Bubbly Collateral and Economic Activity

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Abstract

This paper develops a model of the bubbly economy and uses it to study the effects of bailout policies. In the bubbly economy, weak enforcement institutions do not allow firms to pledge future revenues to their creditors. As a result, 'fundamental' collateral is scarce and this impairs the intermediation process that transforms savings into capital. To overcome this shortage of 'fundamental' collateral, the bubbly economy creates 'bubbly' collateral. This additional collateral supports an intricate array of intra- and inter-generational transfers that allow savings to be transformed into capital and bubbles. Swings in investor sentiment lead to fluctuations in the amount of bubbly collateral, giving rise to bubbly business cycles with very rich and complex dynamics.

Bailout policies can affect these dynamics in a variety of ways. Expected bailouts provide additional collateral and expand investment and the capital stock. Realized bailouts reduce the supply of funds and contract investment and the capital stock. Thus, bailout policies tend to foster investment and growth in normal times, but to depress investment and growth during crisis periods. We show how to design bailout policies that maximize various policy objectives.

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Over the past few years, European governments have devoted trillions of dollars to bailing out their financial institutions. Many of these bailouts have been carried out by countries that are under severe financial stress and are being forced to implement severe spending cuts in health, education and other popular programs. As we write, Spain has just received a €100 billion loan to recapitalize its banking system. The country thus follows in the steps of Ireland, which received €85 billion in 2010 to deal with the costs of government guarantees to the country's banking sector.

These are two examples of the amount of resources that governments are willing to devote in order to protect their financial institutions from further damage, but they are not the only ones. The European Financial Stability Facility, a safety net created by European countries in 2010 with resources of up to €750 billion, was intended in part to cope with the potential recapitalizations of the continent's banks. The European Central Bank, in turn, has also done its share to help troubled banking systems: between December of 2011 and February of 2012, it provided over one trillion euros of cheap long-term loans to more than one thousand eurozone banks.[1]

This large-scale use of public resources to prop up private institutions has prompted a heated debate. First, it is not clear what the purpose of these bailouts is: Do they correct an underlying a new and severe market failure? Or instead, are they just a massive re-distribution to be explained by political economy reasons? Second, less known even about their effects: Do they sustain economic activity in the short run at the expense of distorting long run incentives? Or instead, do they depress economic activity in the short run with the hope of protecting private institutions that are needed to sustain long run growth? In this paper, we develop a framework to think about these questions.

We begin with a simple observation: all of the recent large-scale bailouts can be traced to the crisis that began in 2007, which was characterized by a significant and rapid decline in asset prices in the United States and elsewhere. In previous work (Martin and Ventura 2011), we have argued that this crisis can be modeled as the bursting of a bubble, i.e. a large shock to investor sentiment that led to a drastic reduction of asset prices and wealth. Since various forms of wealth are commonly used as collateral to back financial transactions, the bursting of the bubble has drastically reduced the economy's collateral and thus the ability of financial markets to transform savings into productive capital.

[1] The situation has been somewhat similar in the United States, where the Troubled Assets Relief Program and the Term Asset Backed Lending Facility have devoted huge sums to provide funds for banks and other financial institutions.

We build on this insight to develop a model of the bubbly economy. Entrepreneurs demand funds from bankers to finance their investment projects. Bankers, in turn, demand funds from savers to finance their loans to entrepreneurs. All of this borrowing must be collateralized, that is, it must be backed by credible promises of future payments. This need not pose a problem if entrepreneurs and bankers can pledge future cash-flows as collateral. Weak enforcement institutions, however, limit the extent to which this can be done. As a consequence, the economy suffers from a shortage of collateral and depressed levels of credit and investment. We show how, in this setting, investor optimism leads to bubbles that raise the market value of firms and banks. These bubbles expand the collateral of entrepreneurs and bankers, leading to a credit and investment boom. But a change in investor sentiment might abruptly stop this virtuous cycle. Investor pessimism leads to the bursting of these bubbles, and a contraction in the market value of firms and banks. As the collateral of entrepreneurs and bankers vanishes, credit and investment collapse. The bubbly economy has entered a vicious cycle.

We follow a long tradition of modeling bubbles as pyramid schemes. These schemes are sequences of voluntary and non-negative contributions. Initiators/creators of pyramid schemes receive a pure rent since they obtain the first contribution for free. Later participants make voluntary contributions that entitle them to receive the next voluntary contributions. At first sight, it might seem far-fetched to think that there are pyramid schemes attached to firms and banks. But there are real-world situations that correspond quite well to this concept. Consider, for instance, the stock of a firm/bank that is traded at a price that exceeds its fundamental, i.e. the net present value of the dividends that this stock will generate. This "overvaluation" in stock prices might be part of an equilibrium if buyers rationally expect to sell these stocks in the future at a price that also exceeds the fundamental. Consider also credit given to a firm/bank in excess of the net present value of the cash-flows that this firm/bank will generate. This "excessive" credit might be part of an equilibrium if creditors rationally expect that the firm will be able to raise enough credit in the future to repay them. Real-world phenomenae that are often described as an "overvaluation" of stock prices and "excessive" credit can be usefully modeled as pyramid schemes.

After analyzing the role played by bubbly collateral in sustaining credit and investment, we study the role of public policy. In principle, a government would like to design a stabilization policy that insulates the economy's stock of bubbly collateral from the adverse effects of negative investor sentiment shocks. Whether or not it can do so, however, depends on the resources that it has at its disposal. Consider first the extreme case of a government that has no taxation power at all. Such a government cannot really sustain the stock of bubbly collateral except through another bubble. Without taxation, all transfers must necessarily be financed through the issue of public debt: but public debt itself is a pure bubble if it is not backed by taxes. Consider the other extreme case of a government with access to unlimited and non-distortionary taxes. Then, we show here that the government can effortlessly stabilize the stock of collateral and therefore the economy. This is the only free lunch in what follows though.

The benefits and costs of government intervention are most clearly appreciated in the intermediate case of positive, but limited and/or distortionary, taxation. In this case, a bailout policy implies a commitment by the government to transfer resources from taxpayers to creditors in the event of an adverse shock to the private sector's bubbly collateral. From an ex ante perspective, such a policy raises credit, investment, and the rate of economic growth. It essentially acts as public collateral that complements that of firms and banks, providing incentives for households and banks to expand their lending. Ex post however, when the bubble crashes and the policy is executed, the government uses taxation to divert resources away from taxpayers towards creditors. At this point, the bailout reduces the resources available for credit and investment. Thus, while a bailout policy might certainly raise average growth, it exacerbates the dynamics generated by investor sentiment shocks.

It is useful to briefly comment on the previous result. A widespread rationale for bailouts is that they stimulate economic activity by transferring funds towards those who agents that need them in order to invest. According to this rationale, the negative effects of bailouts are to be found ex ante – say, through the distortion of incentives – but its ex post effects on economic activity are positive. This is the view that transpires, for instance, in the models of bubbly liquidity like Farhi and Tirole (2011). The model developed here suggests a different view, by which bailout policies have positive effects on economic activity ex ante but are costly to execute ex post. We believe that this view resonates well with the recent events in Europe.

The model developed here builds upon previous work by Martin and Ventura (2011a, 2011b). Relative to those papers, we generalize the framework by introducing financial intermediaries. This allows us to study the interaction of fundamental and bubbly collateral at different junctions of the intermediation chain. We also provide a thorough analysis of the different effects of bailout policies. Naturally, this work is also related to the wider literature that has studied the effects of bubbles in the presence of financial frictions: (i) unlike us, Caballero and Krishnamurthy (2006), Farhi and Tirole (2011) and Miao and Wang (2011) focus on the role of bubbles as a useful source of liquidity;[2] (ii) like us, Kocherlakota (2009) focuses on the role of bubbles as collateral or net worth; and (iii) unlike us, Ventura (2011) focuses on the effects of bubbles on the cost of capital. Finally, our model is also related to the vast work on macroeconomic models with financial frictions, in which asset prices play an important role in determining the level of financial intermediation and economic activity.[3] Our theory differs from these models in that asset prices are not only a channel through which traditional or fundamental shocks are transmitted, but they are also the source of shocks themselves.[4]

The paper is or organized as follows. Section 1 develops the basic model of the bubbly economy with entrepreneurs and savers. The former have access to the production technology, while the latter do not. Section 2 discusses the effects of bailout policies in this basic setup. Sections 3 extends the basic setup by introducing bankers, and explores the effects of bailouts in this extended model. Section 4 extends the basic setup by allowing for fundamental collateral, and it shows how fundamental and bubbly collateral interact with one another. Section 5 concludes.

1 The bubbly economy

The bubbly economy is inhabited by a sequence of equal-sized and overlapping generations of young and old. Time starts in period $t = 0$ and then goes on forever. The key feature of the bubbly economy is that weak enforcement institutions do not allow firms to pledge future profits to their creditors and, as a result, there is no 'fundamental' collateral. Despite this, intermediation takes place if there is enough 'bubbly' collateral.

The bubbly economy does not experience technology or preference shocks, but it displays stochastic equilibria with bubble or investor sentiment shocks. Formally, we define h_t as the realization of the bubble shock in period t; h^t as a history of bubble shocks until period t, i.e. $h^t = \{h_0, h_1, ..., h_t\}$; and H_t as the set of all possible histories up to period t, i.e. $h^t \in H_t$. We shall later provide a formal description of the bubble shock in terms of the variables of the model.

[2] There is, of course, a long tradition of papers that view fiat money as a bubble. Indeed, Samuelson (1958) adopted this interpretation. For a recent paper that also emphasizes the liquidity-enhancing role of fiat money in the presence of financial frictions, see Kiyotaki and Moore (2008).

[3] Here we are referring to the huge macroeconomic literature on the financial accelerator that originated with the seminal contributions by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

[4] There is also a well-established literature that analyzes the effects of bailouts. We will include a thorough discussion of this literature in a future draft of this paper.

1.1 Individual maximization

Each generation contains two types, $i \in \{S, E\}$, which we refer to as savers and entrepreneurs. The objective of both types is to maximize their utility U_t^i, which equals expected old-age consumption:

$$U_t^i = E_t C_{t+1}^i \quad \text{for } i \in \{S, E\} \tag{1}$$

where C_{t+1}^i is the old-age consumption of type i of generation t. All variables are indexed by history. For instance, we should write U_{t,h^t}^i instead of U_t^i. To reduce notation, however, we omit the history index whenever this is not confusing. Throughout, we define $E_t\{\cdot\}$ as the conditional expectation operator. The sub-index t means that this expectation is conditional on reaching period t and history h^t.

Savers supply one unit of labor during youth, receive a wage W_t and save it. One option is to purchase credit contracts that offer a, possibly contingent, gross return of R_{t+1} units of output in period $t+1$ for each unit of output in period t. We refer to the average or expected return to these contracts, $E_t R_{t+1}$, as the interest rate. A second option is to build inventories. These yield one unit of output in period $t+1$ for every unit of output stored in period t. Thus, the intertemporal budget constraint of the representative saver is given by:

$$C_{t+1}^S = R_{t+1} \cdot (W_t - I_t) + I_t \tag{2}$$

where I_t are inventories, with $I_t \geq 0$. Equation (2) says that the consumption of old savers is the return to their credit contracts plus inventories.

Savers maximize utility (1) subject to the budget constraint (2). Since they are risk-neutral, they choose the savings option(s) with the highest expected return:

$$I_t \begin{cases} = 0 & \text{if } E_t R_{t+1} > 1 \\ \in [0, W_t] & \text{if } E_t R_{t+1} = 1 \end{cases} \tag{3}$$

If $E_t R_{t+1} > 1$, all savings are used to purchase credit contracts. If $E_t R_{t+1} = 1$, savers are indifferent between credit contracts and inventories, and any portfolio that combines both is consistent with maximization. We shall see later that $E_t R_{t+1} < 1$ never happens in equilibrium.

Entrepreneurs derive all their income from managing firms. During youth, they borrow to purchase existing firms and to produce/purchase capital for them.[5] Capital is produced with consumption goods one-to-one, and it fully depreciates after being used in the production of consumption goods. In old age, entrepreneurs hire workers and produce. Let Y_{t+1}, K_{t+1} and N_{t+1} be the production, capital stock, and labor of the firm(s) owned by the representative entrepreneur of generation t. Then, we have that:

$$Y_{t+1} = K_{t+1}^{\alpha} \cdot \left(\gamma^{t+1} \cdot N_{t+1}\right)^{1-\alpha} \tag{4}$$

with $\alpha \in (0,1)$ and $\gamma > 1$. This is a standard Cobb-Douglas technology with labor productivity that grows at a constant rate, γ. After production takes place, entrepreneurs sell their firms, pay their debts and consume.

To finance their activities, entrepreneurs sell credit contracts. These contracts must be collateralized, that is, they must be backed by credible promises of future payments. This brings us to the key friction that underlies all the analysis of this paper: enforcement institutions are weak. In particular, entrepreneurs can hide/consume their production before enforcement institutions can take over their firms. As a result, young entrepreneurs face the following credit constraint:

$$R_{t+1} \cdot (V_t + K_{t+1}) \leq V_{t+1} \tag{5}$$

where V_t is the price of the firms purchased by the representative entrepreneur. Equation (5) applies for all t and $h^t \in H_t$, and it simply says that the financing obtained in period t, i.e. $V_t + K_{t+1}$, must be such that promised interest payments, i.e. $R_{t+1} \cdot (V_t + K_{t+1})$; do not exceed the price of his/her firms, i.e. V_{t+1}. If promised interest payments exceeded this price, it would be optimal in old age to default on these payments and hide/consume all production before enforcement institutions take over the firms.

We can now write the intertemporal budget constraint of the representative entrepreneur as follows:

$$C_{t+1}^{E} = Y_{t+1} - W_{t+1} \cdot N_{t+1} + V_{t+1} - R_{t+1} \cdot (V_t + K_{t+1}) \tag{6}$$

Equation (6) says that the consumption of old entrepreneurs equals production net of labor costs, $Y_{t+1} - W_{t+1} \cdot N_{t+1}$, plus the proceeds from selling their firms, V_{t+1}, minus the repayment of loan

[5] We allow also entrepreneurs to start new firms at cost zero. But we solve the representative entrepreneur's problem assuming he/she never chooses to do so. Later on, we will indeed show that this is the case in equilibrium.

contracts, $R_{t+1} \cdot (V_t + K_{t+1})$.

Entrepreneurs maximize utility (1) subject to the technological constraint (4), the credit constraint (5), and the budget constraint (6). Solving this problem, we find the following demands for labor and capital:

$$N_{t+1} = \left(\frac{1-\alpha}{W_{t+1}}\right)^{\frac{1}{\alpha}} \cdot \gamma^{\frac{1-\alpha}{\alpha}\cdot(t+1)} \cdot K_{t+1} \tag{7}$$

$$K_{t+1} \begin{cases} = \dfrac{E_t V_{t+1}}{E_t R_{t+1}} - V_t & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} > E_t R_{t+1} \\ \in \left[0, \dfrac{E_t V_{t+1}}{E_t R_{t+1}} - V_t\right] & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = E_t R_{t+1} \end{cases} \tag{8}$$

Equations (7) and (8) show the optimal choices of labor and capital, respectively.[6] As usual, the maximizing choice of labor is obtained by equalizing the marginal product of labor to the wage. The maximizing choice of capital depends on the return to capital and the interest rate. If the return to capital is above the interest rate, the entrepreneur wants to borrow as much as possible to add capital and the credit constraint is binds. If the return to capital equals the interest rate, the entrepreneur is indifferent about how much capital to produce and the credit constraint does not bind. We shall see later that the return to capital is never below the interest rate in equilibrium.

1.2 Markets and prices

Individuals interact within markets. In the labor market, old entrepreneurs hire young savers. Since the supply for labor is one and the demand is given by Equation (7), market clearing implies the following wage:

$$W_t = (1-\alpha) \cdot \gamma^{(1-\alpha)\cdot t} \cdot K_t^{\alpha} \tag{9}$$

Equation (9) says that, as usual, the equilibrium wage equals the marginal product of labor evaluated at the economy-wide aggregate capital-labor ratio.

In the credit market, young entrepreneurs demand credit to purchase their firms and invest and

[6] Here we have assumed that labor and capital choices do not affect the value of the firm, i.e. $\dfrac{\partial V_{t+1}}{\partial N_{t+1}} = \dfrac{\partial V_{t+1}}{\partial K_{t+1}} = 0$. This seems a natural assumption since employment relationships last one period and capital depreciates fully in production. But there are some subtle issues involved as we explain in section 1.2.

young savers supply this credit. The interest rate is determined by matching demand and supply:[7]

$$E_t R_{t+1} = \begin{cases} \min\left\{\dfrac{E_t V_{t+1}}{W_t}, \alpha \cdot \left(\dfrac{W_t - V_t}{\gamma^{t+1}}\right)^{\alpha-1}\right\} & \text{if } W_t < \min\left\{E_t V_{t+1}, \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t\right\} \\ 1 & \text{if } W_t \geq \min\left\{E_t V_{t+1}, \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t\right\} \end{cases} \tag{10}$$

Equation (10) says that there are two possible situations in the credit market. If savings are low, inventories are not used and the interest rate is above one. If entrepreneurs are not credit constrained, the interest rate equals the marginal product of capital. If entrepreneurs are credit constrained, the interest rate is below the marginal product of capital. If savings are high, inventories are used in equilibrium and the interest rate is one.

In the stock market, young entrepreneurs purchase firms from old ones. It is commonplace to impose the restriction that firm prices reflect the market value of the capital in them. Since capital fully depreciates in production, this would imply that the price of a firm is zero. But this restriction is unjustified and we shall not impose it here. Instead, we allow for the possibility of bubbles in firm prices, i.e. $V_t \geq 0$. Formally, we model bubbles as pyramid schemes that entrepreneurs attach to firms. When a young entrepreneur purchases firms, he/she pays V_t for the stock of old bubbles attached to these firms by previous entrepreneurs. Young entrepreneurs attach new bubbles to these firms before selling them in period $t+1$. The expected bubble $E_t V_{t+1}$ reflects the combined value of old and new bubbles. Since old bubbles must grow at the rate of interest, the discounted value of the new bubbles is $\dfrac{E_t V_{t+1}}{E_t R_{t+1}} - V_t$. This is the rent that young entrepreneurs obtain by initiating/creating new pyramid schemes, and we refer to it as bubble creation.[8]

We impose two restrictions on bubbles. The first one is that bubbles must be non-negative: $V_t \geq 0$. This seems quite natural and would follow, for instance, from the assumption that old

[7] Using Equations (8) and (9), we find the demand for credit:

$$K_{t+1} + V_t = \min\left\{\frac{E_t V_{t+1}}{E_t R_{t+1}}, \left(\frac{\alpha}{E_t R_{t+1}}\right)^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t\right\}$$

Using Equation (3), we also find the supply of credit:

$$W_t - I_t \begin{cases} = W_t & \text{if } E_t R_{t+1} > 1 \\ \in [0, W_t] & \text{if } E_t R_{t+1} = 1 \end{cases}$$

Matching demand and supply we find the equilibrium interest rate.

[8] Why should old bubbles grow at the rate of interest? If old bubbles grew slower, owning firms with bubbles would be costly and entrepreneurs would start new firms that have no bubbles attached to them. This would bid the price of old bubbles down until their growth rate equals the interest rate. If old bubbles grew faster, entrepreneurs would make profits when buying a firm and would want to buy all existing firms. This would bid the price of old bubbles down until their growth rate equals the interest rate.

entrepreneurs can close the firm at zero cost. The second restriction is that bubbles are independent of the investment and employment policies of the firm. This is a restriction on the sort of market expectations that are allowed. We adopt it because it implies that $\frac{\partial V_{t+1}}{\partial N_{t+1}} = \frac{\partial V_{t+1}}{\partial K_{t+1}} = 0$ and this simplifies the analysis.

1.3 Competitive equilibrium

We can now define the bubble shock as $h_t = \{V_t, E_t V_{t+1}\}$. It is useful to refer to V_t and $E_t V_{t+1}$ as the realized and expected bubbles, respectively. The 'realized' bubble is the aggregate value in period t of all the bubbles attached to firms by entrepreneurs of earlier generations. Fluctuations in V_t reflect fluctuations in these old bubbles. The 'expected' bubble is the expected value in period $t+1$ of (i) all the old bubbles attached by entrepreneurs of earlier generations, i.e. $V_t \cdot E_t R_{t+1}$, plus (ii) all the new bubbles attached by entrepreneurs of generation t , i.e. $E_t V_{t+1} - V_t \cdot E_t R_{t+1}$. Fluctuations in $E_t V_{t+1}$ therefore reflect fluctuations in both old and new bubbles. As a result, knowing the realization of V_t might not be enough to compute $E_t V_{t+1}$. This is why we treat the latter as a potentially different shock.

As usual, we use lowercase letters to denote variables per efficient worker. So, for instance, k_t and v_t are the capital stock and the bubble per efficient worker, i.e. $k_t \equiv \gamma^{-t} \cdot K_t$ and $v_t \equiv \gamma^{-t} \cdot V_t$. With this notation at hand, we can now state the law of motion of k_t as follows:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha) \cdot k_t^{\alpha} - v_t}{\gamma} & \text{if } \ k_t < \left(\dfrac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t v_{t+1}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t \right\} \right)^{\frac{1}{\alpha}} \\ \min\left\{ E_t v_{t+1} - \dfrac{v_t}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } \ k_t \geq \left(\dfrac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t v_{t+1}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t \right\} \right)^{\frac{1}{\alpha}} \end{cases} \tag{11}$$

The law of motion of the capital stock in Equation (11) has two regions. If the capital stock at time t is not too large, savings are small and they can be intermediated. In this range, the law of motion is concave and the interest rate is higher than one. If the capital stock at time t is sufficiently large, savings are instead too high and they cannot be intermediated. At this point, the law of motion becomes flat and the interest rate is one.

A competitive equilibrium of the bubbly economy consists of a sequence for $\{v_t, E_t v_{t+1}, k_t\}_{t=0}^{\infty}$, such that Equation (11) holds with $v_t \geq 0$ and $k_t \geq 0$, and for all t and $h^t \in H_t$. It is straightforward to see that the bubbly economy has many equilibria. To construct them, we first propose a bubble v_t such that $v_t \geq 0$ for all t and $h^t \in H_t$. Then, we use this bubble together with Equation (11) to determine the dynamics of the capital stock from a given initial condition $k_0 > 0$ for all t and

$h^t \in H_t$. If we find that $k_t \geq 0$ for all $t > 0$ and $h^t \in H_t$, then the proposed bubble is indeed an equilibrium of the bubbly economy. If $k_t < 0$ for some $t > 0$ and $h^t > H_t$, then the proposed bubble is not an equilibrium.

1.4 Bubbly business cycles

In the bubbly economy there are three assets or vehicles to transfer consumption across periods: capital, bubbles and inventories. Capital is the best one of them. But the bubbly economy has no fundamental collateral, making it impossible to convert all savings into capital. Here is where bubbles come in. The bubbly economy uses bubbly collateral to support an intricate array of intra- and inter-generational transfers that affect how savings are transformed into capital, bubbles and inventories. The expected bubble in period $t+1$, provides collateral and allows funds to be transferred from young savers to young entrepreneurs in the form of credit. The funds transferred through the credit market are $K_{t+1} + V_t$. A fraction of these funds are kept by young entrepreneurs and used to finance capital accumulation, i.e. K_{t+1}. The rest of these funds, namely, V_t, are transferred to old entrepreneurs as payment for their firms. Old entrepreneurs then transfer these funds to old savers to cancel the credit contracts that they sold them in period $t-1$. This array of transfers is needed to sustain credit and capital accumulation. Large expected bubbles foster capital accumulation because they provide collateral that is needed to intermediate funds between savers and entrepreneurs. Large realized bubbles depress capital accumulation because they absorb funds that could be used for investment. As bubbles fluctuate, so do these transfers and this gives rise to bubbly business cycles with very rich and complex dynamics.

Perhaps the best way to illustrate how the bubbly economy works is to examine a couple of specific equilibria or examples, which we invoke throughout the paper.

Example 1 (calm bubble) *The calm bubble never changes. Thus,* $v_t = v < \dfrac{\gamma}{\gamma - 1} \cdot \alpha^{\frac{1}{1-\alpha}}$ *for all* t*.*

The calm bubble is an equilibrium of the bubbly economy if the initial capital stock is large enough. We assume this. With this bubble, Equation (11) becomes

$$k_{t+1} = \min \left\{ \frac{\gamma - 1}{\gamma} \cdot v, \frac{(1-\alpha) \cdot k_t^{\alpha} - v}{\gamma} \right\} \tag{12}$$

The law of motion of Equation (12) has a simple shape. Whenever $k_t < \hat{k}_t \equiv \left(\dfrac{\gamma \cdot v}{1 - \alpha} \right)^{\frac{1}{\alpha}}$, the economy's bubbly collateral suffices to intermediate all wages and the limits to investment come from wages themselves: in this range, the law of motion is increasing and concave in k_t and the equilibrium interest rate is higher than one. Once k_t exceeds $\hat{k}$, the economy's bubbly collateral is insufficient to intermediate all wages and inventories are built in equilibrium: in this range, the law of motion is flat and the equilibrium interest rate is equal to one. Figure 1 below depicts the law of motion of the capital stock with the stable bubble.

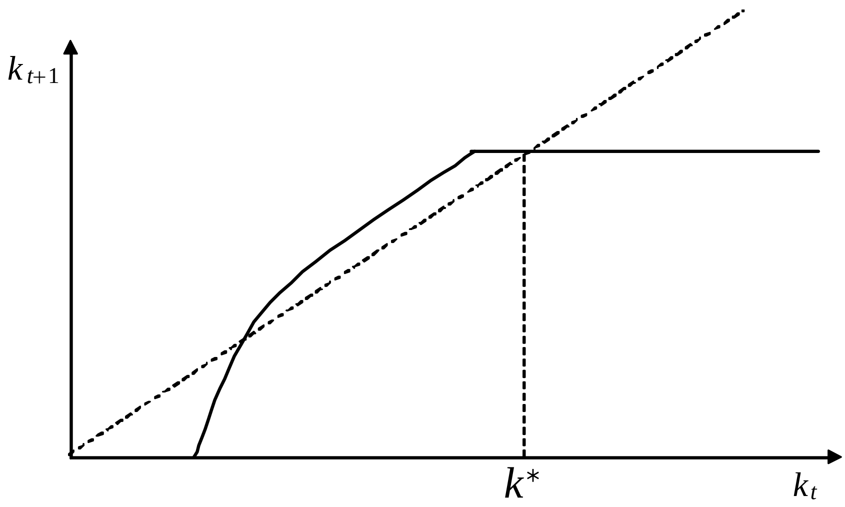

Figure 1

With the calm bubble, the bubbly economy has two steady states.[9] The lower one always corresponds to the increasing segment of the law of motion and it is unstable. The higher one is instead stable, and it could lie on the increasing or on the flat segment of the law of motion. Because it is stable, we focus throughout on this last steady state and call it k^*. Figure 1 depicts the case in which $k^* > \hat{k}$, so that there is insufficient bubbly collateral in steady state and some savings are used to build inventories.[10]

The calm bubble is useful to understand how changes in the size of the bubble affect the economy. The key observation is that this depends on the capital stock. If $k_t \leq \hat{k}$, the economy finds itself in the increasing segment of the law of motion and all wages are already being intermediated.

[9] There is one case in which the economy has a unique steady state, but it is knife-edge and we do not consider it here.

[10] For the calm bubble to be an equilibrium of the bubbly economy the initial capital stock must be weakly higher than the lower steady state. Otherwise, the economy would be expected to shrink and the capital stock would turn negative, which is not possible in equilibrium.

Increases in the bubble are therefore unable to raise credit, but they do reduce the share of credit that can be devoted to investment. Hence, increases in v crowd out investment and generate a downward shift in the law of motion. If instead $k_t > \hat{k}$, the economy finds itself in the flat segment of the law of motion. In this region, inventories are being built in equilibrium. Increases in v raise intermediation and credit, expanding investment and generating an upward shift in the law of motion.

This discussion lays the basis of our next example:

Example 2 (moody bubble) *The moody bubble fluctuates between an optimistic (O) and a pessimistic (P) state. Let* $z_t \in \{O, P\}$ *be investor sentiment, with* $\Pr[z_{t+1} = z_t] = 1 - \pi$ *and* $\Pr[z_{t+1} \neq z_t] = \pi$ *for all* t *and* $h^t \in H_t$*, where* $1 \leq \pi\gamma \leq \dfrac{\gamma - 1}{2}$*. Then,* $v_t = v < \dfrac{\gamma}{(1-\pi) \cdot \gamma - 1} \cdot \alpha^{\frac{1}{1-\alpha}}$ *if* $z_t = P$*, and* $v_t = 0$ *if* $z_t = P$*.*

With the moody bubble, the law of motion of the capital stock and the interest rate both depend on the state of the economy:

$$k_{t+1} = \begin{cases} \min\left\{\dfrac{(1-\pi)\cdot\gamma - 1}{\gamma} \cdot v, \dfrac{(1-\alpha)\cdot k_t^{\alpha} - v}{\gamma}\right\} & \text{if } z_t = O \\ \min\left\{\dfrac{\pi\cdot\gamma - 1}{\gamma} \cdot v, \dfrac{(1-\alpha)\cdot k_t^{\alpha}}{\gamma}\right\} & \text{if } z_t = P \end{cases}, \tag{13}$$

Equation (13) shows that, in each state, the law of motion of the capital stock is analogous to that of the calm bubble. Whether under optimism or pessimism, the law of motion displays an increasing segment as long as the economy's bubbly collateral exceeds wages: beyond this point, the law of motion becomes flat as inventories are built in equilibrium. One interesting aspect of this economy is that optimism has an ambiguous effects on capital accumulation and growth. Optimism today raises the price of firms and, for a given level of intermediation, these higher prices reduce investment. But, if it is persistent, optimism today raises the economy's expected collateral as well: as long as credit is limited by collateral, this expands intermediation and investment. Whether or not optimism can raise investment therefore depends on the capital stock and on the persistence of z_t. Our assumption that $\pi \leq \dfrac{\gamma - 1}{2\gamma}$, however, guarantees that z_t is sufficiently persistent and that optimism raises intermediation and investment, which seems the more relevant case. Figure 2 below shows the optimist and pessimist laws of motion under this assumption.

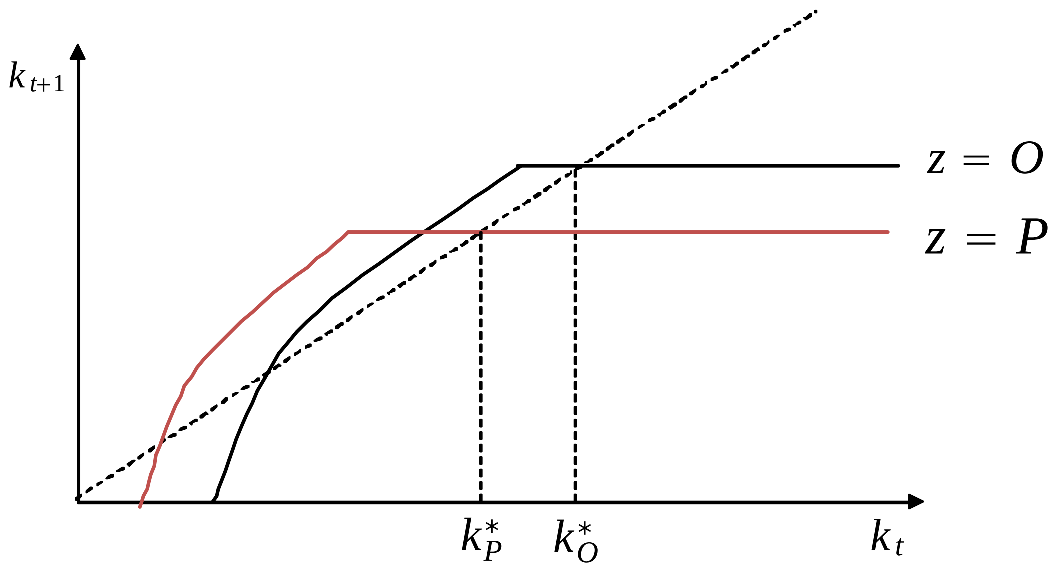

Figure 2

Under optimism or pessimism, the law of motion of the bubbly economy has two steady states. Exactly as before, we focus throughout on the higher of these steady states, which are stable, and denote them by k_z^* for $z \in \{O, P\}$. We assume throughout, as depicted in Figure 2, that k_z^* lies in the flat segment of the law of motion for $z \in \{O, P\}$, so that both steady states are characterized by insufficient bubbly collateral and by the use of resources to build inventories.

With the moody bubble, the bubbly economy nicely illustrates how bubble or investor sentiment shocks can lead to economic fluctuations. When investors are optimistic, expected collateral is high and so is intermediation and investment. The capital stock per efficient worker grows towards k_O^*: as wages increase, collateral becomes less abundant and eventually it turns scarce. In this case, the expansion stops in the flat region of the law of motion, after the economy has moved along the concave region. When investors become pessimistic, however, firm prices collapse, collateral shrinks and the economy enters a recession. At this time, investment falls and the capital stock per efficient worker declines to k_P^*. It is interesting to note that, during the recession, expected collateral is positive even though the price of firms is zero: the reason for this is that collateral depends on the *expected* price of firms, which is positive as there is a likelihood of entering an optimistic state in the future. Thus, bubbly collateral sustains positive levels of intermediation and investment even in the absence of an actual bubble.

The calm and moody bubbles represent different equilibria of the bubbly economy. There are many others, of course. Our discussion so far has provided a positive description of how these equilibria work. But how do they rank in terms of welfare? We turn to this issue next, characterizing the set of Pareto optimal equilibria.

1.5 Pareto optimal equilibria

It is straightforward to show that the set of Constrained Pareto Optimal (CPO) equilibria is that in which the expected bubble is large enough to ensure that all savings are intermediated in all periods and histories:

$$\gamma \cdot E_t v_{t+1} \geq (1-\alpha) \cdot k_t^{\alpha} \quad \text{for all } t \text{ and } h^t \in H_t \tag{14}$$

In other words, an equilibrium is CPO if inventories are never built. In this case, the relevant region of the law of motion is always the increasing and concave one. To prove this result one simply has to show that (i) if the above condition fails, it is possible to implement a pareto improvement; and (ii) if the above condition holds, it is not possible to implement a pareto improvement. The intuition for this result is simple and it follows from the observation that inventories can be interpreted as dynamically inefficient investments. In a CPO equilibrium bubbles eliminate these inefficient investments through two channels. The first one, more direct and classical, is that bubbles need to be financed by savings and this crowds out inventories. The second one, which we emphasize more here, is that bubbles provide collateral and allow for investment: as this investment also needs to be financed by savings, it further crowds out inventories.

CPO equilibria are not necessarily characterized by a high stock of capital and consumption. The CPO equilibrium that maximizes the capital stock features an interest rate equal to one in all periods and histories, i.e. $E_t R_{t+1} = 1$ for all t and $h^t \in H_t$. This steady state has the smallest possible bubble that is compatible with all savings being intermediated. This minimizes the transfer to old savers and maximizes the transfer to young entrepreneurs. Thus, this bubble maximizes investment and the capital stock. The CPO equilibrium that minimizes the average capital stock in steady state features instead an interest rate that is arbitrarily close to the growth rate in all periods and histories, i.e. $E_t R_{t+1} \approx \gamma \cdot \left(\frac{k_{t+1}}{k_t}\right)^{\alpha}$ for all t and $h^t \in H_t$. This equilibrium has the largest possible bubble. This maximizes the transfer to old savers and essentially eliminates the transfer to young entrepreneurs. It therefore minimizes investment and the capital stock.[11]

[11] Why is such a bad-looking equilibrium a CPO? Because the only way to increase the capital stock is by reducing the consumption of the old and, thus, this cannot constitute a pareto improvement.

We can use the calm bubble to illustrate these points. In that economy, CPO equilibria are those that never 'visit' the flat segment of the law of motion. Clearly, this requires that $k^* \leq \hat{k}$. Among these equilibria, the one in which $k^* = \hat{k}$ maximizes the steady state stock of capital: in this case, the steady state lies exactly at the kink of the law of motion, with $E_t R_{t+1} = 1$.[12] If instead $k^* < \hat{k}$, the steady state stock of capital lies below its maximum: in this case, the steady state lies on the increasing segment of the law of motion, with $E_t R_{t+1} > 1$.

CPO equilibria are not characterized by the absence of business cycles either. But these cycles have some special features. Since all savings are intermediated all the time, business cycles result from fluctuations in the fraction of savings that are used to finance bubbles and capital. When the realized bubble is large, investment drops and the capital stock declines. When the realized bubble is small, investment increases and the capital stock grows. The magnitude and frequency of these business cycles depends on the properties of the equilibrium bubble. But in all CPO equilibria, bubbly business cycles are characterized by a negative comovement between bubbles and investment.

This negative comovement seems counterfactual and suggests that, if this theory has any chance to capture relevant aspects of reality, business cycles must not be CPO. The moody bubble discussed above, for instance, features a positive comovement between bubbles and investment. But this requires that the economy operate some times in the flat segment of the law of motion, i.e. it requires that business cycles not be CPO. And, as we have seen, with the moody bubble, the bubbly economy does operate in the flat segment of the law of motion during recessions and sometimes even during expansions. It is the expected bubble that generates the positive comovement between bubbles and investment. When the expected bubble expands, credit expands and so does investment and growth. When the expected bubble contracts, credit contracts and so does investment and growth.

Equilibria with large fluctuations in the expected bubble give theory the best chance to explain real-world events. But these equilibria are not CPO and this suggests a role for policy in them. We turn to this issue next.

[12] This is the steady state when the bubble equals

$$\left(\frac{\gamma-1}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1-\alpha}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}.$$

2 Bailout policies

The bubbly economy has scarcity of collateral and it is natural to ask whether public policy can be used to relieve this scarcity. The answer to this question clearly depends on the policy instruments that the government has at its disposal. We want to think of a government that raises resources through taxation and uses them to back promises made by entrepreneurs. In this manner, the government effectively provides collateral to entrepreneurs and it is in principle able to boost intermediation, investment and growth.

Before formally introducing policy into our analysis, an important disclaimer is warranted. A crucial decision in the study of government intervention is how to model government objectives. We shall follow a long tradition in macroeconomics of dodging this issue by not specifying any objectives for the government. Our justification for this choice is that we are not attempting to conduct a positive analysis of a specific historical episode, but we are instead exploring the theoretical effects of alternative policy scenarios. Nevertheless, we will pay special attention to bailout policies that implement CPO allocations.

2.1 The bubbly economy with bailouts

We introduce a government that can intervene in credit markets in order to provide collateral to firms. We think of this policy as a bailout scheme, in which the government provides resources that can be used to cancel entrepreneurial credit contracts. Formally, we assume that the government promises to subsidize the sale of firms with S_t units (possibly contingent) of the economy's consumption good to prop up the collateral of entrepreneurs in period t. We refer to S_t as a bailout, since it helps to cover the losses that savers experience when the credit contracts made to entrepreneurs are defaulted upon.

The key question, of course, is how to finance these bailouts. Consider that the government has the power to tax young entrepreneurs. If we use $\bar{X}_t$ to denote these tax revenues, the bailout scheme requires that $S_t \leq \bar{X}_t$ in all periods t. In each period, young entrepreneurs are taxed and the revenues from taxation are transferred to old entrepreneurs. Old entrepreneurs, in turn, use these transfers to cancel their credit contracts. From a static perspective, the old gain from the scheme because they receive payments from it whereas young entrepreneurs lose from the scheme because they contribute to it. From a dynamic perspective, however, young entrepreneurs also have a benefit because they expect to receive bailouts during old age.

The introduction of bailouts does not affect the problem of savers, but it does affect the problem of entrepreneurs. In particular, the credit constraint faced by entrepreneurs is now given by,

$$R_{t+1} \cdot \left(V_t + K_{t+1} + \bar{X}_t\right) \leq V_{t+1} + S_{t+1}, \tag{15}$$

as they must borrow also to pay taxes during youth but they can pledge bailouts during old age. The intertemporal budget constraint of the representative entrepreneur becomes:

$$C_{t+1}^{E} = Y_{t+1} - W_{t+1} \cdot N_{t+1} + V_{t+1} + S_{t+1} - R_{t+1} \cdot \left(V_t + K_{t+1} + \bar{X}_t\right). \tag{16}$$

Entrepreneurs maximize utility (1) subject to technology, the new credit constraint (15), and the new budget constraint (16). Solving this problem, we find that the demand for labor is still given by Equation (7) whereas the demand for capital is given by:

$$K_{t+1} \begin{cases} = \dfrac{E_t\left\{V_{t+1} + S_{t+1}\right\}}{E_t R_{t+1}} - \left(V_t + \bar{X}_t\right) & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} > E_t R_{t+1} \\ \in \left[0, \dfrac{E_t\left\{V_{t+1} + S_{t+1}\right\}}{E_t R_{t+1}} - \left(V_t + \bar{X}_t\right)\right] & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = E_t R_{t+1} \end{cases}. \tag{17}$$

Bailout schemes have two effects on the demand for capital. On the one hand, they raise demand for capital by providing entrepreneurs with bailouts during old age, against which they can borrow: on the other hand, though, they reduce the demand for capital because entrepreneurs must contribute to them during youth.

Bailout schemes therefore have no effect on the labor market, as wages are still determined by Equation (9), nor do they affect the stock market. It is through the credit market that these schemes affect the economy, where the equilibrium interest rate becomes:

$$E_t R_{t+1} = \begin{cases} \min\left\{\dfrac{E_t\left\{V_{t+1} + S_{t+1}\right\}}{W_t}, \alpha \cdot \left(\dfrac{W_t - V_t - \bar{X}_t}{\gamma^{t+1}}\right)^{\alpha-1}\right\} & \text{if } W_t < \tilde{W}_t \\ 1 & \text{if } W_t \geq \tilde{W}_t \end{cases}, \tag{18}$$

with $\tilde{W}_t = \min\left\{E_t\left\{V_{t+1} + S_{t+1}\right\}, \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t + \bar{X}_t\right\}$. If $W_t \geq \tilde{W}_t$ and inventories are built in equilibrium, the interest rate is determined by the return to inventories and it is unaffected by bailouts. If $W_t < \tilde{W}_t$, however, bailout schemes raise the equilibrium interest rate. The reason is essentially that bailout schemes raise the demand for funds, as they allow constrained entrepreneurs to expand their borrowing, and they reduce the maximum amount of funds available for investment, as they require some of the income of the young to be taxed.

We can summarize the workings of the bubbly economy with bailouts through the following equilibrium conditions:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha) \cdot k_t^{\alpha} - \bar{x}_t - v_t}{\gamma} & \text{if } \ k_t < \tilde{k}_t \\ \min\left\{ E_t\left\{v_{t+1} + s_{t+1}\right\} - \dfrac{(v_t + \bar{x}_t)}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } \ k_t \geq \tilde{k}_t \end{cases}, \tag{19}$$

$$s_t = \bar{x}_t, \tag{20}$$

where $\tilde{k}_t = \left(\dfrac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t + \bar{x}_t \right\} \right)^{\frac{1}{\alpha}}$.

Equation (19) illustrates the effect of a bailout scheme on capital accumulation. Expected bailouts s_{t+1} complement private collateral and, as long as entrepreneurs are credit constrained, they expand credit, investment and the capital stock. But current bailouts are financed through taxation on young entrepreneurs $\bar{x}_t$, which may reduce capital accumulation. If entrepreneurs are constrained, capital accumulation falls because taxation reduces the demand for credit. But even if entrepreneurs are unconstrained, capital accumulation may fall because taxation reduces total supply of credit by the young. This trade-off between future and current bailouts defines the effects of bailout schemes in the bubbly economy.

A competitive equilibrium of the bubbly economy with bailouts consists of a stochastic process for $\{v_t, E_t v_{t+1}, k_t\}_{t=0}^{\infty}$, and a bailout scheme $\{s_t\}_{t=0}^{\infty}$, such that Equations (19) and (20) hold with $v_t \geq 0$ and $k_t \geq 0$, and for all t and $h^t \in H_t$. As before, the economy may have multiple equilibria. To construct any one of them, once again, we first propose a bubble v_t such that $v_t \geq 0$ for all t and $h^t \in H_t$. Then, we use Equation (19) together with a bailout policy satisfying Equation (20) and an initial condition k_0 to determine the dynamics of the capital stock for all t and $h^t \in H_t$. If we find that $k_t \geq 0$ for all t and $h^t \in H_t$, then the proposed bubble is indeed an equilibrium of the bubbly economy.

In the bubbly economy with bailouts, the competitive equilibrium cannot be defined independently of the bailout scheme. In fact, one can think of different schemes as selecting or implementing different equilibrium allocations. This interaction between bailout schemes and the equilibrium allocation is central to our analysis, and we now explore it in more detail.

2.2 Bubbly business cycles with bailouts

To simplify the discussion that follows, we focus throughout on situations in which entrepreneurs are credit constrained. With this assumption at hand, we can rewrite Equation (19) as follows:

$$k_{t+1} = \min\left\{E_t\left\{v_{t+1} + s_{t+1}\right\}, \frac{(1-\alpha)\cdot k_t^{\alpha}}{\gamma}\right\} - \frac{v_t + s_t}{\gamma}, \tag{21}$$

where we have already substituted that $s_t = \overline{x}_t$ for all t.

Equation (21) illustrates the set of equilibrium transfers implemented in the bubbly economy with bailouts. The expected bubble and bailouts in period $t+1$ provide collateral and, by doing so, they allow funds to be transferred from young savers to young entrepreneurs in the form of credit. The funds transferred are $\min\left\{\gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, (1-\alpha)\cdot k_t^{\alpha}\right\}$. A fraction of these funds, namely $\min\left\{\gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, (1-\alpha)\cdot k_t^{\alpha}\right\} - (v_t + s_t)$, are kept by young entrepreneurs and used to finance capital accumulation. The rest of these funds are used by young entrepreneurs to purchase firms and/or to pay taxes. In either case, these funds end up in the hands of old entrepreneurs and they are used to cancel credit contracts with old savers.

The law of motion of Equation (21) also illustrates how different bailout schemes affect the equilibrium allocation. To see this, consider an economy in which the stochastic process for the bubble shock is given by v_t for all t, while the bailout scheme specifies bailouts s_t for all t. The competitive equilibrium of this economy is identical to the competitive equilibrium of an alternative economy without bailouts but with a bubble shock $\hat{v}_t = v_t + s_t$ for all t. A corollary of this statement is that, through the design of an appropriate bailout scheme, it is possible to replicate any equilibrium of the bubbly economy.

We can illustrate these points with the help of the quiet and moody bubbles of Section 1.4. In the quiet bubble, with $v_t = v$ for all t, the equilibrium is not CPO if the economy visits the flat part of the law of motion. This could happen if v is too small, so that the economy's bubbly collateral is insufficient to intermediate all wages in the steady state. In this case, the government could design a bailout scheme to replicate a CPO allocation: in particular, we focus on the scheme that maximizes the value of the capital stock, in which $k^* = \hat{k}$ and $E_t R_{t+1} = 1$ in steady state. To implement this allocation, the expected collateral of entrepreneurs must equal aggregate wages in steady state, so that $\hat{v} = \dfrac{w}{\gamma}$. This can be attained by setting bailouts $s = \hat{v} - v$, where

$$\hat{v} = \left(\frac{\gamma - 1}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\alpha}{\gamma}\right)^{\frac{1}{1-\alpha}}. \tag{22}$$

Through this bailout scheme, the government guarantees that the expected collateral of entrepreneurs equals $\gamma \cdot \hat{v}$ in steady-state. Figure 3 illustrates the macroeconomic effects of the scheme. The dashed lines represent the law of motion of the capital stock when $v_t = v$, in the absence of bailouts. The solid line depicts the law of motion of the capital stock after the adoption of the scheme described above. As can be seen in the figure, bailouts raise the steady state stock of capital and they eliminate the use of inventories.

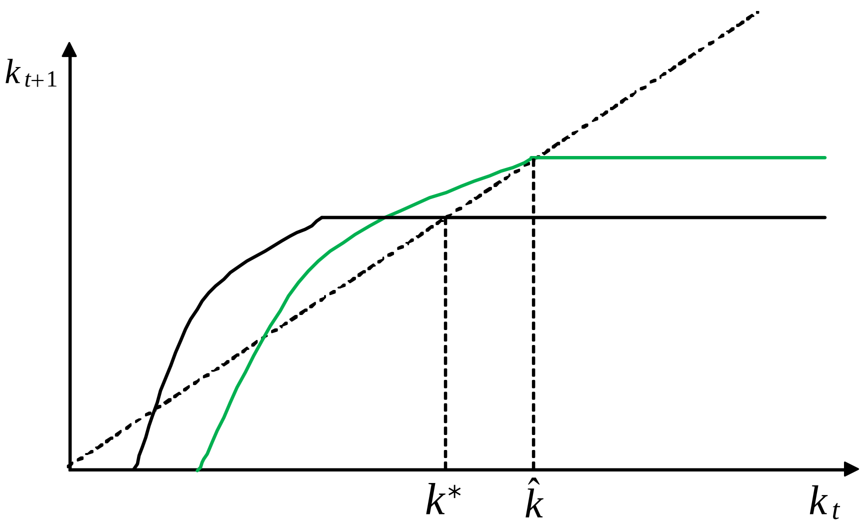

Figure 3

We can perform a similar analysis for the moody bubble, which oscillates between v and zero. In the absence of bailouts, this economy experiences fluctuations, inventories are built along some histories and the competitive equilibrium is not a CPO allocation. Here, once again, the government can design a bailout scheme to maximize the steady-state stock of capital. To do so, it now has to set contingent bailouts, which vary according to the state of the economy: $s_P = \hat{v} > \hat{v} - v = s_O$. This scheme guarantees that the expected collateral of entrepreneurs equals $\gamma \cdot \hat{v}$ regardless of the actual value taken by the bubble. When investors are pessimistic and expected collateral is low, the government complements it by raising expected bailouts in the future: at the same time, it taxes young entrepreneurs in order to compensate old individuals for losses on their credit contracts.

When investors are optimistic and the bubble equals v, collateral is higher and so the government bailout is lower. In both cases, the government bailout scheme manages to completely eliminate inventories.

Figure 4 below depicts the effects of this bailout scheme. The dashed lines represent the laws of motion of the capital stock for $z \in \{O, P\}$, in the absence of bailouts. The solid line depicts the unique law of motion of the capital stock after the adoption of the scheme described above. As can be seen in the figure, bailouts raise the steady state stock of capital, they eliminate the use of inventories and they completely stabilize investment and output.

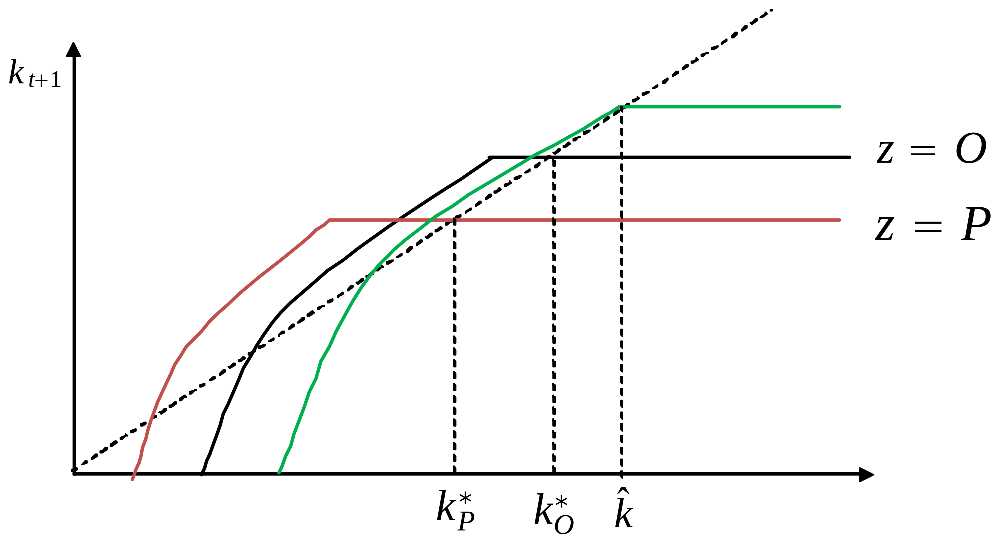

Figure 4

At first glance, the idea that the government stabilizes credit through expected and current bailouts might seem strange or impractical. But it is actually quite natural if one thinks of this policy as an insurance fund for entrepreneurs, who contribute during their youth and – should a crisis occur – are assisted by the government during old age. An alternative interpretation of the policy is that it amounts to a price stabilization scheme, by which the government intervenes in the market for firms in order to counteract fluctuations in their value. In our baseline model, the market price of all firms in the economy in a given period t is given by v_t. Through the stabilization policy described above, the government essentially guarantees that it will intervene to sustain a market price of $\hat{v}$: whenever the realized price of firms is lower than this, the difference is transferred to old entrepreneurs in the form of bailouts. The high expected "price" of firms enables entrepreneurs to borrow more during youth, thereby expanding investment and the capital stock.

But the current "price" of firms also increases as a consequence of the policy, which relies on taxing young entrepreneurs, and this has a contractionary effect on economy activity.

2.3 Mandatory vs. voluntary bailouts

The bubbly economy seems to call for government intervention. By properly designing a bailout scheme, as we have argued, the government can provide collateral, maximize the steady-state stock of capital and fully stabilize the economy. By participating in such a scheme, all generations stand to gain. Both in the quiet and in the moody bubble examples analyzed above, government intervention provides an expected transfer of resources to young entrepreneurs and raises investment regardless of the state.

But if all entrepreneurs benefit from joining the bailout scheme, why is the government needed at all? Can't entrepreneurs run their own scheme, to which each generation may contribute in exchange for future bailouts? This scheme would look exactly like the one analyzed above except in one important respect: contributions would be voluntary rather than mandatory. If we use x_t to denote the contribution of the young at time t, a voluntary scheme thus entails $x_t \in [0, \bar{x}_t]$; in exchange for this contribution, generation t would be entitled to a bailout financed pro-rata with the contribution of generation $t+1$. Relative to the mandatory scheme, in which each generation is forced to set $x_t = \bar{x}_t$, contributions to the voluntary scheme are freely chosen by each generation.

Consider a voluntary scheme like the one just described and assume initially that $E_t R_{t+1} = 1$. In this case, each generation t will set $x_t = \min\{\gamma \cdot E_t x_{t+1}, \bar{x}_t\}$. The reason is simple: as long as they don't exceed expected bailouts, contributions are profitable for entrepreneurs and they will be as high as possible. But this expression for the equilibrium contributions encapsulates the shortcomings of a voluntary scheme. As long as each generation expects future contributions to be high, it will chose to make high contributions as well. Nothing guarantees that this will be the case, however. Once a generation expects future contributions to be low, it will abstain from contributing itself and the scheme will fail.

What guarantees the success of a bailout scheme, then, is that it be mandatory. It is then that contributions cannot be affected by expectations or sentiment. Formally, we can say that a mandatory scheme implements specific contributions and bailouts for each realization of the bubble shocks v_t and $E_t v_{t+1}$. Hence, it can be designed to implement a particular equilibrium allocation. In voluntary bailout schemes, instead, the contributions and bailouts themselves are also subject to shocks. No matter how well designed, the equilibrium allocation that such schemes ultimately implement depends – exactly as in the case of market bubbles – on sentiment.

This discussion suggests that voluntary schemes may collapse at the whim of entrepreneurs. But what about schemes that are only partially mandatory, i.e. schemes in which part of the contribution is mandatory but part of it is not? To think about this, consider that the government can impose a (small) mandatory contribution of $\underline{x}$ from each generation, so that contributions must now satisfy $x_t \in [\underline{x}, \bar{x}_t]$. In this case, it follows that $x_t = \min\{\underline{x}, \min\{\gamma \cdot E_t x_{t+1}, \bar{x}_t\}\}$, since there is a minimum contribution that everyone must comply with. But note that the new constraint on the contribution can never be binding: since $E_t x_{t+1} \geq \underline{x}$, generation t always wants to contribute more than it is obliged to do, setting $x_t \geq \gamma \cdot \bar{x}_t$. Since the same must be true for generation $t+1$, however, it follows that $E_t x_{t+1} \geq \gamma \cdot \underline{x}$ as well. Generation t anticipates this, however, so that its contribution will be even higher, satisfying $x_t \geq \gamma^2 \cdot \underline{x}$; by repeatedly iterating this reasoning, we obtain that $x_t = \min\{\gamma \cdot E_t \bar{x}_{t+1}, \bar{x}_t\}$. That is, a partially mandatory scheme ends up behaving exactly like a fully mandatory scheme! All generations set their contributions to the maximum and this does not depend on sentiment shocks. Partially mandatory schemes can thus be designed to implement any CPO: in particular, they can implement the CPO allocation that maximizes the capital stock, validating our initial assumption that $E_t R_{t+1} = 1$.[13]

The previous argument seems surprising. A partial mandatory contribution, it says, is all it takes to sustain a bailout scheme. Moreover, since the argument has been developed for an arbitrary value of $\underline{x}$, the mandatory contribution can be as small as desired. As long as the government can mandate some positive contribution, it can use it to sustain a full bailout scheme! The trick is that the mandatory contribution is small relative to the economy, but the economy grows at a rate that is higher than the interest rate: this implies that, no matter how small future mandatory contributions are, their discounted value suffices to sustain large voluntary contributions today. Thus, unlike a voluntary bailout scheme, a partially mandatory scheme is not a bubble and it is sustained by the discounted value of future contributions.

In the bubbly economy, the scarcity of collateral creates a role for government intervention. We have shown how bailout schemes can be used to complement entrepreneurial collateral, and argued that these schemes can be sustained as long as the government can mandate some level of contributions. Of course, these results have been obtained under heavily stylized assumptions regarding the government's actions. Most notably, we have assumed throughout that the govern-

[13] Of course, the same reasoning would have been valid for any value of $E_t R_{t+1}$ as long as it does not exceed γ. But we know that it never does in a steady state of the bubbly economy.

ment taxes and distributes bailouts in a fully efficient manner. Moreover, we have assumed that the government can commit to extending a bailout scheme indefinitely into the future. We now relax these assumptions.

2.4 Limits on bailouts

We extend our analysis of bailout schemes by relaxing two restrictions that we have imposed so far. First, we allow for the possibility that schemes be inefficient, in the sense that they require $\lambda \geq 1$ units of contributions per unit of bailouts paid to entrepreneurs. This parameter can be directly interpreted as a cost of government intervention in terms of corruption or waste. Or it can be interpreted more subtly as representing the informational costs of implementing a bailout policy, in the sense that only a fraction $\frac{1}{\lambda}$ of bailouts end up in the hands of their intended recipients, i.e. entrepreneurs. Second, we assume that there is a probability $1-\mu \geq 0$ that promised bailouts are not paid out in each period. This can be interpreted as a limitation on the government's ability to commit to the scheme. This limited commitment could arise, for instance, because there is a probability that the young run the government and decide not to contribute to the scheme.

Under these two assumptions, Equations (19) and (20) become

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^{\alpha} - \bar{x}_t - v_t}{\gamma} & \text{if } k_t < \tilde{k}_t \\ \min\left\{ E_t\left\{v_{t+1} + s_{t+1}\right\} - \dfrac{(v_t + \bar{x}_t)}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } k_t \geq \tilde{k}_t \end{cases}, \tag{23}$$

$$\lambda \cdot s_t = \bar{x}_t, \tag{24}$$

where $\tilde{k}_t = \left(\dfrac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t + \bar{x}_t \right\} \right)^{\frac{1}{\alpha}}$. Clearly, both $\mu < 1$ and $\lambda > 1$ reduce the effectiveness of bailout schemes in transferring resources to young entrepreneurs. In this regard, they reduce the set of equilibria that can be implemented through bailouts.

Given a scheme $\{s_t, \bar{x}_t\}_{t=0}^{\infty}$, low values of μ reduce the expected collateral provided by bailouts, while high values of λ raise the taxes required to sustain them. As long as $\mu \cdot \gamma > \lambda$, however, it is still possible to implement bailout schemes that raise intermediation and investment in steady state. But the maximum level of capital that can be sustained through such schemes is lower than what the market could sustain through bubbly collateral. Once $\mu \cdot \gamma < \lambda$, though, bailout schemes become so ineffective that they can no longer be used to raise intermediation and investment in steady state. Under those conditions, any scheme must necessarily transfer resources away from entrepreneurs, as it generates less than one unit of expected collateral for each unit of taxation. Obviously, no such scheme can be sustained on a voluntary basis.

This discussion shows that whether or not bailout schemes can be used to alleviate the economy's scarcity of collateral depends on the characteristics of the corresponding government. Governments that are efficient and that can credibly commit to the future bailouts implied by any scheme will be able to complement the private collateral of entrepreneurs. In contrast, governments that are inefficient and that are unable to commit to delivering future bailouts as promised will be unable to provide collateral. If they attempt to do so nonetheless, the result will be a decrease in the collateral of entrepreneurs and a fall in intermediation and investment.

2.5 Public debt

We have thus far assumed that the only source of funding for bailout schemes are the contributions of young entrepreneurs. This seems like an odd restriction, given that the rationale for these schemes is to complement the collateral of entrepreneurs. If bailout schemes were financed through public debt, for instance, wouldn't they be more effective in channeling resources to entrepreneurs? By issuing public debt, after all, the government could use the resources of savers to fund entrepreneurial bailouts. We explore this possibility in our baseline scenario of efficient bailout schemes, with $\lambda = \mu = 1$.

The two sets of agents directly affected by government debt are savers and the government. If we use D_t to denote the debt payments made in period t, the budget constraint of the representative saver becomes

$$C_{t+1}^S = R_{t+1} \cdot \left(W_t - I_t - \frac{E_t D_{t+1}}{E_t R_{t+1}} \right) + D_{t+1} + I_t.$$

The optimal choice of inventories can therefore be expressed as

$$I_t \begin{cases} = 0 & \text{if } E_t R_{t+1} > 1 \\ \in \left[0, W_t - \dfrac{E_t D_{t+1}}{E_t R_{t+1}} \right] & \text{if } E_t R_{t+1} = 1 \end{cases},$$

so that, whenever $E_t R_{t+1} > 1$, all savings are used to purchase credit contracts or public debt. By issuing this debt, of course, the government can finance part of its expenses, and its budget constraint is given by

$$S_t + D_t \geq \bar{X}_t + \frac{E_t D_{t+1}}{E_t R_{t+1}}. \tag{25}$$

Equation (25) simply notes that, once debt is issued, government expenditures comprise both bailouts and debt payments, whereas government revenues include both taxes and issues of new debt.

The introduction of public debt does not affect the labor market, where the equilibrium wage is still given by Equation (9), nor does it affect the stock market. In the credit market, however, the interest rate is now given by:

$$E_t R_{t+1} = \begin{cases} \min\left\{ \frac{E_t\{V_{t+1} + S_{t+1} + D_{t+1}\}}{W_t}, \alpha \cdot \left(\frac{W_t - V_t - S_t - D_t}{\gamma^{t+1}} \right)^{\alpha - 1} \right\} & \text{if } W_t < \tilde{W}_t \\ 1 & \text{if } W_t \geq \tilde{W}_t \end{cases}, \quad (26)$$

where $\tilde{W}_t = \min\left\{ E_t\{V_{t+1} + S_{t+1} + D_{t+1}\}, \gamma^{t+1} \cdot \alpha^{\frac{1}{1-\alpha}} + V_t + S_t + D_t \right\}$ and we have used the government budget constraint (25) to substitute for $\bar{X}_t$. Equation (26) shows that the introduction of public debt weakly raises the equilibrium interest rate. If $W_t < \tilde{W}_t$ and aggregate wages are being fully intermediated, public debt raises the interest rate through two channels. First, newly issued debt $E_t D_{t+1}$ is purchased by young savers and this reduces the resources available for investment. Second, debt payments D_t must be financed either through taxation or through issues of new debt, and they also reduce the resources available for investment. As usual, once $W_t \geq \tilde{W}_t$ the interest rate is determined by the return to inventories and it is unaffected by public debt.

We can summarize the workings of the bubbly economy with public debt through the law of motion

$$k_{t+1} = \begin{cases} \frac{(1-\alpha) \cdot k_t^{\alpha} - s_t - d_t - v_t}{\gamma} & \text{if } k_t < \tilde{k}_t \\ \min\left\{ E_t\{v_{t+1} + s_{t+1}\} - \frac{(v_t + s_t)}{\gamma} + E_t d_{t+1} - \frac{d_t}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } k_t \geq \tilde{k}_t \end{cases}, \quad (27)$$

where $\tilde{k}_t = \left(\frac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t\{v_{t+1} + s_{t+1} + d_{t+1}\}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t + s_t + d_t \right\} \right)^{\frac{1}{\alpha}}$ and, once more, we have used the government's budget constraint to substitute for $\bar{x}_t$. Equation (27) illustrates the effects of public debt on capital accumulation for a given bailout scheme $\{s_t\}_{t=0}^{\infty}$. If $k_t \geq \tilde{k}_t$ and entrepreneurial collateral is scarce, the use of debt financing increases capital accumulation: as long as $E_t d_{t+1} > \frac{d_t}{\gamma}$, bailouts are partially financed through debt and there is less need for entrepreneurial taxes. If $k_t < \tilde{k}_t$ and aggregate wages are fully intermediated, however, debt payments divert resources away from investment and they slow down capital accumulation.

Although debt affects capital accumulation for a given bailout scheme $\{s_t\}_{t=0}^{\infty}$, it does not change the set of allocations that can be implemented through bailouts. Equation (27) also provides a clear illustration of this: any sequence of bailouts $\{s_t\}_{t=0}^{\infty}$ and debt payments $\{d_t\}_{t=0}^{\infty}$ is formally equivalent to an alternative sequence of bailouts $\{s_t'\}_{t=0}^{\infty}$, with $s_t' = s_t + d_t$, that is financed solely through taxation. The only real difference between a bailout scheme financed through taxation and an equivalent scheme financed through debt is the voluntary nature of the demand for public debt. Because of this, and for essentially the same reasons that we discussed in Section 2.3, a scheme financed through debt is prone to sentiment shocks in a way that a scheme financed through taxation is not.

3 Financial intermediaries

We have thus far considered the bubbly economy without financial intermediaries, in which savers lend directly to entrepreneurs. Although it provides a useful benchmark to study the role of bubbly collateral, that model misses an important ingredient: financial intermediaries. We now introduce such intermediaries and allow them to play a crucial role in the credit market, borrowing from savers and lending to entrepreneurs. Like entrepreneurs, intermediaries must collateralize their borrowing; like that of entrepreneurs, their collateral may be bubbly. The main message of this section is that, in the bubbly economy with intermediaries, it is not only the overall size of the bubble that matters, but also its distribution between intermediaries and entrepreneurs.

3.1 The bubbly economy with financial intermediaries

We introduce a new type of agent, bankers, which are denoted by B. Bankers have the same preferences as savers and entrepreneurs, but they derive all their income from managing a bank. Just as entrepreneurs are the subset of individuals that have access to the production technology, one can think of bankers as the subset of individuals that have access to a screening or monitoring technology that is necessary to provide credit profitably.

During youth, bankers raise deposits and use them to finance the purchase of their banks and loans to entrepreneurs. In old age, bankers collect their loans, pay their deposits, sell their bank and then consume. Deposit contracts must be collateralized but, once again, enforcement institutions might be weak and, in the event of default, these institutions are not able to grab loan repayments.

As a result, bankers operate under the following credit constraint:

$$R^B_{t+1} \cdot \left(V^B_t + L_t + \bar{X}^B_t\right) \leq V^B_{t+1} \tag{28}$$

where V^B_t is price of the banks purchased by the representative banker, L_t is the total amount of loan contracts bought from entrepreneurs, and $\bar{X}^B_t$ are the taxes that bankers must pay. Equation (28) says that the the representative banker can pledge only the price of the bank. This must be true in all periods and histories.

The intertemporal budget constraint of the representative banker is given by:

$$C^B_{t+1} = \left(R^E_{t+1} - R^B_{t+1}\right) \cdot L_t + V^B_{t+1} + S^B_{t+1} - R^B_{t+1} \cdot \left(V^B_t + \bar{X}^B_t\right) \tag{29}$$

Equation (29) says that the consumption of old bankers equals the profits/losses from their loan positions, $\left(R^E_{t+1} - R^B_{t+1}\right) \cdot L_t$, plus the capital gains from holding the banks, $V^B_{t+1} - R^B_{t+1} \cdot V^B_t$, minus fiscal transfers, i.e. $S^B_{t+1} - R^B_{t+1} \cdot \bar{X}^B_t$.

Banks maximize utility (1) subject to their credit and budget constraints (28) and (29). This implies the following loan supply:

$$L_t \begin{cases} = \dfrac{E_t\left\{V^B_{t+1} + S^B_{t+1}\right\}}{E_t R^B_{t+1}} - \left(V^B_t + \bar{X}^B_t\right) & \text{if } E_t R^B_{t+1} > E_t R^E_{t+1} \\ \in \left[0, \dfrac{E_t\left\{V^B_{t+1} + S^B_{t+1}\right\}}{E_t R^B_{t+1}} - \left(V^B_t + \bar{X}^B_t\right)\right] & \text{if } E_t R^E_{t+1} = E_t R^E_{t+1} \end{cases} \tag{30}$$

Equation (30) shows the optimal choice of loans. The maximizing choice of loans depends on the interest rate spread, i.e. $E_t R^E_{t+1} - E_t R^B_{t+1}$. If this spread is positive, the banker wants to lend as much as possible and the credit constraint binds. If the spread is zero, the banker is indifferent about how much to lend to produce and the credit constraint does not bind. We shall see later that the spread is never negative in equilibrium.

The presence of banks do not affect savers and entrepreneurs, other than now they interact with one another through banks. Optimal policies regarding inventories and capital choice thus become

$$I_t \begin{cases} = 0 & \text{if } E_t R^B_{t+1} > 1 \\ \in [0, W_t] & \text{if } E_t R^B_{t+1} = 1 \end{cases}, \tag{31}$$

and

$$K_{t+1}\begin{cases} = \dfrac{E_t\left\{V_{t+1}^E+S_{t+1}^E\right\}}{E_tR_{t+1}^E}-\left(V_t^E+\bar{X}_t^E\right) & \text{if } \alpha\cdot\left(\dfrac{1-\alpha}{\gamma^{-(t+1)}\cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}}>E_tR_{t+1}^E \\ \in\left[0,\dfrac{E_t\left\{V_{t+1}^E+S_{t+1}^E\right\}}{E_tR_{t+1}^E}-\left(V_t^E+\bar{X}_t^E\right)\right] & \text{if } \alpha\cdot\left(\dfrac{1-\alpha}{\gamma^{-(t+1)}\cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}}=E_tR_{t+1}^E \end{cases}. \tag{32}$$

Equations (31) and (32) are almost identical to Equations (3) and (8). The only difference is that now the interest rates paid by entrepreneurs might not coincide with those received by savers, as banks do charge an spread. Another minor, cosmetic change is that now we refer to the price of firms as V_{t+1}^E to distinguish it from the price of banks, which we denote as V_{t+1}^B. The same applies to transfer variables.

The presence of bankers does not affect the labor market and the wage is still determined by Equation (9). It does affect the stock and the credit markets, however. The stock market is trivially affected because both banks and firms are traded in it. Young entrepreneurs purchase firms from old entrepreneurs at a price of V_t^E, while young bankers purchase banks from old bankers at a price of V_t^B. As before, we impose the condition that $V_t^E\geq 0$ and $V_t^B\geq 0$ for all t.

But it is in the credit market, which is now characterized by two interest rates, $E_tR_{t+1}^E$ and $E_tR_{t+1}^B$, where the introduction of banks has the greatest effect. Equilibrium requires that $V_t^E+\bar{X}_t^E+K_{t+1}=L_t$ and $V_{t+1}^B+\bar{X}_{t+1}^B+L_t=W_t-I_t$. There are three possibilities depending on how much collateral there is and on how it is distributed between entrepreneurs and bankers:

1. Both entrepreneurs and bankers have enough collateral: $E_tR_{t+1}^E>E_tR_{t+1}^B>1$. In this case, we have that $E_tR_{t+1}^E$ and $E_tR_{t+1}^B$ are determined as follows:

$$\min\left\{\frac{E_t\left\{V_{t+1}^E+S_{t+1}^E\right\}}{E_tR_{t+1}^E},\left(\frac{\alpha}{E_tR_{t+1}^E}\right)^{\frac{1}{1-\alpha}}\cdot\gamma^{t+1}+V_t^E+\bar{X}_t^E\right\}=\frac{E_t\left\{V_{t+1}^B+S_{t+1}^B\right\}}{E_tR_{t+1}^B}-(V_t^B+\bar{X}_t^B), \tag{33}$$

and

$$\frac{E_t\left\{V_{t+1}^B+S_{t+1}^B\right\}}{E_tR_{t+1}^B}=W_t. \tag{34}$$

Equation (33) says that entrepreneurs have enough collateral to borrow the maximum amount of loans that bankers can supply, while Equation (34) says that bankers have enough collateral to fully absorb the economy's aggregate wages as deposits. Equation (33) implies that there is a spread between the rate on loans $E_tR_{t+1}^E$ and the rate on deposits $E_tR_{t+1}^B$, and Equation

(34) implies that there is a spread between the rate on deposits $E_t R^B_{t+1}$ and the return to inventories. This means, in particular, that there are no inventories and savings are allocated as follows:

$$I_t = 0, \tag{35}$$

$$K_{t+1} = W_t - (V_t^B + \bar{X}_t^B) - \left(V_t^E + \bar{X}_t^E\right). \tag{36}$$

2. Entrepreneurs have enough collateral but bankers do not: $E_t R^E_{t+1} > E_t R^B_{t+1} = 1$. In this case, we have that $E_t R^E_{t+1}$ is determined by:

$$\min\left\{\frac{E_t\left\{V^E_{t+1} + S^E_{t+1}\right\}}{E_t R^E_{t+1}}, \left(\frac{\alpha}{E_t R^E_{t+1}}\right)^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t^E + \bar{X}_t^E\right\} = E_t\left\{V^B_{t+1} + S^B_{t+1}\right\} - (V_t^B + \bar{X}_t^B). \tag{37}$$

Equation (37) says that, as in the previous case, entrepreneurs have enough collateral to borrow the maximum amount of loans that bankers can supply. In this case, however, bankers do not have enough collateral to fully absorb the economy's aggregate wages as deposits. Hence, there is a spread between $E_t R^E_{t+1}$ and $E_t R^B_{t+1}$, but the latter equals one. Inventories are built in equilibrium and savings are allocated as follows:

$$I_t = W_t - E_t\left\{V^B_{t+1} + S^B_{t+1}\right\}, \tag{38}$$

and

$$K_{t+1} = E_t\left\{V^B_{t+1} + S^B_{t+1}\right\} - (V_t^B + \bar{X}_t^B) - \left(V_t^E + \bar{X}_t^E\right). \tag{39}$$

3. Banks have enough collateral but entrepreneurs do not: $E_t R^E_{t+1} = E_t R^B_{t+1} = 1$. In this case, bankers would have enough collateral to absorb the economy's aggregate wages as deposits, but entrepreneurs do not have enough collateral to borrow these funds. Hence, bankers only take in deposits what they can lend to entrepreneurs. Inventories are build in equilibrium, and savings are allocated a follows:

$$I_t = W_t - \min\left\{E_t\left\{V^E_{t+1} + S^E_{t+1}\right\}, \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t^E + \bar{X}_t^E\right\}, \tag{40}$$

and

$$K_{t+1} = \min\left\{E_t\left\{V^E_{t+1} + S^E_{t+1}\right\} - \left(V_t^E + \bar{X}_t^E\right), \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1}\right\}. \tag{41}$$

These three cases clearly illustrate what we already mentioned: in the economy with financial intermediaries, the total amount of collateral held by entrepreneurs and bankers is important, but so is the distribution of collateral between both types of agents. This becomes clearer once we characterize the competitive equilibria of this economy.

3.2 Competitive Equilibrium

We can now define the bubble shock as $h_t = \left\{V_t^E, V_t^B, E_t V_{t+1}^E, E_t V_{t+1}^B\right\}$. Naturally, the shock now includes the realized and expected bubbles in both, the entrepreneurial and banking sectors. As before, we can write the law of motion of the system as follows:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^{\alpha} - \left(v_t^B + \bar{x}_t^B\right) - \left(v_t^E + \bar{x}_t^E\right)}{\gamma} & \text{if } k_t < \tilde{k}_t \\ \min\left\{E_t\left\{v_{t+1}^E + s_{t+1}^E\right\} - \dfrac{v_t^E + \bar{x}_t^E}{\gamma}, E_t\left\{v_{t+1}^B + s_{t+1}^B\right\} - \dfrac{v_t^B + \bar{x}_t^B}{\gamma} - \dfrac{v_t^E + \bar{x}_t^E}{\gamma}, \alpha^{\frac{1}{1-\alpha}}\right\} & \text{if } k_t \geq \tilde{k}_t \end{cases}, \tag{42}$$

where

$$\tilde{k} = \left(\frac{1}{1-\alpha}\cdot\min\left\{\gamma\cdot E_t\left\{v_{t+1}^E + s_{t+1}^E\right\}, \gamma\cdot E_t\left\{v_{t+1}^B + s_{t+1}^B\right\} - v_t^B - \bar{x}_t^B, \gamma\cdot\alpha^{\frac{1}{1-\alpha}} + v_t^B + \bar{x}_t^B + v_t^E + \bar{x}_t^E\right\}\right)^{\frac{1}{\alpha}}.$$

The law of motion of the capital stock in Equation (42) has the usual two regions. If the capital stock at time t is not too large, savings are small and there is enough bank and entrepreneurial collateral to intermediate them fully. In this range, the law of motion is concave and the interest rate is higher than one. If the capital stock at time t is sufficiently large, savings are instead too high and they cannot be intermediated. The only novelty relative to the basic law of motion of Equation (11) is that savings may fail to be intermediated even if entrepreneurial collateral is high, simply because the collateral of banks is not. At this point, the law of motion becomes flat and the interest rate is one.

We can return to our examples of the quiet and moody bubbles in order to illustrate the interaction between entrepreneurial and bank collateral. We do so by combining them in the following example:

Example 3 (double bubble) *The double bubble combines a quiet bubble in the entrepreneurial sector with a moody bubble in the banking sector. The quiet entrepreneurial bubble never changes,* $v_t^E = v^E < \dfrac{\gamma}{\gamma-1}\cdot\alpha^{\frac{1}{1-\alpha}}$. *The moody banking bubble banking instead fluctuates between an optimistic (O) and a pessimistic (P) state. If* $z_t \in \{O, P\}$ *denotes the state, it is assumed that* $\Pr\left[z_{t+1} \neq z_t\right] =$

π for all t and $h^t \in H_t$, where $1 \leq \pi\gamma \leq \frac{\gamma-1}{2}$. Then, $v_t^B = v^B$ if $z_t = O$, where $\frac{v^B}{v^E} \in \left(\frac{\gamma}{(1-\pi)\cdot\gamma - 1}, \frac{1}{\pi}\right)$, and $v_t^B = 0$ if $z_t = P$.

With the double bubble and in the absence of any bailout schemes, the law of motion of the capital stock depends on the state of the economy:

$$k_{t+1} = \begin{cases} \min\left\{\frac{\gamma-1}{\gamma}\cdot v^E, \frac{(1-\alpha)\cdot k_t^\alpha - v^E - v^B}{\gamma}\right\} & \text{if } z_t = O \\ \min\left\{\pi\cdot v^B - \frac{v^E}{\gamma}, \frac{(1-\alpha)\cdot k_t^\alpha - v^E}{\gamma}\right\} & \text{if } z_t = P \end{cases}, \tag{43}$$

The law of motion of the capital stock under the double bubble is very similar to the one that emerges under the moody bubble. The only difference is that the relevant collateral constraint now varies with the state of nature. When investors are optimistic, the bubbly collateral of banks is so high that it would allow them to extend more loans than what entrepreneurs can borrow: in this situation, which corresponds to case 3 above, the limit to capital accumulation is the scarcity of entrepreneurial collateral. When investors are instead pessimistic, the bubbly collateral of banks is low and it becomes the factor that constrains capital accumulation. This situation corresponds to case 2 above: even though the bubbly collateral of entrepreneurs could in principle enable them to expand their investment, banks are unable to provide them with the necessary loans.

Clearly, everything we have said regarding bailout schemes extends directly to the double bubble. The only novel aspect is that the effects of bailout schemes depend not only on the total amount of collateral that they provide, but also on how this collateral is distributed among bankers and entrepreneurs. Consider, for instance, the implementation of a scheme that affects only entrepreneurs, taxing them during their youth and bailing them out during old age. Whereas such a scheme might be useful in times of optimism, it is obvious from Equation (43) that it is contractionary in pessimistic times! The reason is that, in those times, banks are already extending as many loans as they can: any scheme targeted exclusively to entrepreneurs will require them to pay taxes during their youth but it will have no effect on the total supply of loans. The result must therefore be a fall in investment.

It is simple to characterize the bailout scheme that maximizes the steady-state stock of capital in the double bubble example. Such a scheme equalizes entrepreneurial demand for loans with the supply offered by banks, i.e. it sets $E_t\left\{v^E + s^E\right\} = E_t\left\{v_{t+1}^B + s_{t+1}^B\right\} - \frac{v_t^B + \bar{x}_t^B}{\gamma}$ regardless of the state. This can be achieved by setting entrepreneurial bailouts equal to $s^E = \bar{x}_t^E = \hat{\upsilon} - \upsilon^E$, where

$$\hat{\upsilon} = \left(\frac{\gamma - 1}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\alpha}{3\gamma - 2}\right)^{\frac{\alpha}{1-\alpha}}. \tag{44}$$

As for bank bailouts, they need to be state contingent: a scheme that sets $s_P^B = \bar{x}_t^B = \frac{\gamma}{\gamma - 1} \cdot \hat{\upsilon}$ and $s_O^B = \bar{x}_t^O = \frac{\gamma}{\gamma - 1} \cdot \hat{\upsilon} - \upsilon^B$, manages to stabilize the bubbly collateral of banks and to equalize the supply and demand for loans at $\frac{\gamma - 1}{\gamma} \cdot \hat{\upsilon}$. Note that this policy entails no cross-subsidization, in the sense that both entrepreneurs and bankers fiance their respective bailouts. It is not possible, however, to raise the steady-state level of investment by redistributing the costs of bailouts across both types of agents, since doing so would necessarily reduce either the supply or the demand of loans.

This completes our analysis of the bubbly economy with financial intermediaries. Before concluding, there is an aspect of this economy that is worthy of mention: it can never attain the same level of capital than the benchmark economy of Section 1. This is illustrated by our examples. The value of $\hat{\upsilon}$ in Equation (44), which corresponds to the double bubble example, is strictly lower than its corresponding value in in Equation (22), which corresponds to the quiet and moody bubbles. The reason is simple: there is a maximum amount of bubble creation that the economy can sustain in steady state. In the absence of financial intermediaries, this creation can be undertaken solely by entrepreneurs. In the economy with financial intermediaries, however, both deposits and loans must be collateralized. This implies that the economy's bubble creation must be spread out across both entrepreneurs and intermediaries, reducing the total amount of investment that can be sustained in equilibrium.

4 Fundamental collateral

The bubbly economy that we have analyzed throughout the paper is somewhat extreme, in the sense that entrepreneurs (and bankers) have no wealth or collateral other than the one provided by the bubble. This assumption has been useful to illustrate the role of bubbly collateral in the simplest possible setting, but it may raise questions as to how that role would change if there were other sources of collateral or entrepreneurial wealth. We now address this question.

We modify the bubbly economy of Section 1 along two dimensions. First, we assume that entrepreneurs make up a fraction ε of each generation and that, like savers, they are endowed with one unit of labor during youth. This assumption implies that entrepreneurs have a labor income that enables them to finance some investment on their own. Second, we assume that capital depreciates at rate $\delta \in (\varepsilon, 1)$. To simplify matters, we assume that capital can be converted back into consumption goods at a rate of one-to-one. This means that the stock market value of a firm now includes the capital left after production, which has a price of $(1-\delta) \cdot K_t$. If we use B_t to denote the bubble component of firm prices, we define it as

$$B_t = V_t - (1-\delta) \cdot K_t. \tag{45}$$

The bubbly economy that we have analyzed so far corresponds to the case in which $\delta = 1$ and the stock market is a pure bubble, i.e. $B_t = V_t$.[14]

These changes to the bubbly economy affect the workings of the credit market. The credit constraint faced by the representative entrepreneur is exactly as in Equation (5), with two important differences: (i) as entrepreneurs now have a labor income, they do not need to borrow the full amount $B_t + K_{t+1}$ required to purchase the firm and invest in it, and; (ii) because entrepreneurs can borrow against the future value of their firm, which now includes undepreciated capital, there is now fundamental collateral alongside bubbly collateral.

Entrepreneurial demand for labor, which is obtained by equalizing the marginal product of labor to the wage, remains unaffected by these changes and it is still given by Equations (7). Entrepreneurial demand for capital, though, is now given by

$$K_{t+1} \begin{cases} = \frac{1}{\delta} \cdot \left[\frac{E_t B_{t+1}}{E_t R_{t+1}} - B_t + \varepsilon \cdot W_t \right] & \text{if } \alpha \cdot \left(\frac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta > E_t R_{t+1} \\ \in \left[0, \frac{E_t B_{t+1}}{E_t R_{t+1}} - B_t + \varepsilon \cdot W_t \right] & \text{if } \alpha \cdot \left(\frac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta = E_t R_{t+1} \end{cases}. \tag{46}$$

A comparison of Equations (8) and (46) shows how the credit constraint of entrepreneurs has changed. The maximum that entrepreneurs can borrow is now given by the bubble creation plus their wages, leveraged $\frac{1}{\delta}$ times. The reason is that investment can initially be financed both directly through wages or indirectly by borrowing against bubble creation. Each unit invested, however, raises the future capital stock and thus the firm's fundamental collateral by $1-\delta$, allowing for even greater borrowing and investment.

[14] We maintain our previous assumptions on bubbles, namely, $B_t \geq 0$ and $\frac{\partial B_{t+1}}{\partial N_{t+1}} = \frac{\partial B_{t+1}}{\partial K_{t+1}} = 0$.

In the labor market, equilibrium wages are still given by Equation (9). In the stock market, we have already mentioned how the value of firms reflects both a bubbly component and the capital stock. In the credit market, the interest rate that matches the supply and demand of funds is now:

$$E_tR_{t+1} = \begin{cases} \min\left\{\dfrac{E_tV_{t+1}}{(1-\varepsilon)\cdot W_t}, \alpha\cdot\left(\dfrac{W_t - B_t}{\gamma^{t+1}}\right)^{\alpha-1} + 1-\delta\right\} & \text{if } W_t < \min\left\{\dfrac{E_tV_{t+1}}{1-\varepsilon}, \left(\dfrac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\cdot\gamma^{t+1} + B_t\right\} \\ 1 & \text{if } W_t \geq \min\left\{\dfrac{E_tV_{t+1}}{1-\varepsilon}, \left(\dfrac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\cdot\gamma^{t+1} + B_t\right\} \end{cases}, \tag{47}$$

which is a minor modification of Equation (10). As before, inventories are not used when savings are low and the interest rate is above one. If entrepreneurs are not credit constrained, the interest rate equals the return to capital accumulation, which is now given by the sum of the marginal product of capital and $1-\delta$. If entrepreneurs are instead credit constrained, the interest rate lies below the marginal product of capital and it equals the ratio between the expected collateral of entrepreneurs and the wages of savers. If savings are high enough, inventories are built in equilibrium and the interest rate is one.

We can summarize the workings of the bubbly economy with fundamental and bubbly collateral through the law of motion of k_t:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^{\alpha} - b_t}{\gamma} & \text{if } k_t < \tilde{k}_t \\ \min\left\{\dfrac{1}{\delta}\cdot\left[\dfrac{\varepsilon\cdot(1-\alpha)\cdot k_t^{\alpha}}{\gamma} + E_tb_{t+1} - \dfrac{b_t}{\gamma}\right], \left(\dfrac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right\} & \text{if } k_t \geq \tilde{k}_t \end{cases} \tag{48}$$

where $\tilde{k}_t = \left(\dfrac{1}{1-\alpha}\cdot\min\left\{\dfrac{\gamma\cdot E_tb_{t+1} - (1-\delta)\cdot b_t}{\delta-\varepsilon}, \gamma\cdot\left(\dfrac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}} + b_t\right\}\right)^{\frac{1}{\alpha}}$.

The law of motion of the capital stock in Equation (27) has two regions. The first region, which is the relevant one whenever the capital stock at time t is not too large and all savings are invested, is concave exactly as before. It is in the second region where the introduction of fundamental collateral makes a difference. Whenever the credit constraint is binding, capital accumulation is given by the sum of entrepreneurial wages and bubble creation, leveraged $\frac{1}{\delta}$ times. The law of motion is no longer flat, but rather increasing and concave. The reason is that a higher capital stock implies higher wages, which raises the labor income of entrepreneur and allows them to expand their borrowing and investment.

This discussion shows that the introduction of fundamental collateral does not significantly alter our analysis of the bubbly economy, nor does it undermine the important role of bubbly collateral.

On the contrary, the possibility of collateralizing capital actually increases the power of bubbly collateral in sustaining intermediation and investment. This is clearly illustrated by the law of motion in Equation (27): when credit constraints are binding, the creation of an additional unit of bubbly collateral has a more than proportional effect on capital accumulation. The reason, of course, is that when $\delta < 1$ each unit of capital serves in part as fundamental collateral, so that bubbly collateral boosts capital accumulation both directly and indirectly through the creation of fundamental collateral.

5 Concluding remarks

Financial markets constitute the backbone of modern economies, channeling savings towards its most productive uses. In order to function properly, however, financial markets need collateral. What determines the creation of collateral? The standard answer is that enforcement institutions do, as they enable economic agents to pledge future profits or cash flows and use them to back financial promises. But enforcement institutions are not always strong, in which case the creation of collateral is hampered, depressing intermediation and investment.

We have studied an economy in which weak enforcement institutions lead to a scarcity of collateral. In this context, we have shown how investor optimism generates bubbles that raise the market value of assets. This creates bubbly collateral, which is used to sustain credit and productive investment. Being governed by sentiment, however, this collateral is volatile and it may experience sudden drops when the expectations of investors change. In fact, we have shown how investor pessimism leads to the bursting of bubbles, the destruction of bubbly collateral, and the collapse of credit and investment.

What, if any, is the role for policy in such an environment? We have argued that policy can play an important role in sustaining and stabilizing the value of bubbly collateral. It can do so, for instance, by providing public bailouts that complement the collateral of bankers and entrepreneurs. Contrary to widespread notions, we have argued that this type of bailout schemes are expansionary ex-ante but contractionary ex-post. From an ex-ante perspective, future bailouts provide expected transfers to entrepreneurs, complementing private collateral and helping to sustain borrowing and investment. From an ex-post perspective, however, bailouts are used by entrepreneurs to pay their creditors and they divert resources away from productive investment.

The economic effects of bubbles, and the optimal policies to deal with them, rank high on the concerns of economists and policymakers. Yet we lack consistent models to think about these issues, let alone to provide answers. This paper has developed a macroeconomic framework of bubbles and has taken a necessary first step in characterizing the role of policy. Much remains to be done.

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