Bubbly Collateral and Economic Activity

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Introduction

- Recent years: trillions of government dollars to financial institutions
  - Spain: €100 billion three months ago
  - Portugal: €78 billion in 2011
  - Ireland: €85 billion in 2010
  - Greece: €110 billion in 2010, €130 billion in 2011
  - EFSF: €750 billion, partly to recapitalize banks
  - ECB: over a trillion € of cheap loans to banks between Dec. 2011 and Jan. 2012
  - in the US: TARP and Term Asset Backed Lending Facility
- What is the rationale for these bailout schemes, even by governments that are under financial stress?
  - do they correct underlying market failure?
  - do they boost activity in the short run at the expense of long run incentives?

This paper

- Start with simple observation: bailouts date back to 2007 crisis
  - crisis characterized by significant and rapid decline in asset prices and wealth
  - can be modeled as bursting of bubble, i.e. large shock to investor sentiment that destroys wealth (Martin-Ventura 2011)
  - in real-world financial markets: because of financial frictions wealth used as collateral
  - collapse of investor sentiment destroys collateral and credit
- To model this insight
  - macroeconomic model with financial frictions
  - key role of credit markets: savers $\leftrightarrow$ financial intermediaries $\leftrightarrow$ entrepreneurs
  - but lending must be collateralized, and collateral is scarce (weak enforcement institutions)
- In this setting
  - investor optimism leads to bubbles that raise value of firms or banks
  - bubbles expand stock of collateral, raising credit and investment
  - when bubbles burst, collateral falls and credit and investment contract

Main insights

- Firms and banks combine fundamental and bubbly collateral
  - bubbly collateral: pyramid schemes
    - valued today only because expected to be valued in the future
  - far fetched?
    - stocks traded at price above NPV of dividends
    - credit raised by firm/bank in excess of the cash flows it may generate
- Analysis of the bubbly economy
  - characterization of equilibrium
  - Pareto optimality
  - interaction of bubbly and fundamental collateral

Main insights

- Role of bailouts
  - in principle, policy useful to preserve or complement bubbly collateral
  - bailout policies that guarantee private promises
    - ex-ante: this policy creates collateral, boosts credit, investment and growth
    - ex-post: this policy needs to pay for bailout, taxation lowers credit, investment and growth
    - different from standard view: bailouts lower growth ex-ante, raise it ex-post
    - collateral vs. liquidity?
  - how feasible are these policies?
  - why government? mandatory vs. voluntary bailout schemes
  - which bailouts? deposit vs. loan guarantees

Related literature

- Rational bubbles
  - Samuelson (1958), Tirole (1985)
  - Samuelson (1958), Kiyotaki and Moore (2008): fiat money as a bubble
- Bubbles and economic growth
  - Saint-Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), Olivier (2000)
- Bubbles and financial frictions: macroconomic implications
  - Azariadis and Smith (1993): existence
  - Caballero and Krishnamurthy (2006), Farhi and Tirole (2010), Miao and Wang (2011), Aoki and Nikolov (2011): liquidity
  - Kocherlakota (2010), Martin and Ventura (2011): collateral
  - Ventura (2011): cost of capital
- Financial accelerator
  - Bernanke and Gertler (1989), Kiyotaki and Moore (1997)
- Bailouts (preliminary)
  - Tornell and Schneider (2004), Ranciere, Tornell and Westermann (2008), Bianchi (2012)

Roadmap

- Objective for the talk:

  1. Present model in which bubbles provide useful collateral
  2. Describe its implications for business cycles
  3. Explore the role of stabilization policy in the model
  4. Introduce financial intermediaries
  5. Robustness
  6. Conclusions

Model

- OLG: young and old
- Each generation: composed of $i \in \{S, E\}$, savers and entrepreneurs
- *Preferences*: all generations maximize expected consumption when old (i.e. they are patient and risk neutral!)

$$U_t^i = E_t^i \{c_{t+1}\}$$

- Savers (measure one) supply one unit of labor when young, $N_t = 1$; and receive wage W_t.
- Portfolio problem: inventories or credit?
  - Inventories (I_t): storage, gross return of one per unit invested
  - Credit ($W_t - I_t$): gross return of R_{t+1} per unit invested
  - Optimal portfolio: $I_t \begin{cases} = 0 & \text{if } E_t R_{t+1} > 1 \\ \in [0, W_t] & \text{if } E_t R_{t+1} = 1 \end{cases}$

Entrepreneurs

- Derive all income from managing firms
  - During youth, borrow $V_t + K_{t+1}$ to purchase firms and/or invest
  - During old age, hire workers to produce: production technology

$$Y_t = K_t^{\alpha} \cdot (A_t \cdot N_t)^{1-\alpha} \quad \text{with} \quad A_{t+1} = \gamma^{t+1},\ \gamma > 1$$

    where K fully depreciates.
  - After producing, sell firm at price V_{t+1}, pay debts $R_{t+1} \cdot (V_t + K_{t+1})$, and consume

- Investment in capital is productive: but loans need to be collateralized
  - Weak enforcement institutions: firm profits cannot be pledged
  - Borrowing constraint

$$R_{t+1}\,(V_t + K_{t+1}) \leq V_{t+1}$$

- Optimal capital/labor demands:

$$N_{t+1} = \left(\frac{1-\alpha}{W_{t+1}}\right)^{\frac{1}{\alpha}} \cdot \gamma^{\frac{1-\alpha}{\alpha}\cdot(t+1)} \cdot K_{t+1}$$

$$K_{t+1} \begin{cases} = \dfrac{E_t V_{t+1}}{E_t R_{t+1}} - V_t & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot A_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} > E_t R_{t+1} \\ \in \left[0, \dfrac{E_t V_{t+1}}{E_t R_{t+1}} - V_t\right] & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot A_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = E_t R_{t+1} \end{cases}$$

Markets and prices

- Labor market competitive and frictionless:

$$W_t = (1-\alpha) \cdot \gamma^{(1-\alpha)\cdot t} \cdot K_t^{\alpha}$$

- Credit market: entrepreneurs sell credit contracts to savers at interest rate

$$E_t R_{t+1} = \begin{cases} \min\left\{\dfrac{E_t V_{t+1}}{W_t}, \alpha \cdot \left(\dfrac{W_t - V_t}{\gamma^{t+1}}\right)^{\alpha-1}\right\} & \text{if } W_t < \min\left\{E_t V_{t+1}, \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t\right\} \\ 1 & \text{if } W_t \geq \min\left\{E_t V_{t+1}, \alpha^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V_t\right\} \end{cases}$$

  depending on whether collateral, productivity or resources limit credit

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  depending on whether collateral, productivity or resources limit credit

- Stock market: young entrepreneurs purchase firms that are a pure bubble
  - young entrepreneur pays V_t for stock of old bubbles: also attaches new bubbles to firm
  - discounted value of new bubbles: $\dfrac{E_t V_{t+1}}{E_t R_{t+1}} - V_t$

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- Two restrictions regarding bubbles

$$V_t \geq 0$$

$$\frac{\partial V_{t+1}}{\partial N_{t+1}} = \frac{\partial V_{t+1}}{\partial K_{t+1}} = 0$$

Competitive equilibrium

- Bubble shock: $h_t = \{V_t, E_t V_{t+1}\}$
- Let $k_t \equiv \gamma^{-t} \cdot K_t$ and $v_t \equiv \gamma^{-t} \cdot V_t$
- Law of motion of k_t:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha) \cdot k_t^{\alpha} - v_t}{\gamma} & \text{if } \; k_t < \left(\dfrac{1}{1-\alpha} \cdot \min \left\{ \gamma \cdot E_t v_{t+1}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t \right\} \right)^{\frac{1}{\alpha}} \\ \min \left\{ E_t v_{t+1} - \dfrac{v_t}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } \; k_t \geq \left(\dfrac{1}{1-\alpha} \cdot \min \left\{ \gamma \cdot E_t v_{t+1}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t \right\} \right)^{\frac{1}{\alpha}} \end{cases}$$

  - If k_t small: investment determined by supply of funds, i.e. wages
    - law of motion increasing
  - If k_t large: investment determined by demand of funds, i.e. collateral or efficient investment
    - law of motion flat

- Competitive equilibrium: sequence $\{v_t, E_t v_{t+1}, k_t\}_{t=0}^{\infty}$ satisfying law of motion with $v_t \geq 0$ and $k_t \geq 0$, and for all t and $h^t \in H_t$.

What are bubbles doing?

- Bubbly economy: three assets to transfer consumption across periods: capital, bubbles, inventories
  - shortage of collateral
  - bubbly collateral used to sustain transfers
- Bubbly collateral
  - useful to sustain transfers
  - may fluctuate randomly across periods and histories (alongside investor sentiment)
- Transfers from young savers to young entrepreneurs and old savers:
  - transfer from young savers to young entrepreneurs: $K_{t+1} + V_t$
  - kept by young entrepreneurs and used to finance capital: K_{t+1}
  - transferred to old savers: V_t

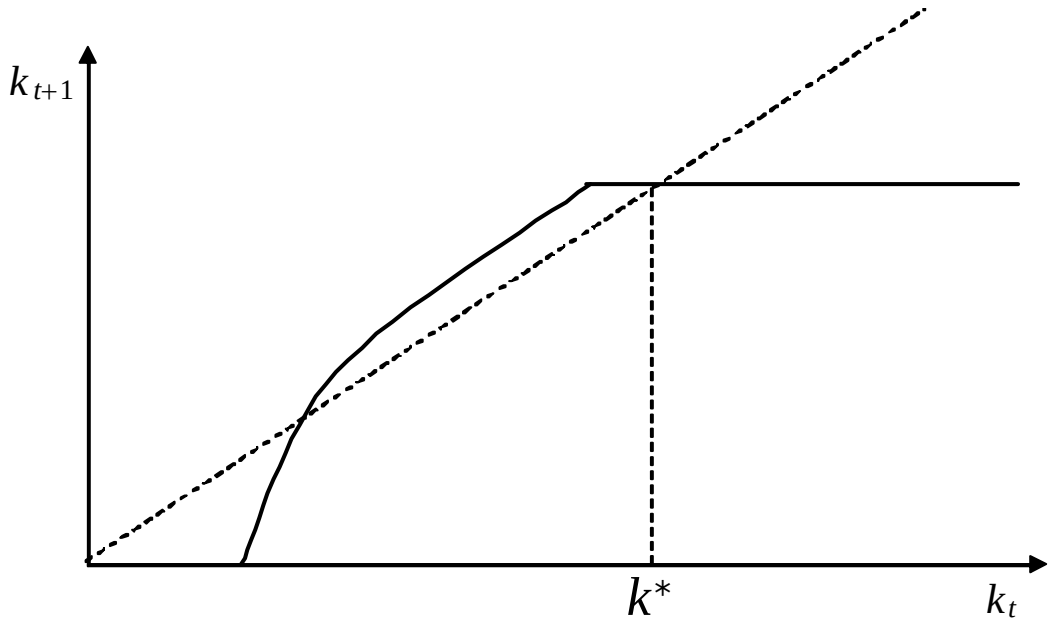
Bubbly business cycles

Example 1 (calm bubble) *The calm bubble never changes. Thus,* $v_t = v < \dfrac{\gamma}{\gamma - 1} \cdot \alpha^{\frac{1}{1-\alpha}}$ *for all* t.

- With calm bubble, law of motion becomes

$$k_{t+1} = \min \left\{ \frac{\gamma - 1}{\gamma} \cdot v, \frac{(1-\alpha) \cdot k_t^{\alpha} - v}{\gamma} \right\}$$

- Graphically,



Bubbly business cycles

Example 2 (moody bubble) *The moody bubble fluctuates between an optimistic* (O) *and a pessimistic* (P) *state. Let* $z_t \in \{O, P\}$ *be investor sentiment, with* $\Pr[z_{t+1} = z_t] = 1 - \pi$ *and* $\Pr[z_{t+1} \neq z_t] = \pi$ *for all* t *and* $h^t \in H_t$*. Then,* $v_t = v < \dfrac{\gamma}{\pi \cdot \gamma - 1} \cdot \alpha^{\frac{1}{1-\alpha}}$ *if* $z_t = P$*, and* $v_t = 0$ *if* $z_t = P$*.*

- With moody bubble, law of motion depends on state of the economy:

$$k_{t+1} = \begin{cases} \min\left\{\dfrac{(1-\pi)\cdot\gamma - 1}{\gamma} \cdot v, \dfrac{(1-\alpha)\cdot k_t^\alpha - v}{\gamma}\right\} & \text{if } z_t = O \\ \min\left\{\dfrac{\pi\cdot\gamma - 1}{\gamma} \cdot v, \dfrac{(1-\alpha)\cdot k_t^\alpha}{\gamma}\right\} & \text{if } z_t = P \end{cases}$$

Bubbly business cycles

Example 3 (moody bubble) *The moody bubble fluctuates between an optimistic (O) and a pessimistic (P) state. Let $z_t \in \{O, P\}$ be investor sentiment, with $\Pr[z_{t+1} = z_t] = 1 - \pi$ and $\Pr[z_{t+1} \neq z_t] = \pi$ for all t and $h^t \in H_t$. Then, $v_t = v < \dfrac{\gamma}{\pi \cdot \gamma - 1} \cdot \alpha^{\frac{1}{1-\alpha}}$ if $z_t = P$, and $v_t = 0$ if $z_t = P$.*

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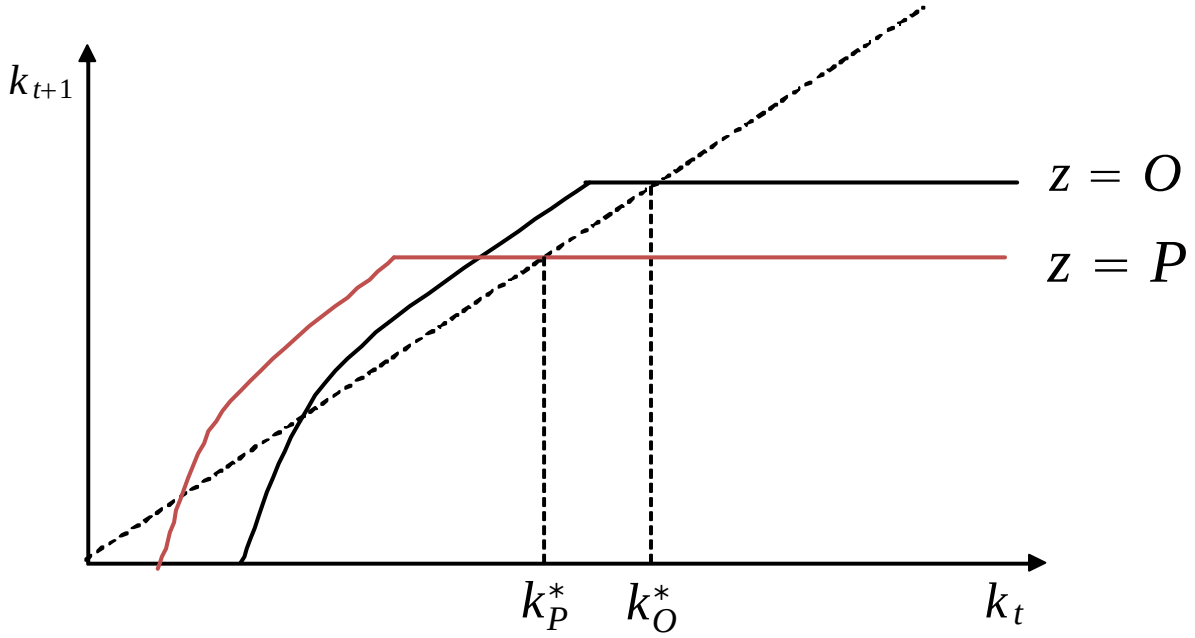
- Graphically,



where $\pi \leq \dfrac{\gamma - 1}{2\gamma}$, i.e. z_t is sufficiently persistent.

Discussion on Pareto optimality...

- CPO equilibria: economy has enough collateral to intermediate all savings of young savers:

$$\gamma \cdot E_t v_{t+1} \geq (1-\alpha) \cdot k_t^{\alpha} \geq v_t \text{ for all } t \text{ and } h^t \in H_t$$

  i.e., inventories eliminated in all histories

- In any CPO equilibrium

$$E_t R_{t+1} \in \left[1, \gamma \cdot \left(\frac{k_{t+1}}{k_t}\right)^{\alpha}\right)$$

- CPO not necessarily characterized by
  - high stock of capital and consumption
  - absence of fluctuations

- All savings intermediated. Where does intermediation go?
  - In CPO equilibria with low interest rates: to young entrepreneurs
    - high capital stock and consumption
  - In CPO equilibria with high interest rates: to old savers
    - low capital stock and consumption

A Role for Policy?

- Bubbly economy characterized by lack of collateral
  - can public policy be used to relieve this scarcity?
- It depends on what the government can do
- Introduce government that
  - raises taxes
  - backs promises made by entrepreneurs
  - government provision of collateral
- Disclaimer: no objective function for the government
  - explore theoretical effects of different policies

Bubbly economy with bailouts

- Introduce government that can provide bailout S_t to firms
  - formally: in each period t, government provides S_t that can be used to cancel credit contracts
  - could be contingent
- Financed through taxation $\bar{x}_t$ on young entrepreneurs
- Equilibrium entails,

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^{\alpha} - \bar{x}_t - v_t}{\gamma} & \text{if } \; k_t < \widetilde{k} \\ \min\left\{ E_t\left\{v_{t+1} + s_{t+1}\right\} - \dfrac{(v_t + \bar{x}_t)}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } \; k_t \geq \widetilde{k} \end{cases},$$

$$s_t = \bar{x}_t,$$

where $\widetilde{k} = \left(\dfrac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t + \bar{x}_t \right\} \right)^{\frac{1}{\alpha}}$.

- Competitive equilibrium: sequence $\{v_t, E_t v_{t+1}, k_t\}_{t=0}^{\infty}$ and bailout scheme and a bailout scheme $\{s_t, \bar{x}_t\}_{t=0}^{\infty}$ satisfying law of motion and $s_t = \bar{x}_t$ with $v_t \geq 0$ and $k_t \geq 0$, and for all t and $h^t \in H_t$.
- Note: equilibrium cannot be defined independently of scheme

Bubbly business cycles with bailouts

- Assuming credit constraints bind, law of motion can be written as:

$$k_{t+1} = \min\left\{E_t\left\{v_{t+1} + s_{t+1}\right\}, \frac{(1-\alpha)\cdot k_t^{\alpha}}{\gamma}\right\} - \frac{v_t + s_t}{\gamma}$$

- Set of transfers in economy with bailouts:
  - expected bubble and bailouts in period $t+1$ provide collateral, transfer funds to young entrepreneurs

$$\min\left\{\gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, (1-\alpha)\cdot k_t^{\alpha}\right\}$$

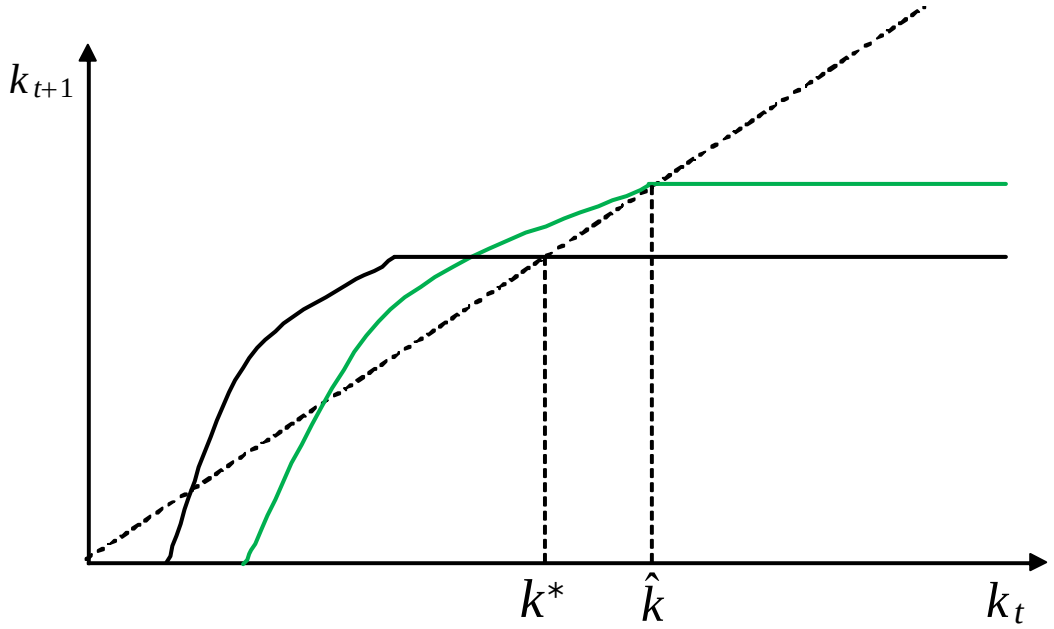
  - of these funds:
    - $\min\left\{\gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, (1-\alpha)\cdot k_t^{\alpha}\right\} - (v_t + s_t)$ used for investment
    - $v_t + s_t$ used to purchase firms and pay taxes, i.e. transfers to the old
- From law of motion:
  - economy with process for bubble shock v_t and bailouts s_t for all t
  - competitive equilibrium identical to alternative economy, without bailouts but bubble shock $\hat{v}_t = v_t + s_t$ for all t

Bubbly business cycles with bailouts (I)

- Calm bubble: bailout scheme to replicate CPO allocation that maximizes k^*
  - expected collateral of entrepreneurs must equal wages, i.e. $\hat{v} = \dfrac{w}{\gamma}$
  - attained by setting bailouts $s = \hat{v} - v$, with

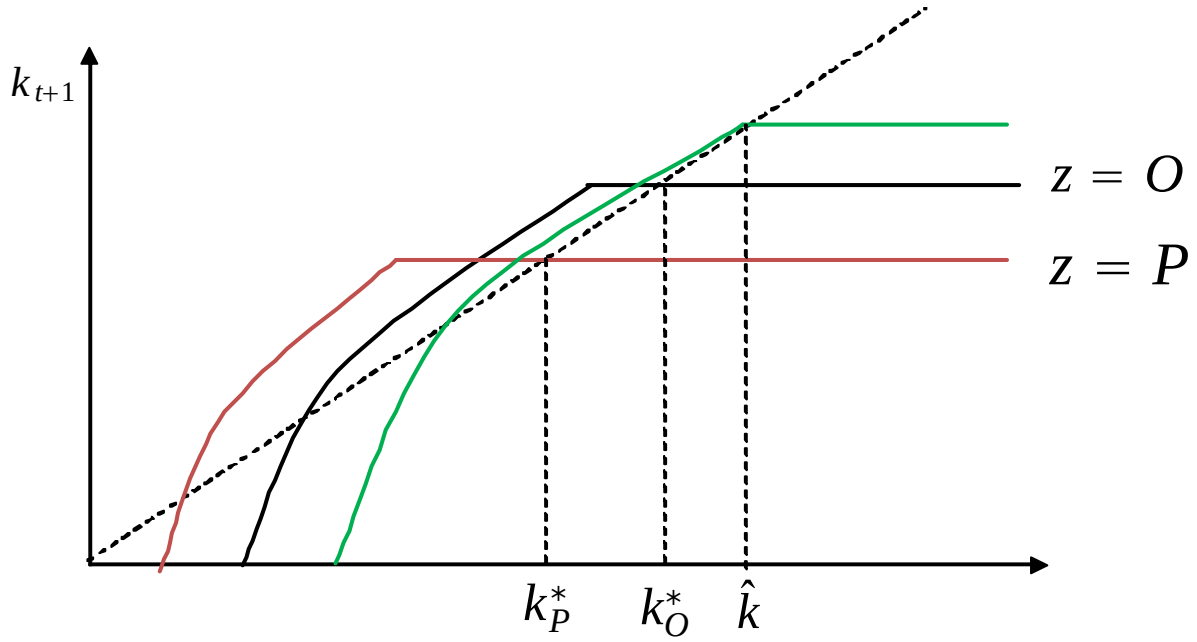
$$\hat{v} = \left(\frac{\gamma - 1}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\alpha}{\gamma}\right)^{\alpha}$$

- Graphically,



Bubbly business cycles with bailouts (II)

- Moody bubble: bailout scheme to replicate CPO allocation that maximizes k^*
  - expected collateral of entrepreneurs must be raised and stabilized at $\hat{v} = \frac{w}{\gamma}$
  - attained by setting contingent bailouts: $s_P = \hat{v} > \hat{v} - v = s_O$
- Graphically,



- At first glance: government stabilization of credit might seem strange or impractical. But:
  - insurance fund: entrepreneurs contribute during youth and – if crisis – receive transfer in old age
  - price stabilization scheme: government intervenes in market for firms

Mandatory vs. voluntary bailout schemes

- Bailout schemes benefit all
  - why is government needed? can't entrepreneurs run scheme on their own?
- Voluntary scheme: contribution $x_t \in [0, \bar{x}]$ entitles young to pro-rata share of future contributions
  - assume $E_t R_{t+1} = 1$ for all t
  - optimal contribution sets $x_t = \min\{\gamma \cdot E_t x_{t+1}, \bar{x}\}$
  - present contribution depends on (voluntary) expected ones
  - like bubbles, subject to "sentiment shocks" $\neq$ mandatory scheme

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- Partially mandatory schemes: $x_t \in [\underline{x}, \bar{x}]$
  - in this case: $x_t = \min\{\underline{x}, \min\{\gamma \cdot E_t x_{t+1}, \bar{x}\}\}$, but constraint not binding!
    - $E_t x_{t+1} \geq \underline{x} \;\rightarrow x_t \geq \gamma \cdot \bar{x}$...
    - ...the same holds for generation $t+1$: $E_t x_{t+1} \geq \gamma \cdot \underline{x} \;\Rightarrow\; x_t \geq \gamma^2 \cdot \underline{x}$
    - by iterating, $x_t = \min\{\gamma \cdot E_t \bar{x}_{t+1}, \bar{x}_t\}$, exactly like mandatory scheme!
    - argument can be made for any equilibrium interest rate

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    - by iterating, $x_t = \min\{\gamma \cdot E_t \bar{x}_{t+1}, \bar{x}_t\}$, exactly like mandatory scheme!
    - argument can be made for any equilibrium interest rate
- This argument holds for arbitrarily small mandated contribution $\underline{x}$
  - key: growth rate exceeds the interest rate

Limits to bailouts

- Relax two restrictions in policy:
  - allow for inefficiencies: each unit of bailouts requires $\lambda \geq 1$ units of contributions
    - direct cost of corruption or waste
    - informational costs of bailouts: only fraction $\frac{1}{\lambda}$ go to intended recipients
  - allow for interruption of policy: young do not contribute with prob. $1 - \mu \geq 0$
    - limit on the government's ability to commit to the scheme
    - probability that young run the government

- Law of motion becomes,

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha) \cdot k_t^{\alpha} - \lambda \cdot s_t - v_t}{\gamma} & \text{if } \; k_t < \widetilde{k} \\ \min\left\{ E_t\left\{v_{t+1} + s_{t+1}\right\} - \dfrac{(v_t + \lambda \cdot s_t)}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } \; k_t \geq \widetilde{k} \end{cases},$$

  where $\widetilde{k} = \left(\dfrac{1}{1-\alpha} \cdot \min\left\{ \gamma \cdot E_t\left\{v_{t+1} + s_{t+1}\right\}, \gamma \cdot \alpha^{\frac{1}{1-\alpha}} + v_t + \bar{x}_t \right\} \right)^{\frac{1}{\alpha}}$.

- Both μ and λ reduced the effectiveness of policy
  - if $\mu \cdot \gamma > \lambda$: qualitatively, same as before
  - if $\mu \cdot \gamma \leq \lambda$: no expansionary scheme can be implemented in steady state
  - in this last case, scheme would be contractionary and subject to sentiment shocks

Role of government debt

- Back to efficient policy ($\lambda = \mu = 1$)
- Bailout schemes can also be implemented through debt d_t
  - intuitively, bailouts should be more effective if financed through debt
- Government budget constraint becomes

$$d_t + s_t = \bar{x}_t + \frac{\gamma \cdot E_t d_{t+1}}{E_t R_{t+1}}$$

- Law of motion now becomes

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^{\alpha} - s_t - d_t - v_t}{\gamma} & \text{if } \; k_t < \widetilde{k} \\ \min\left\{ E_t\left\{v_{t+1} + s_{t+1} + d_{t+1}\right\} - \dfrac{(v_t + s_t + d_t)}{\gamma}, \alpha^{\frac{1}{1-\alpha}} \right\} & \text{if } \; k_t \geq \widetilde{k} \end{cases},$$

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- Contributions could now be set to zero, all financed through debt
  - no change to allocations that can be implemented: set $s'_t = s_t + d_t$
  - once again, debt subject to roll over crisis unless some contribution is mandatory

Financial intermediaries

- Introduce financial intermediaries: bankers B
  - same preferences as savers or entrepreneurs
- Derive all income from managing banks: endowed with screening/monitoring technology
  - during youth, raise deposits, purchase banks and make loans L_t
  - during old age, collect payment $R^E_{t+1} \cdot L_t$ from entrepreneurs
  - after producing, sell bank at price V^B_{t+1}, pay debts, and consume
- Deposits need to be collateralized
  - Weak enforcement institutions: loan repayments cannot be pledged
  - Borrowing constraint

$$R^B_{t+1} \cdot \left(V^B_t + L_t + \bar{X}^B_t\right) \leq V^B_{t+1}$$

- Note: banks contain no real assets and their price consists only of the bubble component
- Budget constraint of representative banker given by

$$C^B_{t+1} = \left(R^E_{t+1} - R^B_{t+1}\right) \cdot L_t + V^B_{t+1} + S^B_{t+1} - R^B_{t+1} \cdot \left(V^B_t + \bar{X}^B_t\right)$$

  where we allow for bailout scheme $\left\{S^B_t, \bar{X}^B_t\right\}_{t=0}^{\infty}$ for banks.

Financial intermediaries (II)

- Banker maximizes utility s.t. budget constraint: set

$$L_t \begin{cases} = \dfrac{E_t\left\{V_{t+1}^B + S_{t+1}^B\right\}}{E_t R_{t+1}^B} - \left(V_t^B + \bar{X}_t^B\right) & \text{if } E_t R_{t+1}^B > E_t R_{t+1}^E \\ \in \left[0, \dfrac{E_t\left\{V_{t+1}^B + S_{t+1}^B\right\}}{E_t R_{t+1}^B} - \left(V_t^B + \bar{X}_t^B\right)\right] & \text{if } E_t R_{t+1}^E = E_t R_{t+1}^E \end{cases}$$

- Savers and entrepreneurs unaffected, but face different interest rates

$$I_t \begin{cases} = 0 & \text{if } E_t R_{t+1}^B > 1 \\ \in [0, W_t] & \text{if } E_t R_{t+1}^B = 1 \end{cases}$$

$$K_{t+1} \begin{cases} = \dfrac{E_t\left\{V_{t+1}^E + S_{t+1}^E\right\}}{E_t R_{t+1}^E} - \left(V_t^E + \bar{X}_t^E\right) & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} > E_t R_{t+1}^E \\ \in \left[0, \dfrac{E_t\left\{V_{t+1}^E + S_{t+1}^E\right\}}{E_t R_{t+1}^E} - \left(V_t^E + \bar{X}_t^E\right)\right] & \text{if } \alpha \cdot \left(\dfrac{1-\alpha}{\gamma^{-(t+1)} \cdot W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = E_t R_{t+1}^E \end{cases}$$

- Here, once again, bubbly collateral sustains intermediation:
  - bubble in financial intermediaries helps sustain deposits
  - bubble in firms helps sustain loans

Credit market equilibrium with intermediaries

- Labor market as before
- Credit market: two interest rates, $E_t R^E_{t+1}$ and $E_t R^B_{t+1}$. In equilibrium,

$$\begin{aligned} V^E_t + \bar{X}^E_t + K_{t+1} &= L_t, \\ V^B_t + \bar{X}^B_t + L_t &= W_t - I_t \end{aligned}$$

Three possibilities, depending on collateral:

1. Both entrepreneurs and banks have enough collateral: $E_t R^E_{t+1} > E_t R^B_{t+1} > 1$. In this case:

$$\min \left\{ \frac{E_t \left\{ V^E_{t+1} + S^E_{t+1} \right\}}{E_t R^E_{t+1}}, \left(\frac{\alpha}{E_t R^E_{t+1}} \right)^{\frac{1}{1-\alpha}} \cdot \gamma^{t+1} + V^E_t + \bar{X}^E_t \right\} = \frac{E_t \left\{ V^B_{t+1} + S^B_{t+1} \right\}}{E_t R^B_{t+1}} - (V^B_t + \bar{X}^B_t)$$

$$\frac{E_t \left\{ V^B_{t+1} + S^B_{t+1} \right\}}{E_t R^B_{t+1}} = W_t$$

so that

$$I_t = 0$$

$$K_{t+1} = W_t - (V^B_t + \bar{X}^B_t) - \left(V^E_t + \bar{X}^E_t \right)$$

Credit market equilibrium with intermediaries

- Cases in which either banks or entrepreneurs have insufficient collateral

2. Entrepreneurs have enough collateral but banks do not: $E_t R^E_{t+1} > E_t R^B_{t+1} = 1$. In this case:

$$\min\left\{\frac{E_t\left\{V^E_{t+1}+S^E_{t+1}\right\}}{E_t R^E_{t+1}}, \left(\frac{\alpha}{E_t R^E_{t+1}}\right)^{\frac{1}{1-\alpha}}\cdot\gamma^{t+1}+V^E_t+\bar{X}^E_t\right\} = E_t\left\{V^B_{t+1}+S^B_{t+1}\right\}-(V^B_t+\bar{X}^B_t)$$

and

$$I_t = W_t - E_t\left\{V^B_{t+1}+S^B_{t+1}\right\}$$

$$K_{t+1} = E_t\left\{V^B_{t+1}+S^B_{t+1}\right\}-(V^B_t+\bar{X}^B_t)-\left(V^E_t+\bar{X}^E_t\right)$$

3. Banks have enough collateral but entrepreneurs do not: $E_t R^E_{t+1} = E_t R^B_{t+1} = 1$. In this case:

$$I_t = W_t - \min\left\{E_t\left\{V^E_{t+1}+S^E_{t+1}\right\}, \alpha^{\frac{1}{1-\alpha}}\cdot\gamma^{t+1}+V^E_t+\bar{X}^E_t\right\}$$

$$K_{t+1} = \min\left\{E_t\left\{V^E_{t+1}+S^E_{t+1}\right\}-\left(V^E_t+\bar{X}^E_t\right), \alpha^{\frac{1}{1-\alpha}}\cdot\gamma^{t+1}\right\}$$

Competitive equilibrium

- Bubble shock: $h_t = \left\{V_t^E, V_t^B, E_t V_{t+1}^E, E_t V_{t+1}^B\right\}$
- Law of motion of k_t:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^\alpha - \left(v_t^B + \bar{x}_t^B\right) - \left(v_t^E + \bar{x}_t^E\right)}{\gamma} \\ \min\left\{E_t\left\{v_{t+1}^E + s_{t+1}^E\right\} - \dfrac{v_t^E + \bar{x}_t^E}{\gamma}, E_t\left\{v_{t+1}^B + s_{t+1}^B\right\} - \dfrac{v_t^B + \bar{x}_t^B}{\gamma} - \dfrac{v_t^E + \bar{x}_t^E}{\gamma}, \alpha^{\frac{1}{1-\alpha}}\right\} \end{cases}$$

  depending on whether k_t is less than or greater than a threshold $\widetilde{k}_t$

- What changes?
  - If k_t small: investment determined by supply of funds, i.e. wages
    - law of motion increasing
  - If k_t large: investment determined by demand of funds, i.e. **collateral of banks or firms** or efficient investment
    - law of motion flat
- Main insight: dynamics affected not only by amount of collateral, but also by its distribution
  - role for cross-subsidization: using bank contributions to bailout firms

Bubbly business cycle with financial intermediaries

Example 4 (double bubble) *Combines quiet bubble in firms with moody bubble in banks. Quiet bubble never changes,* $v_t^E = v^E < \frac{\gamma}{\gamma - 1} \cdot \alpha^{\frac{1}{1-\alpha}}$. *Moody bubble fluctuates between an optimistic* (O) *and a pessimistic* (P) *state. If* $z_t \in \{O, P\}$ *denotes the state, it is assumed that* $\Pr[z_{t+1} \neq z_t] = \pi$ *for all* t *and* $h^t \in H_t$, *where* $1 \leq \pi\gamma \leq \frac{\gamma - 1}{2}$. *Then,* $v_t^B = v^B$ *if* $z_t = O$, *where* $\frac{v^B}{v^E} \in \left(\frac{\gamma}{(1-\pi) \cdot \gamma - 1}, \frac{1}{\pi} \right)$, *and* $v_t^B = 0$ *if* $z_t = P$.

- Law of motion of the capital stock depends on the state of the economy:

$$k_{t+1} = \begin{cases} \min \left\{ \frac{\gamma - 1}{\gamma} \cdot v^E, \frac{(1-\alpha) \cdot k_t^\alpha - v^E - v^B}{\gamma} \right\} & \text{if } z_t = O \\ \min \left\{ \pi \cdot v^B - \frac{v^E}{\gamma}, \frac{(1-\alpha) \cdot k_t^\alpha - v^E}{\gamma} \right\} & \text{if } z_t = P \end{cases},$$

  - in optimistic times, capital accumulation constrained by entrepreneurial collateral
  - in pessimistic times, capital accumulation constrained by bank collateral

- Implications for bailout schemes

  - not just size, also distribution of bailouts matter
  - in example: capital stock maximized with right combination of loan and deposits guarantees
  - note: maximum capital stock lower than in baseline economy without banks

Fundamental collateral

- All results extend to economy with fundamental collateral
  - assume entrepreneurs are measure ε of young and have labor
  - capital depreciates at rate δ and that it can be pledged
- Stock market value of firms contains fundamental bubbly component

$$B_t = V_t - (1-\delta)\cdot K_t$$

- Law of motion of k_t:

$$k_{t+1} = \begin{cases} \dfrac{(1-\alpha)\cdot k_t^{\alpha} - b_t}{\gamma} & \text{if } \; k_t < \tilde{k}_t \\ \min\left\{\dfrac{1}{\delta}\cdot\left[\dfrac{\varepsilon\cdot(1-\alpha)\cdot k_t^{\alpha}}{\gamma} + E_t b_{t+1} - \dfrac{b_t}{\gamma}\right], \left(\dfrac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right\} & \text{if } \; k_t \geq \tilde{k}_t \end{cases}$$

- What changes?
  - entrepreneurs now can pledge bubble creation and wages
  - these resources are leveraged $\dfrac{1}{\delta}$
- Fundamental collateral does not change our analysis
  - if anything, bubbly collateral becomes more powerful
  - direct and indirect effect (through the creation of fundamental collateral) on capital accumulation

Discussion and final thoughts

- Simple, theoretical model of bubbly business cycles
- Financial markets need collateral: how is it created?
  - Fundamental collateral: enforcement institutions
  - Bubbly collateral: investor sentiment
- Bubbly episodes: fluctuations in asset prices, collateral, credit and productive investment
- Connection with models of the financial accelerator (Martin Ventura 2011)
  - like them: higher asset prices raise credit, efficiency and growth
  - unlike them: asset prices decoupled from fundamentals
- Policy implications:
  - in economy with scarce collateral, government can use taxation power to sustain and stabilize value of public collateral
    - ex-ante: this policy creates collateral, boosts credit, investment and growth
    - ex-post: this policy needs to pay for bailout, taxation lowers credit, investment and growth
- At the end of the day revisit old and fascinating question
  - should the authority target asset prices? (Bernanke and Gertler (1999))
  - but do so in a theoretically consistent model in which asset prices are endogenous

Where do we go from here?

- Can these mechanisms really be quantitatively important?
  - let's see
  - embed investor sentiment shock into quantitative model
    - sophisticated model with rich demographics / preferences
  - can we distinguish between TFP and investor sentiment shocks in recent past?
  - Carvalho et al. (2011)