediaries to the first  $\pi$  investors who withdraw and the tax rate. Since all intermediaries face the same decision problem, I omit the  $j$  subscript and use  $c_E$  to denote the payment offered by a representative intermediary. The equilibrium value of  $c_E$  solves

$$\max_{\{c_E, c_L\}} (1 - q) (\pi u(c_E) + (1 - \pi)u(c_L)) + q (\pi u(c_E) + (1 - \pi)\hat{V}) \quad (15)$$

subject to

$$\pi c_E + (1 - \pi) \frac{c_L}{R} = 1 - \tau, \quad \text{and} \quad (16)$$

$$c_L \geq c_E. \quad (17)$$

Intermediaries and their investors anticipate the fact that, in the event of a crisis, the consumption of each remaining investor will depend only on the aggregate amount of resources in the economy and not on the condition of the investor's own intermediary. For this reason, an intermediary takes the value  $\hat{V}$  as given when choosing the payment  $c_E$ .

The first-order conditions that characterize the solution to this problem when the incentive-compatibility constraint (17) does not bind are

$$u'(c_E) = (1 - q) \mu = (1 - q) R u'(c_L), \quad (18)$$

where  $(1 - q) \mu$  is the multiplier on the resource constraint (16). Comparing the first inequality with (4) illustrates the distortion of incentives: the equilibrium payment  $c_E$  balances the marginal value of resources in the early period against the marginal value of resources in the late period *in the no-run state*, ignoring the value of resources in the event of a run. The larger the probability

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<sup>7</sup> Note that, in principle, a similar incentive problem could arise in state  $s_1$  if the policy maker made bailout payments to intermediaries that chose an unusually high level of  $c_E^j$  in that state as well. I assume that bailout payments are only made in the event of a financial crisis. This assumption could be justified by reputation concerns, which will be significant for decisions made in normal times but much less important for a policy maker facing a rare event like a financial crisis.



of a run  $q$  is, the more distorted the allocation of resources becomes. We can also see from this expression that the incentive compatibility constraint will be satisfied at the interior solution as long as

$$q \leq \frac{R-1}{R},$$

but will otherwise be violated. When the constraint does bind, the equilibrium values are determined by the condition  $c_L = c_E$  together with the resource constraint (16).

Define the value function

$$V^D(\tau) \equiv \pi u(c_E) + (1-q)((1-\pi)u(c_L) + v(\tau)) + q \left( (1-\pi) \widehat{V} \left( \frac{1-\tau-\pi c_E + b}{1-\pi} \right) + v(\tau - b) \right) \quad (19)$$

where  $c_E$  and  $c_L$  are the solution to problem (15) and  $b$  is given by (14). The policy maker will choose the tax rate  $\tau$  in the initial period to maximize the function  $V^D$ . Notice that (19) differs from the objective in (15) because the policy maker recognizes that the value  $\widehat{V}$  depends on the total quantity of resources remaining after the first  $\pi$  withdrawals have taken place, whereas individual intermediaries and investors taken this value as given.

The first-order condition characterizing the equilibrium tax rate can be written as

$$v'(\tau) = \mu + \frac{q}{1-q} \widehat{\mu} \pi \frac{dc_E}{d\tau}. \quad (20)$$

This equation shows that if the probability of a crisis  $q$  were equal to zero, the tax rate would be set to equate the marginal utility of the public good with the marginal value of goods used for private consumption,  $\mu$ . When  $q$  is positive, however, the policy maker must also take into account the fact that changes in  $\tau$  will lead to changes in the equilibrium level of  $c_E$ , which in turn affects the total quantity of resources available in the event of a run. This effect is captured by the second term on the right-hand side of (20).

Let  $c^D$  denote the complete equilibrium allocation. It is straightforward to show that this solution varies continuously with the probability of a crisis  $q$  and converges to the efficient allocation as  $q$  goes to zero. Welfare in the competitive equilibrium is given by

$$W^D \equiv \max_{\{\tau\}} V^D(\tau).$$

### 4.3 Illiquidity and fragility

The distortion created by the bailout policy gives each intermediary an incentive to become more illiquid by offering a larger return to its investors who withdraw early. The next proposition shows that, in the aggregate, this effect increases illiquidity in the financial sector as a whole.

**Proposition 3**  $\rho^D > \rho^*$  holds for all  $q > 0$ . In addition,  $\rho^D$  is strictly increasing in  $q$  for  $q < (R - 1) / R$  and constant for larger values of  $q$ .

Recall that under the  $q$ -efficient allocation of resources, an increase in the probability of a crisis leads to a more liquid financial system (see Proposition 1). Proposition 3 shows that the opposite occurs in the competitive equilibrium. When a financial crisis – and the associated bailout – is more likely, investors prefer a higher short-run return and intermediaries become less liquid. Together, the propositions show that the gap between the efficient level of illiquidity and the level that emerges in equilibrium becomes wider as the probability of a crisis increases.

This higher degree of illiquidity increases the scope for financial fragility in the model. The strategy profile studied here – all investors attempt to withdraw early in state  $s_2$  – is an equilibrium of the decentralized economy if and only if  $c_E^D \geq \widehat{c}_L^D$  holds. Letting  $\Phi^D$  denote the set of economies  $e$  for which this condition holds, the following strict inclusion relationship obtains.

**Proposition 4**  $\Phi^D \supset \Phi^*$ .

This result gives a precise sense in which the incentive problem caused by bailouts make the financial system more fragile. Consider an economy that is not in the set  $\Phi^*$ . For these parameter values, the  $q$ -efficient allocation of resources is such that a patient investor has no incentive to withdraw early, even if he believes everyone else will try to do so. As a result, the financial system is stable in the sense that a self-fulfilling run cannot occur in equilibrium. In the competitive equilibrium, however, intermediaries become more illiquid than in the  $q$ -efficient allocation and investors would find themselves in a worse position in the event of a run. This fact increases the incentive for a patient investor to withdraw early if he believes other investors will run. In some cases, this increase is large enough to make joining the run an optimal response, so that there exists an equilibrium in which all investors attempt to withdraw early with probability  $q$ . In these cases, the distortions created by the bailout policy introduce the possibility of a self-fulfilling financial

crisis.

In the next two sections, I analyze two policy measures designed to mitigate the incentive problem and potentially improve welfare compared to this discretionary policy regime.

## 5 Committing to No Bailouts

I now examine a policy regime that has received considerable attention in the financial press and elsewhere: a commitment to not providing any bailout payments, that is, to setting  $b = 0$  in all states of nature. A very limited form of commitment is being introduced here, in the sense that the policy maker can commit to follow this simple rule but not a more intricate plan. Whether or not it is feasible to commit to this rule in reality is debatable. The question I ask here is whether such a policy – if feasible – would be desirable.<sup>8</sup>

### 5.1 Equilibrium

In the event of a run, each intermediary responds by implementing the efficient allocation of its remaining resources among its investors, as in (8). These resources will be allocated according to the first-order condition (10), and their value is measured by the function  $\widehat{V}$ . The equilibrium values of  $c_E$  and  $c_L$  will solve

$$\max_{\{c_E, c_L\}} \pi u(c_E) + (1 - \pi) \left( (1 - q) u(c_L) + q \widehat{V} \left( \frac{1 - \tau - \pi c_E}{1 - \pi} \right) \right) \quad (21)$$

subject to

$$\begin{aligned} \pi c_E + (1 - \pi) \frac{c_L}{R} &\leq 1 - \tau, \quad \text{and} \\ c_L &\geq c_E. \end{aligned}$$

Note that in this problem the function  $\widehat{V}$  is evaluated at the level of resources (per investor) that the intermediary will have after  $\pi$  withdrawals, a quantity that depends on the intermediary's choice of  $c_E$ . Intermediaries and investors now recognize that, in the event of a run, the only resources that will be available for the private consumption of the remaining investors will be those funds held by the intermediary.

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<sup>8</sup> Note that committing to a pre-specified bailout size  $b > 0$  would not correct the incentive problem that arises in the discretionary regime. The distortion in the model comes not from the size of the bailout payment *per se*, but from the distribution of the bailout payment across intermediaries according to (13).

The solution to this problem is characterized by the first-order conditions

$$u'(c_E) = (1 - q) \mu + q \hat{\mu} \quad (22)$$

and

$$Ru'(c_L) = \mu, \quad (23)$$

where  $(1 - q) \mu$  is the multiplier on resource constraint and the first equation uses the envelope condition  $\hat{V}' = \hat{\mu}$ . Comparing (22) with (18) shows the effect of the no-bailout policy and how it mitigates the incentive problem. Under this policy, an intermediary must balance the value of the early payment  $c_E$  not only against the value of late consumption in the no-run state  $\mu$ , but also against the value of resources in the run state  $\hat{\mu}$ .

Define the value function

$$V^{NB}(\tau) \equiv \pi u(c_E) + (1 - \pi) \left( (1 - q) u(c_L) + q \hat{V} \left( \frac{1 - \tau - \pi c_E}{1 - \pi} \right) \right) + v(\tau),$$

where  $c_E$  and  $c_L$  are the solution to (21). As indicated in this expression, the level of the public good is equal to tax revenue  $\tau$  in both states. The policy maker will choose the tax rate to maximize  $V^{NB}$ . The first-order condition for this problem can be written as

$$v'(\tau) = (1 - q) \mu + q \hat{\mu}. \quad (24)$$

Let  $\mathbf{c}^{NB}$  denote the equilibrium allocation under a no-bailout policy. Welfare under this policy regime is given by

$$W^{NB} \equiv \max_{\{\tau\}} V^{NB}(\tau).$$

## 5.2 Illiquidity and fragility

One can show that the equilibrium degree of illiquidity under the no-bailout regime is strictly decreasing in  $q$ . Recall that this result is the opposite of that obtained in the previous section. When intermediaries and investors anticipate a bailout in the event of a run, an increase in the probability of a run leads them to adopt a more illiquid position. Here, in contrast, an increase in the probability of a run leads intermediaries to adopt a more *liquid* position. In this sense, the no-bailout policy is successful in eliminating the distortion of ex ante incentives.

Comparing  $\rho^{NB}$  to the degree of illiquidity in the  $q$ -efficient allocation, however, shows that the

no-bailout policy actually leads intermediaries to be *too liquid*. These results are summarized in the following proposition.

**Proposition 5**  $\rho^{NB} < \rho^*$  holds for all  $q > 0$  and  $\rho^{NB}$  is strictly decreasing in  $q$ .

This proposition shows that the no-bailout policy introduces a new distortion in ex ante incentives. Instead of performing too much maturity transformation, and taking on too much illiquidity, intermediaries perform too little under this policy. The reason is that intermediaries must now completely self-insure against the possibility of a run. In the  $q$ -efficient allocation, in contrast, the bailout policy provides intermediaries with some insurance against this event.

Despite encouraging financial intermediaries to be liquid, the no-bailout policy still generates greater scope for financial fragility than the  $q$ -efficient allocation. Letting  $\Phi^{NB}$  denote the set of economies for which  $c_E^{NB} \geq \hat{c}_L^{NB}$  holds, and hence there is an equilibrium in which all investors attempt to withdraw early in state  $s_2$ , we have the following result.

**Proposition 6**  $\Phi^{NB} \supset \Phi^*$ . Moreover, there exist economies in  $\Phi^{NB}$  that are not in  $\Phi^D$ .

The intuition behind this result can be seen by considering the limiting case as  $q$  goes to zero. The components of the allocation that apply to the no-run state ( $c_E^{NB}$ ,  $c_L^{NB}$ , and  $g^{NB}$ ) converge to the corresponding components of the  $q$ -efficient allocation, but the post-run components of the allocation ( $\hat{c}_E^{NB}$ ,  $\hat{c}_L^{NB}$ , and  $\hat{g}^{NB}$ ) do not. Because no bailout payments are made, the level of the public good is higher than in the  $q$ -efficient allocation and the private consumption levels  $\hat{c}_E^{NB}$  and  $\hat{c}_L^{NB}$  are lower. It follows that the fragility condition  $c_E \geq \hat{c}_L$  must hold for a strictly larger set of parameter values.

The second part of Proposition 6 demonstrates that some economies that are not fragile under the discretionary policy regime *become* fragile when a no-bailout policy is implemented. This result is somewhat surprising in light of the arguments made by many commentators during the recent financial crisis and the subsequent debate over financial regulatory reform. The intuition behind this result is clear: by increasing  $\hat{c}_L$ , a bailout reduces the cost to an investor of leaving her funds deposited in the event of a run. In other words, the anticipation of a bailout also has a *positive* effect on ex ante incentives by encouraging investors to keep their funds deposited in the financial system. The no-bailout policy removes this positive effect and, as a result, can create

financial fragility.

### 5.3 Welfare

In cases where the economy is fragile under both the policy regimes, the desirability of a no-bailout commitment will depend on how it affects equilibrium welfare. In general, a no-bailouts policy may either raise or lower welfare compared to the discretionary regime, depending on parameter values. As the next proposition shows, however, a sharp comparison is possible when the value of  $q$  is small, that is, when a financial crisis is sufficiently unlikely. In such situations, committing to a no-bailout policy (i) never enhances financial stability and (ii) necessarily leads to lower welfare.

**Proposition 7** *For any  $(R, \pi, u, v)$ , there exists  $\bar{q} > 0$  such that  $q < \bar{q}$  and  $e \in \Phi^D$  implies both  $e \in \Phi^{NB}$  and  $W^D > W^{NB}$ .*

### 5.4 An example

A numerical example can be used to illustrate the results presented above. The utility functions for this example are

$$u(c) = \frac{(c)^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g) = \delta \frac{(g)^{1-\gamma}}{1-\gamma},$$

and the fundamental parameter values are given by  $(R, \pi, \gamma, \delta) = (1.1, 0.5, 6, 0.01)$ . When  $q$  is small, the financial system is fragile under the  $q$ -efficient allocation of resources for these values and, hence, is fragile under both the discretionary and the no-bailout policy regimes. Panel (a) in Figure 1 shows the degree of illiquidity  $\rho$  in each regime as a function of the probability of a crisis  $q$ . When  $q = 0$ , the first-best value of  $\rho$  obtains in all three scenarios. As a crisis becomes more likely, the degree of illiquidity in the efficient allocation declines, in accordance with Proposition 1. Under the no-bailout policy, illiquidity declines even faster as intermediaries adopt more conservative positions, in line with Proposition 5. Under the discretionary policy, in contrast, illiquidity rises as  $q$  increases. The kink in this curve corresponds to point where the incentive compatibility constraint begins to bind in problem (15). Beyond this point the degree of illiquidity stays constant, in line with Proposition 3.

Panel (b) of the figure compares equilibrium welfare under the discretionary and no-bailout regimes. The curve plotted in the figure represents the benefit of the discretionary regime over the no-bailouts regime,  $W^D - W^{NB}$ . Two competing forces are at work in determining the shape of

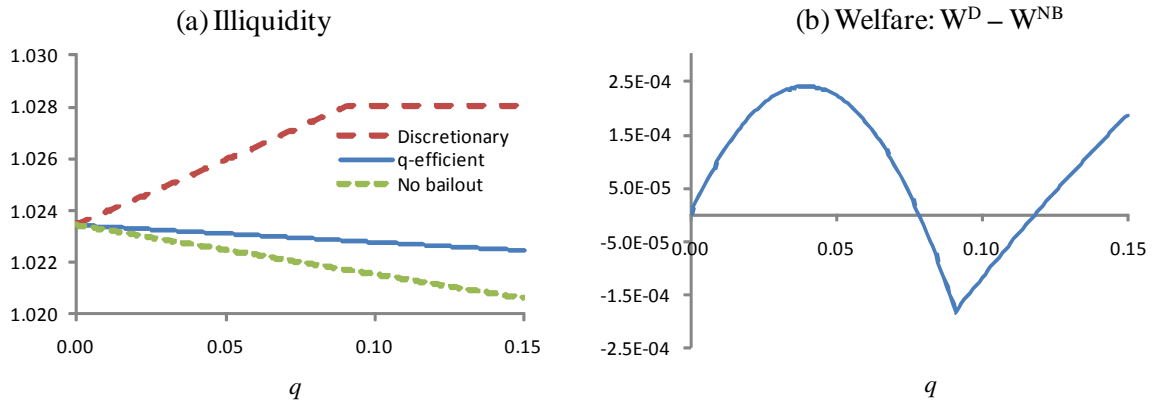


Figure 1: A numerical example

this curve. The ex ante distortion – as depicted in panel (a) – is larger in the discretionary case; this fact tends to make the no-bailout policy attractive. However, the no-bailout regime also leads to an ex post inefficient allocation of resources in the event of a run. For small enough values of  $q$ , these ex post concerns dominate and the discretionary policy yields higher welfare, in line with Proposition 7. As  $q$  increases further and the ex ante distortions become larger, however, the former effect eventually dominates. For values of  $q$  above approximately 0.08, the curve becomes negative and welfare is higher under the no-bailouts policy. Once  $q$  passes the threshold level  $(R - 1) / R$ , however, the incentive compatibility constraint binds in the discretionary equilibrium. As a result, the ex ante distortion in the discretionary case remains constant as  $q$  increases further. For the no-bailout policy, however, the welfare loss from having an inefficient allocation of resources in the event of a run continues to grow as the probability of this event increases. For values of  $q$  above 0.12, the curve becomes positive and the discretionary policy again yields higher welfare.

Figure 2 illustrates how financial fragility differs across policy regimes by presenting a projection of the sets  $\Phi^*$ ,  $\Phi^D$ , and  $\Phi^{NB}$  onto a two-dimensional diagram. The horizontal axis of the figure corresponds to the probability of a crisis,  $q$ , while the vertical axis measures one of the fundamental parameters,  $\pi$ . Different shades are used to represent economies that are fragile under the different policy regimes. The darkest area in the figure represents the economies belong to all three sets. For these combinations of parameter values, the financial system is fragile even under the  $q$ -efficient allocation of resources. As the probability of a crisis  $q$  rises, illiquidity falls in this allocation (Proposition 3) and, as a result, the set of values of  $\pi$  leading to fragility becomes smaller, as shown in the figure.

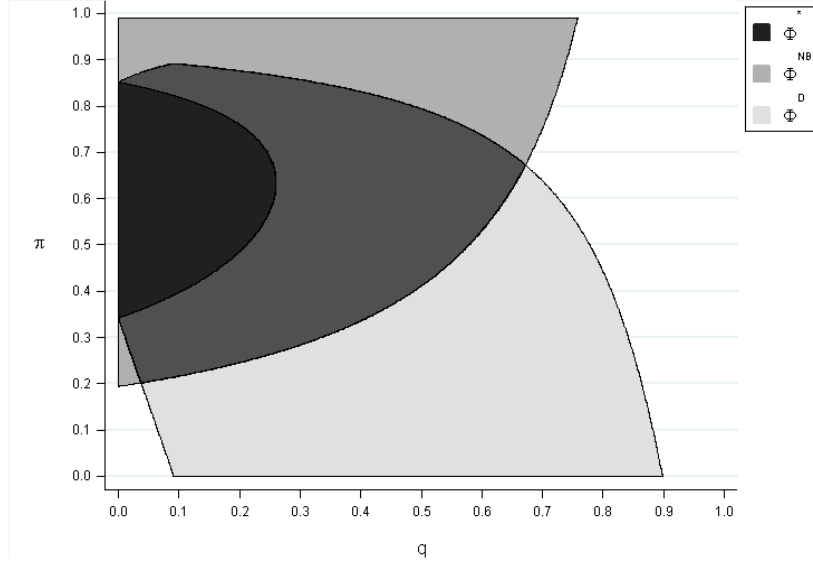


Figure 2: The sets  $\Phi^*$ ,  $\Phi^D$  and  $\Phi^{NB}$ . Darker areas indicate the intersection of sets.

The set  $\Phi^D$  is represented by the lightest colored (and lower most) area, together with the two darkest areas where it overlaps with the other sets. Notice that economies with low values of  $\pi$  tend to be fragile under the discretionary policy regime. This pattern reflects the fact that intermediaries tend to take on more illiquidity when there are relatively few impatient investors, which implies that the magnitude of the distortion under the discretionary regime is largest when  $\pi$  is small. The set  $\Phi^{NB}$  is represented by the next-lightest colored (and upper most) area, together with the two darkest areas. Under this regime, economies with low values of  $\pi$  tend to be stable, but those with high values of  $\pi$  tend to be fragile. If  $\pi$  is large, there are relatively few remaining investors when a bailout payment is made, which implies that even a moderate-sized bailout payment will have a large effect on investors' incentives. Hence, the destabilizing effect of removing this insurance is largest when  $\pi$  is close to one.

Figure 3 presents this same diagram for a variety of different parameter values, showing how changes in the parameters  $\gamma$  and  $\delta$  affect the size and shape of the sets  $\Phi^*$ ,  $\Phi^D$ , and  $\Phi^{NB}$ .

## 6 Taxing Short-term Liabilities

Another policy option is to place no restrictions on the bailout policy, but to offset the distortion through regulation or some other ex ante intervention. To illustrate the effects of such an intervention, I now allow the policy maker to impose a tax on intermediaries' short-term liabilities; this policy can also be thought of as a tax on the activity of maturity transformation. This particular tax



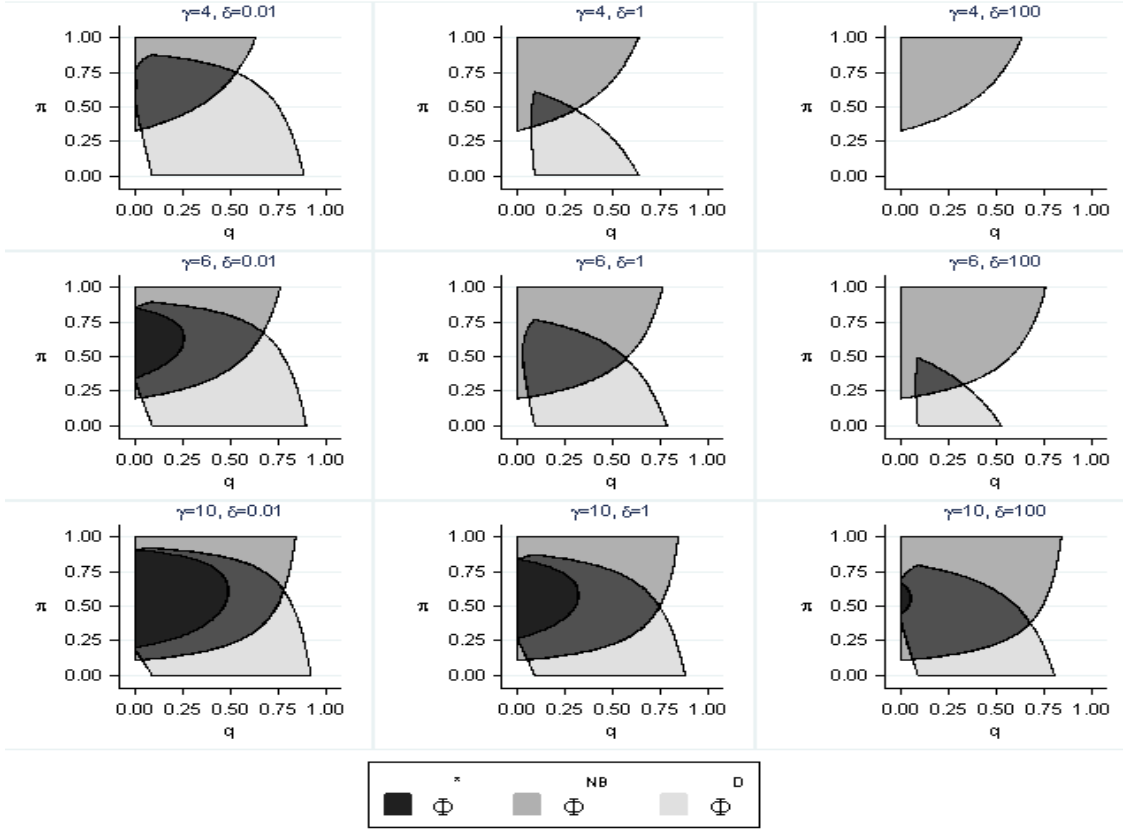


Figure 3: The sets  $\Phi^*$ ,  $\Phi^D$  and  $\Phi^{NB}$  for different parameter values

is one of several possible policies that would have equivalent effects in the simple model studied here, including directly imposing an appropriately-chosen cap on short-term liabilities. The goal is to investigate the effectiveness of a policy regime that aims to influence intermediaries' choices through ex ante intervention rather than through restrictions on the ex post bailout payments. A Pigouvian tax on short-term liabilities is one way to illustrate the results of such an approach.

Suppose each intermediary must pay a fee that is proportional to the total value of its short-term liabilities,

$$\text{fee}_j = \eta\pi\sigma_j C_E,$$

where, as above,  $\sigma_j$  denotes the fraction of investors who deposit with intermediary  $j$ . The tax rate in this policy is  $\eta\pi$ , where  $\eta$  is chosen by the policy maker. For simplicity, I make the policy revenue neutral by giving each intermediary a lump-sum transfer  $N\sigma_j(1 - \tau)$ , where  $N$  is equal to the average fee collected per unit of deposits. This assumption is only to facilitate comparison with the earlier cases.

## 6.1 Equilibrium

Under this policy, the equilibrium payment  $c_E$  will maximize the objective in (15), but subject to the modified resource constraint

$$\pi c_E + (1 - \pi) \frac{c_L}{R} \leq 1 - \tau - \eta \pi c_E + N(1 - \tau). \quad (25)$$

The first-order conditions of this modified problem are

$$u'(c_E) = (1 + \eta)(1 - q)\mu = (1 + \eta)(1 - q)Ru'(c_L),$$

where  $(1 - q)\mu$  is again the multiplier on the resource constraint. We know that the post-run allocation of resources will be efficient, and hence will satisfy the usual first-order conditions (6). Revenue neutrality implies

$$N(1 - \tau) = \eta \pi c_E.$$

Substituting this condition into (25) yields the standard resource constraint for the no-run state.

## 6.2 The optimal tax rate

Can the tax rate  $\eta$  be set so that the equilibrium allocation with ex ante intervention matches the  $q$ -efficient allocation? In the  $q$ -efficient allocation, we have

$$u'(c_E^*) = (1 - q)Ru'(c_L^*) + qRu'(\hat{c}_L^*)$$

In order for the equilibrium allocation to be efficient, therefore we need

$$\eta(1 - q)Ru'(c_L^*) = qRu'(\hat{c}_L^*)$$

or

$$\eta = \frac{q\hat{\mu}^*}{(1 - q)\mu^*} \equiv \eta^*, \quad (26)$$

where  $(1 - q)\mu^*$  and  $q\hat{\mu}^*$  are the multipliers on the resource constraints (2) and (3), respectively, evaluated at the  $q$ -efficient allocation. In other words, the tax rate  $\eta^*$  induces each intermediary to place an additional value on period-2 resources that is based on the marginal social value of resources in the event of a run, rather than in the no-run state. Note that when a crisis is unlikely – that is,  $q$  is close to zero – the optimal tax rate is correspondingly small. When  $\eta$  is set equal

to  $\eta^*$ , the competitive equilibrium allocation will satisfy all of the conditions characterizing the  $q$ -efficient allocation. Since these conditions uniquely determine the efficient allocation, we have the following result.

**Proposition 8** *When the tax rate  $\eta$  is set according to (26), the equilibrium allocation with a tax on short-term liabilities is equal to the  $q$ -efficient allocation.*

This result shows how ex ante intervention can be a powerful policy tool in the environment studied here. An appropriately chosen tax rate allows the policy maker to follow the efficient bailout policy while correcting the distortion created by this policy. The policy maker is thus able to provide investors with the optimal level of insurance against the losses associated with a financial crisis without leading intermediaries to choose excessively high levels of illiquidity. Importantly, the set of economies for which the financial system is fragile is the same as that in the  $q$ -efficient allocation,  $\Phi^*$ . In other words, the optimal tax policy decreases financial fragility relative to either the discretionary or the no-bailouts regime.

Of course, other types of ex ante intervention could be equally effective in the simple environment studied here. The policy maker could, for example, simply impose a ceiling of  $c_E^*$  on the level of short-term liabilities per investor. The model is not designed to distinguish between different types of ex ante policy interventions; a richer environment in which intermediaries face a higher-dimensional decision problem would be needed for that purpose. Rather, the model here highlights the benefits of using *some* ex ante intervention together with the ex post optimal bailout policy. Compared to a no-bailouts regime, this combination not only leads to a more efficient allocation of resources, it also increases financial stability.

## 7 Concluding Remarks

There is widespread agreement that the anticipation of receiving a public-sector bailout in the event of a crisis distorts the incentives of financial institutions and other investors. By partially insulating these agents from the effects of a negative outcome, bailouts diminish their incentive to provision for such outcomes and encourage excessively risky behavior. Such concerns have featured prominently in the recent debate on financial regulatory reform and have lead some commentators to argue that governments and central banks should aim to make credible commitments to not providing any future bailouts.

The model presented here shows that there is another side to this issue, however, and that the anticipation of a bailout can have *positive* ex ante effects as well. These positive effects appear in two distinct forms. First, bailouts are part of an efficient insurance arrangement. A financial crisis leads to a misallocation of resources that raises the marginal social value of private consumption. The optimal response for a policy maker is to decrease public consumption, using these resources to augment the private consumption of agents facing losses. This “bailout” policy raises ex ante welfare by providing risk-averse agents with insurance against the losses associated with a crisis.

In addition, the insurance provided by a bailout policy can have a stabilizing effect on the financial system. Financial crises are commonly thought to have an important self-fulfilling component, with individual investors each withdrawing funds in part because they fear the withdrawals of others will deepen the crisis and create further losses. The anticipation of a bailout lessens the potential loss an investor faces if she does not withdraw her funds. As such, it decreases the incentive for investors to withdraw, which, in turn, makes the financial system less susceptible to a crisis. Committing to a no-bailouts policy removes this insurance and, in some cases, can actually create fragility in the financial system

It should be emphasized that the bailout policies studied here are efficient; they do not lead to rent-seeking behavior, nor are they motivated by outside political considerations. In reality, these types of distortions are important concerns. The message of the paper is *not* that any type of bailout policy is acceptable as long as the ex ante effects are offset through taxation. Limits on the ability of policy makers to undertake inefficient redistribution during a crisis may well be desirable. Rather, the message is that restrictions on bailouts are not *sufficient* to ensure that investors face the correct ex ante incentives. In a reasonably standard economic environment, the efficient allocation of resources requires that investors receive some insurance in the form of a bailout. Providing this insurance distorts incentives, and some form of regulation or other ex ante policy intervention is needed to offset this distortion.

Extending the analysis to richer environments may generate insight into the relative merits of different types of ex ante intervention. In the model presented here, taxing short-term liabilities and imposing a cap on such liabilities are equally effective policies. In a setting where intermediaries make additional decisions and, perhaps, take unobserved actions (such as portfolio allocations, effort in monitoring investments, etc.), this equivalence may no longer hold. Studying such environments using the approach developed here seems a promising avenue for future research.

## Appendix A. Proofs of Propositions

**Proposition 1:**  $\rho^* > 1$  holds for all  $q \geq 0$  and  $\rho^*$  is strictly decreasing in  $q$ .

*Proof:* The proof is divided into three steps.

**Step 1:** Show that  $\rho^* > 1$  holds. If this were not true,  $c_E^* \leq 1 - g^*$  would hold for some value of  $q$ . Straightforward algebra would then yield

$$\frac{1 - \pi c_E^* - g^*}{1 - \pi} \geq 1 - g^*. \quad (27)$$

Note that the first-order conditions (4) – (6) imply

$$(c_E^*, c_L^*, g^*) \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} (\hat{c}_E^*, \hat{c}_L^*, \hat{g}^*) \quad \text{as} \quad \mu^* \left\{ \begin{array}{l} < \\ = \\ > \end{array} \right\} \hat{\mu}^*, \quad (28)$$

where the first comparison is a vector inequality. The resource constraints (2) and (3) will both hold with equality at the  $q$ -efficient allocation and can be written as

$$\begin{aligned} \pi c_E^* + (1 - \pi) \frac{c_L^*}{R} &= 1 - g^* & \text{and} \\ \pi \hat{c}_E^* + (1 - \pi) \frac{\hat{c}_L^*}{R} &= \frac{1 - \pi c_E^* - \hat{g}^*}{1 - \pi}. \end{aligned}$$

Comparing these expressions with (27) shows that setting  $(\hat{c}_E, \hat{c}_L, \hat{g}) = (c_E^*, c_L^*, g^*)$  would necessarily satisfy (3). Condition (28) then implies  $\mu^* \geq \hat{\mu}^*$  would hold. In other words, if  $c_E^* \leq 1 - g^*$  held then each component of the post-run allocation would be at least as large as the corresponding component of the no-run allocation and, as a result, the marginal value of resources would be no larger in the run state than in the no-run state. Finally, note that  $\mu^* \geq \hat{\mu}^*$  would, together with the first-order conditions (4) and (5), imply

$$u'(c_E^*) \leq Ru'(c_L^*). \quad (29)$$

However, the assumption that the coefficient of relative risk aversion is greater than 1 implies<sup>9</sup>

$$u'(1 - g) > Ru'(R(1 - g)). \quad (30)$$

<sup>9</sup> This is a well-know property of Diamond-Dybvig preferences; see footnote 3 of Diamond and Dybvig [8] for a proof.

Together, (29) and (30) would imply  $c_E^* > 1 - g^*$ , contradicting the original supposition. Hence,  $\rho^* > 1$  must hold.

**Step 2:** Show that  $\mu^* < \hat{\mu}^*$  holds. The reasoning is similar to that used in Step 1. Given that  $c_E^* > 1 - g^*$  holds, straightforward algebra yields

$$\frac{1 - \pi c_E^* - g^*}{1 - \pi} < 1 - g^*.$$

Setting  $(\hat{c}_E, \hat{c}_L, \hat{g}) = (c_E^*, c_L^*, g^*)$  would then violate the resource constraint (3), which, together with condition (28), implies that  $\mu^* < \hat{\mu}^*$  must hold.

**Step 3:** Show that  $\rho^*$  is strictly decreasing in  $q$ . Note that the resource constraint (2) can be written as

$$\rho^{-1} = \pi + (1 - \pi) \frac{1}{R} \left( \frac{c_E}{c_L} \right)^{-1}. \quad (31)$$

Thus  $\rho^*$  is strictly decreasing in  $q$  if and only if the ratio  $c_E^*/c_L^*$  is strictly decreasing in  $q$ . The first-order conditions (4) – (6) together with the resource constraints (2) and (3) implicitly define the  $q$ -efficient allocation as a function of  $q$  in a neighborhood of the solution  $\mathbf{c}^*$ . Differentiating (2) and (5) with respect to  $q$  and combining the resulting equations yields

$$\frac{dc_L^*}{dq} = -b_1 \frac{dc_E^*}{dq}, \quad \text{where } b_1 \equiv \frac{\pi}{(1 - \pi) \frac{1}{R} + \frac{Ru''(c_L^*)}{v''(g^*)}} > 0. \quad (32)$$

To show that the ratio  $c_E^*/c_L^*$  is strictly decreasing in  $q$ , therefore, it suffices to show that  $c_E^*$  is strictly decreasing in  $q$ . Differentiating (3) and (6) with respect to  $q$  yields

$$\frac{d\hat{c}_L^*}{dq} = -b_2 \frac{dc_E^*}{dq}, \quad (33)$$

where

$$b_2 \equiv \frac{\pi}{(1 - \pi) \left( \pi \frac{Ru''(\hat{c}_L^*)}{u''(\hat{c}_E^*)} + (1 - \pi) \frac{1}{R} \right) + \frac{Ru''(\hat{c}_L^*)}{v''(\hat{g}^*)}} > 0.$$

Differentiating (4) with respect to  $q$  yields

$$u''(c_E^*) \frac{dc_E^*}{dq} - (1 - q) Ru''(c_L^*) \frac{dc_L^*}{dq} - q Ru''(\hat{c}_L^*) \frac{d\hat{c}_L^*}{dq} = R(u'(\hat{c}_L^*) - u'(c_L^*)).$$

Define

$$b_3 = R(u'(\hat{c}_L^*) - u'(c_L^*)) > 0.$$

The fact that this expression is strictly positive follows from  $\widehat{\mu}^* > \mu^*$  and the first-order conditions (5) and (6). Combining the previous equation with (32) and (33) yields

$$\frac{dc_E^*}{dq} = \frac{b_3}{u''(c_E^*) + (1-q)Ru''(c_L^*)b_1 + qRu''(\widehat{c}_L^*)b_2} < 0,$$

as desired. ■

**Proposition 2:**  $\widehat{g}^* < g^*$  holds for all  $q \geq 0$

*Proof:* The proof of Proposition 1 establishes that  $\mu^* < \widehat{\mu}^*$  holds for all  $q$  (see Step 2 of the proof). The first-order conditions (5) and (6) then immediately imply  $\widehat{g}^* < g^*$ . ■

**Proposition 3:**  $\rho^D > \rho^*$  holds for all  $q > 0$ . In addition,  $\rho^D$  is strictly increasing in  $q$  for  $q < (R-1)/R$  and constant for larger values of  $q$ .

*Proof:* First, since the multipliers  $\mu^*$  and  $\widehat{\mu}^*$  are always strictly positive, we clearly have

$$(1-q) + q\frac{\widehat{\mu}^*}{\mu^*} > (1-q).$$

This inequality implies

$$\frac{(1-q)\mu^* + q\widehat{\mu}^*}{R^{-1}\mu^*} > \frac{(1-q)\mu^D}{R^{-1}\mu^D}$$

or

$$\frac{u'(c_E^*)}{u'(c_L^*)} > \frac{u'(c_E^D)}{u'(c_L^D)}.$$

Because the function  $u$  is of the constant-relative-risk-aversion form, preferences over pairs  $(c_E, c_L)$  are homothetic and the above inequality implies

$$\frac{c_E^*}{c_L^*} < \frac{c_E^D}{c_L^D}.$$

Using (31), this inequality immediately implies  $\rho^* < \rho^D$ , as desired.

Next, from the first-order conditions (18) we have

$$\frac{u'(c_E^D)}{u'(c_L^D)} = \frac{1-q}{R} \quad \text{for } q < \frac{R-1}{R}.$$

Using the homotheticity of preferences, this equation implies that the ratio  $c_E^D/c_L^D$  is strictly increasing in  $q$ . Equation (31) then shows that  $\rho^D$  is also strictly increasing in  $q$  over this range.

For larger values of  $q$ , the incentive compatibility constraint  $c_E \leq c_L$  binds in the equilibrium allocation. In this case, (31) implies

$$\rho^D = \left( \pi + (1 - \pi) \frac{1}{R} \right)^{-1},$$

independent of  $q$ . ■

**Proposition 4:**  $\Phi^D \supset \Phi^*$ .

*Proof:* The proof is divided into two steps.

**Step 1:** Show that  $c_E^D > c_E^*$  holds for all  $q > 0$ . To begin, consider the first-order condition for the policy maker's choice of  $\tau$  in the discretionary regime, which is given by (20). The effect of  $\tau$  on the equilibrium value of  $c_E$  can be derived by substituting the resource constraint (16) into the first-order condition (18) and differentiating with respect to  $\tau$ ,

$$\frac{dc_E}{d\tau} = \frac{1}{\pi + (1 - \pi) \frac{u''(c_E)}{(1-q)R^2(u''(c_L))}}.$$

This expression can be used to show

$$-1 < \pi \frac{dc_E}{d\tau} < 0.$$

Combined with (20), this inequality implies

$$v'(g^D) > \mu^D - \frac{q}{1-q} \widehat{\mu}^D \tag{34}$$

whenever  $q > 0$ .

Now, suppose  $c_E^D \leq c_E^*$  held for some  $q > 0$ . Then the first-order conditions (4) and (18) would imply

$$(1 - q) \mu^D \geq (1 - q) \mu^* + q \widehat{\mu}^*. \tag{35}$$

In addition, the fact that  $1 - \pi c_E^D \geq 1 - \pi c_E^*$  holds combined with the conditions (6) and (10) would imply

$$\widehat{\mu}^D \leq \widehat{\mu}^*. \tag{36}$$



Together, (35) and (36) would imply

$$(1 - q) \mu^D \geq (1 - q) \mu^* + q \hat{\mu}^D$$

or

$$\mu^D - \frac{q}{1 - q} \hat{\mu}^D \geq \mu^*. \quad (37)$$

This inequality, combined with (34) and (5), implies  $v'(g^D) > v'(g^*)$ , or

$$g^D < g^*. \quad (38)$$

Also note that (37) implies  $\mu^D > \mu^*$ , which through (5) and (18) implies  $Ru'(c_L^D) > Ru'(c_L^*)$  or

$$c_L^D < c_L^*. \quad (39)$$

However, combining  $c_E^D \leq c_E^*$  with (38), (39), the resource constraint (16) and the equilibrium condition  $g = \tau$  shows that the resource constraint (2) is violated at the  $q$ -efficient allocation, a contradiction. Hence,  $c_E^D > c_E^*$  must hold.

**Step 2:** Show  $\Phi^D \supset \Phi^*$ . Using the fact that  $c_E^D > c_E^*$  for all  $q > 0$ , the first-order conditions (4) and (18) imply

$$(1 - q) \mu^D < (1 - q) \mu^* + q \hat{\mu}^*.$$

Similarly,  $1 - \pi c_E^D < 1 - \pi c_E^*$  implies

$$\hat{\mu}^D > \hat{\mu}^*.$$

Consider any economy in  $\Phi^*$ . The fact that  $c_E^* \geq \hat{c}_L^*$  holds is equivalent to condition (7), which can also be written as

$$(1 - q) \mu^* + q \hat{\mu}^* \leq \frac{1}{R} \hat{\mu}^*.$$

Combining the three inequalities above yields

$$(1 - q) \mu^D < \frac{1}{R} \hat{\mu}^D, \quad (40)$$

which implies  $c_E^D > \hat{c}_L^D$  and, hence, the economy is also in  $\Phi^D$ . Moreover, the fact that the inequality in (40) is strict implies that the inclusion relationship is also strict: there exist economies for which (7) is violated by a small amount, but (40) still holds. Alternatively, it is easy to find examples of economies that belong to  $\Phi^D$  but not to  $\Phi^*$ ; see Figure 2. ■

**Proposition 5:**  $\rho^{NB} < \rho^*$  holds for all  $q > 0$  and  $\rho^{NB}$  is strictly decreasing in  $q$ .

*Proof:* The proof, which is similar to that of Proposition 1, is divided into three steps.

**Step 1:** Show that  $\rho^{NB} < \rho^*$  holds. The resource constraints (2) and (3) can be written as

$$\begin{aligned} (1 - \pi) \frac{c_L}{R} &= 1 - \pi c_E - g \\ (1 - \pi) \left( \pi \widehat{c}_E + (1 - \pi) \frac{\widehat{c}_L}{R} \right) &= 1 - \pi c_E - \widehat{g}. \end{aligned}$$

Proposition 2 shows that  $\widehat{g}^* > g^*$  for all  $q$ , which implies

$$\frac{c_L^*}{R} < \pi \widehat{c}_E^* + (1 - \pi) \frac{\widehat{c}_L^*}{R}$$

or

$$1 < R\pi \frac{\widehat{c}_E^*}{c_L^*} + (1 - \pi) \frac{\widehat{c}_L^*}{c_L^*}.$$

Under a no-bailout policy,  $\widehat{g} = g$  holds by definition and we have

$$1 = R\pi \frac{\widehat{c}_E^{NB}}{c_L^{NB}} + (1 - \pi) \frac{\widehat{c}_L^{NB}}{c_L^{NB}}. \quad (41)$$

It must be the case, therefore, that at least one of the following two inequalities holds:

$$\frac{\widehat{c}_E^*}{c_L^*} < \frac{\widehat{c}_E^{NB}}{c_L^{NB}} \quad \text{or} \quad \frac{\widehat{c}_L^*}{c_L^*} < \frac{\widehat{c}_L^{NB}}{c_L^{NB}}. \quad (42)$$

If the first of these inequalities holds, then by the homotheticity of preferences we have

$$\frac{u'(\widehat{c}_E^*)}{Ru'(c_L^*)} < \frac{u'(\widehat{c}_E^{NB})}{Ru'(c_L^{NB})}$$

or

$$\frac{\widehat{\mu}^*}{\mu^*} < \frac{\widehat{\mu}^{NB}}{\mu^{NB}}. \quad (43)$$

Note that the second inequality in (42) would lead to the same conclusion. Working from (43), we have

$$(1 - q) + q \frac{\widehat{\mu}^*}{\mu^*} < (1 - q) + q \frac{\widehat{\mu}^{NB}}{\mu^{NB}}$$

or

$$\frac{(1 - q) \mu^* + q \widehat{\mu}^*}{R^{-1} \mu^*} < \frac{(1 - q) \mu^{NB} + q \widehat{\mu}^{NB}}{R^{-1} \mu^{NB}}$$

or

$$\frac{u'(c_E^*)}{u'(c_L^*)} < \frac{u'(c_E^{NB})}{u'(c_L^{NB})}.$$

Again using the homotheticity of preferences, this last inequality implies

$$\frac{c_E^*}{c_L^*} > \frac{c_E^{NB}}{c_L^{NB}}.$$

Using (31), this inequality immediately implies  $\rho^* > \rho^{NB}$ , as desired.

**Step 2:** Show that  $\mu^{NB} < \hat{\mu}^{NB}$  holds. The assumption that the coefficient of relative risk aversion in  $u$  is greater than one implies  $R\hat{c}_E^{NB} > c_L^{NB}$ . Equation (41) then implies that the ratio  $\hat{c}_L^{NB}/c_L^{NB}$  must be smaller than one, which through the first-order conditions (10) and (23) implies  $\hat{\mu}^{NB} > \mu^{NB}$ , as desired.

**Step 3:** Show that  $\rho^{NB}$  is strictly decreasing in  $q$ . Using the relationship (31), we know that  $\rho$  is strictly decreasing in  $q$  if and only if the ratio  $c_E/c_L$  is strictly decreasing in  $q$ . The first-order conditions (4) – (6) together with the resource constraints (2) and (3) implicitly define the  $q$ -efficient allocation as a function of  $q$  in a neighborhood of the solution  $\mathbf{c}^*$ . Using (2) and (24) together yields

$$\frac{dc_L^{NB}}{dq} = -b_1 \frac{dc_E^{NB}}{dq}, \quad \text{where } b_1 \equiv \frac{\pi + \frac{u''(c_E^{NB})}{v''(g^{NB})}}{(1-\pi)/R} > 0. \quad (44)$$

To show that the ratio  $c_E^{NB}/c_L^{NB}$  is strictly decreasing in  $q$ , therefore, it suffices to show that  $c_E^{NB}$  is strictly decreasing in  $q$ . Differentiating (3) and (23) with respect to  $q$  yields

$$\frac{d\hat{c}_L^{NB}}{dq} = -b_2 \frac{dc_E^{NB}}{dq}, \quad (45)$$

where

$$b_2 \equiv \frac{\pi + \frac{u''(c_E^{NB})}{v''(g^{NB})}}{(1-\pi) \left( (1-\pi)/R + \pi \frac{Ru''(\hat{c}_L^{NB})}{u''(\hat{c}_E^{NB})} \right)} > 0.$$

Differentiating (22) with respect to  $q$  yields

$$u''(c_E^{NB}) \frac{dc_E^{NB}}{dq} - (1-q) Ru''(c_L^{NB}) \frac{dc_L^{NB}}{dq} - q Ru''(\hat{c}_L^{NB}) \frac{d\hat{c}_L^{NB}}{dq} = R(u'(\hat{c}_L^{NB}) - u'(c_L^{NB})).$$

Define

$$b_3 = R(u'(\hat{c}_L^{NB}) - u'(c_L^{NB})) > 0.$$

The fact that this expression is strictly positive follows from  $\widehat{\mu}^{NB} > \mu^{NB}$  and the first-order conditions (10) and (23). Combining the previous equation with (44) and (45) yields

$$\frac{dc_E^{NB}}{dq} = \frac{b_3}{u''(c_E^{NB}) + (1-q)Ru''(c_L^{NB})b_1 + qRu''(\widehat{c}_L^{NB})b_2} < 0,$$

as desired. ■

**Proposition 6:**  $\Phi^{NB} \supset \Phi^*$ . Moreover, there exists economies in  $\Phi^{NB}$  that are not in  $\Phi^D$ .

*Proof:* The proof of the first statement is divided into three steps.

**Step 1:** Show that  $\mu^{NB} < \mu^*$  holds. Suppose this were not true. If  $\mu^{NB} \geq \mu^*$  held, then the first-order conditions for each case would imply

$$c_L^{NB} \leq c_L^*. \quad (46)$$

From the resource constraint (2), we would then have

$$1 - g^{NB} - \pi c_E^{NB} \leq 1 - g^* - \pi c_E^* < 1 - \widehat{g}^* - \pi c_E^*,$$

where the latter inequality follows from Proposition 2. The post-run resource constraint (3) (with the no-bailout restriction  $\widehat{g}^{NB} = g^{NB}$ ) would then imply

$$\pi \widehat{c}_E^{NB} + (1-\pi) \frac{\widehat{c}_L^{NB}}{R} < \pi \widehat{c}_E^* + (1-\pi) \frac{\widehat{c}_L^*}{R},$$

and, through the first-order conditions,

$$\widehat{\mu}^{NB} > \widehat{\mu}^*.$$

Combined with the supposed relationship  $\mu^{NB} \geq \mu^*$ , this last inequality would imply that for any  $q > 0$ , we must have

$$(1-q)\mu^{NB} + q\widehat{\mu}^{NB} > (1-q)\mu^* + q\widehat{\mu}^* > \mu^*,$$

where the second inequality follows from the proof of Proposition 1 (see Step 2). The first inequality on the line above implies

$$c_E^{NB} \leq c_E^*, \quad (47)$$

while the second inequality implies

$$g^{NB} < g^*. \quad (48)$$

However, combining (46), (47) and (48) would then imply that the  $q$ -efficient allocation violates the resource constraint (2), a contradiction. Hence,  $\mu^{NB} < \mu^*$  must hold.

**Step 2:** Show that  $\widehat{\mu}^{NB} > \widehat{\mu}^*$  holds. Suppose this were not true. If  $\widehat{\mu}^{NB} \leq \widehat{\mu}^*$  held, then together with the result from Step 1 it would imply

$$(1 - q)\mu^{NB} + q\widehat{\mu}^{NB} < (1 - q)\mu^* + q\widehat{\mu}^* < \widehat{\mu}^*,$$

where the second inequality again follows from Step 2 in the proof of Proposition 1. The first inequality above would imply

$$c_E^{NB} > c_E^*,$$

while the two inequalities together would imply

$$g^{NB} > \widehat{g}^*.$$

Together, these inequalities would imply

$$1 - g^{NB} - c_E^{NB} < 1 - \widehat{g}^* - \pi c_E^*.$$

Combining this inequality with the post-run resource constraint (3) would then yield

$$\pi \widehat{c}_E^{NB} + (1 - \pi) \frac{\widehat{c}_L^{NB}}{R} < \pi \widehat{c}_E^* + (1 - \pi) \frac{\widehat{c}_L^*}{R}.$$

The first-order conditions for each case would then imply

$$\widehat{\mu}^{NB} > \widehat{\mu}^*,$$

contradicting the original supposition. Hence,  $\widehat{\mu}^{NB} > \widehat{\mu}^*$  must hold.

**Step 3:** Show that  $\Phi^{NB} \supset \Phi^*$ . For any economy in  $\Phi^*$ , we know that condition (7) holds. The results of the two steps above establish that

$$\frac{\mu^{NB}}{\widehat{\mu}^{NB}} < \frac{\mu^*}{\widehat{\mu}^*}.$$

Combined with condition (7), this implies

$$\frac{\mu^{NB}}{\widehat{\mu}^{NB}} < \frac{R^{-1} - q}{1 - q}, \quad (49)$$

establishing that the economy is also in  $\Phi^{NB}$ .

Moreover, the fact that the inequality in (49) is strict implies that the inclusion relationship is also strict: there exist economies for which (7) is violated by a small amount, but (49) still holds. Alternatively, it is easy to find examples of economies that belong to  $\Phi^{NB}$  but not to  $\Phi^*$ ; see Figure 2.

Finally, Figure 2 also presents examples of economies that are in  $\Phi^{NB}$  but not in  $\Phi^D$ . ■

**Proposition 7:** For any  $(R, \pi, u, v)$ , there exists  $\bar{q} > 0$  such that  $q < \bar{q}$  and  $e \in \Phi^D$  implies both  $e \in \Phi^{NB}$  and  $W^D > W^{NB}$ .

*Proof:* For any  $(R, \pi, u, v)$ , in the limit as  $q$  goes to zero, there are no ex ante distortions and the value of  $c_E$  is the same under each of the policy regimes,

$$\lim_{q \rightarrow 0} c_E^{NB}(q) = \lim_{q \rightarrow 0} c_E^D(q) = \lim_{q \rightarrow 0} c_E^*(q).$$

However, it follows from Proposition 2 and the resource constraint (3) that  $\widehat{c}_L$  will be lower in the no-bailouts regime,

$$\lim_{q \rightarrow 0} \widehat{c}_L^{NB}(q) < \lim_{q \rightarrow 0} \widehat{c}_L^D(q) = \lim_{q \rightarrow 0} \widehat{c}_L^*(q).$$

Therefore, there exists some  $\bar{q} > 0$  such that

$$\frac{c_E^{NB}(q)}{\widehat{c}_L^{NB}(q)} > \frac{c_E^D(q)}{\widehat{c}_L^D(q)} \quad \text{for all } q < \bar{q}.$$

If  $e \in \Phi^D$  for any such value of  $q$ , then  $c_E^D(q) \geq \widehat{c}_L^D(q)$  holds by definition. The inequality above then implies  $c_E^{NB}(q) > c_E^D(q)$  and, hence,  $e \in \Phi^{NB}$  also holds, establishing the first part of the proposition.

It is straightforward to show that when  $e \in \Phi^D$  the two policy regimes yield the same equilibrium welfare in the limit

$$\lim_{q \rightarrow 0} W^{NB}(q) = \lim_{q \rightarrow 0} W^D(q).$$

When there is essentially no possibility of a crisis, there is no distortion of ex ante incentives and

both policy regimes deliver the first-best allocation of resources. The proposition will, therefore, be established if we can show that welfare initially falls faster under the no-bailouts regime as  $q$  rises, that is, if we can show

$$\lim_{q \rightarrow 0} \frac{dW^{NB}(q)}{dq} < \lim_{q \rightarrow 0} \frac{dW^D(q)}{dq}. \quad (50)$$

In the limiting case, the effect of an increase in  $q$  on equilibrium welfare in the discretionary regime can be written as

$$\lim_{q \rightarrow 0} \frac{dW^D(q)}{dq} = -(1 - \pi) u(c_L^*) - v(g^*) + (1 - \pi) (\pi u(\hat{c}_E^*) + (1 - \pi) u(\hat{c}_L^*)) + v(\hat{g}^*).$$

This expression uses the fact that the equilibrium allocation  $\mathbf{c}^D$  converges to the efficient allocation  $\mathbf{c}^*$  as  $q$  goes to zero. Under the no-bailouts regime, this derivative can be written as

$$\lim_{q \rightarrow 0} \frac{dW^{NB}(q)}{dq} = -(1 - \pi) u(c_L^*) - v(g^*) + (1 - \pi) (\pi u(\hat{c}_E^{NB}) + (1 - \pi) u(\hat{c}_L^{NB})) + v(\hat{g}^{NB}).$$

Notice that the first two terms in these derivatives are the same, but the last two terms differ. This difference reflects the fact that only the no-run component of the no-bailouts equilibrium allocation converges to  $\mathbf{c}^*$ ; the components associated with the run state are different precisely because no bailout takes place. Moreover, note that  $(\hat{c}_E^*, \hat{c}_L^*, \hat{g}^*)$  maximizes continuation utility

$$(1 - \pi) (\pi u(\hat{c}_E) + (1 - \pi) u(\hat{c}_L)) + v(\hat{g})$$

subject to the resource constraint (3), while  $(\hat{c}_E^{NB}, \hat{c}_L^{NB}, \hat{g}^{NB})$  does not. It follows that (50) holds, which establishes the result. ■

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