BANK COMPETITION AND FINANCIAL STABILITY: A GENERAL EQUILIBRIUM EXPOSITION*

Gianni De Nicolò International Monetary Fund and CESifo gdenicolo@imf.org

Marcella Lucchetta University of Venice Ca' Foscari, Department of Economics lucchett@unive.it

November 23, 2012

Abstract

We study the welfare properties of a general equilibrium banking model with moral hazard that includes essential features of set-ups studied in a large partial equilibrium literature. We show that perfect bank competition is constrained Pareto optimal and delivers an optimal level of banks' risk of failure. This result holds even though the risk of failure of competitive banks is higher than that of banks enjoying monopoly rents, and is robust to the introduction of social costs of bank failures. In this model there is no trade-off between bank competition and financial stability.

* The views expressed in this paper are those of the authors and do not necessarily represent the views of the International Monetary Fund.

I. INTRODUCTION

The theoretical banking literature offers contrasting results on the relationship between bank competition and banks' risk of failure. In models where banks raise funds from insured depositors and choose the risk of their investment portfolio, more competition results in a higher risk of bank failure, as higher funding costs erode banks' charter values (expected profits): under limited liability and unobservable risk choices, banks have an incentive to choose riskier projects.¹ However, when banks compete a la Cournot in both loan and deposit markets and the loan returns are perfectly correlated, then banks' risk of failure could decline as the number of banks increases, while if loan returns are not perfectly correlated, there might exist a U-shaped relationship between the number of banks and banks' risk of failure.²

The issue of whether bank competition should be restrained has a long history in the bank regulatory debate, having resurfaced with urgency in the aftermath of the recent financial crisis. Yet, the current banking literature does not offer a clear guidance to this debate, since it is based on implications derived from partial equilibrium set-ups. These setups establish rankings of banks' risk of failure depending on particular assumptions about banks competitive interactions, but are not equipped to address the key normative issue of whether there is a trade-off between bank competition and financial stability. Is a lower level of risk of bank failure necessarily undesirable in a welfare sense? This paper addresses the following key normative question: What are the welfare-maximizing levels of bank competition and the relevant optimal levels of banks' risk of failure?

Policy prescriptions suggesting that bank competition should be restrained seem at variance with the welfare results of some general equilibrium banking models. Allen and Gale (2004b) demonstrate that perfect competition among intermediaries is Pareto optimal under complete markets, and constrained Pareto optimal under incomplete markets. Importantly, in their model a certain degree of financial"instability" is a necessary condition of optimality. Analogous results are obtained under low inflation in the general equilibrium monetary economy with aggregate liquidity risk analyzed by Boyd, De Nicolò and Smith

¹ See Keeley (1990), Matutes and Vives (1996), Hellmann, Murdock and Stiglitz (2000), Allen and Gale (2000 and 2004a), Cordella and Levi-Yeyati (2002), and Repullo (2004), among others.

² See Boyd and De Nicolò (2005) and Martinez-Miera and Repullo (2010).

(2004). Yet, these general equilibrium models do not feature the type of moral hazard in investment associated with financing choices considered by the partial equilibrium literature of banking we have just mentioned. This motivates our explicit consideration of these features in a general equilibrium set-up.

In our model, the size of the banking sector and the resource allocated to productive investment are endogenously determined, as risk-neutral agents choose to become either bankers or depositors, with banks established as coalitions of bankers financed by depositors. An important feature of our model is that setting up banks has a resource cost: as a result, any welfare criterion will balance the costs of bank intermediation with the benefits of increasing available resources for productive investment.³ As in the partial equilibrium literature, in our model banks choose the riskiness of their investment incurring higher effort costs to select lower risk investments. As is standard, we interpret these costs as characterizing an intermediation technology that embeds screening and/or monitoring costs. Furthermore, bank risk choices are unobservable, hence there is moral hazard. However, depositors take into account banks' optimal risk choices in their decision to accept deposit terms. Differences in competitive conditions in the economy are simply modeled assuming that banks can choose to operate as monopolists or competitive banks, while depositors incur switching costs to be served by competitive banks. Thus, different degrees of imperfect bank competition are proxied by the fraction of bank deposit contracts in the economy priced monopolistically and competitively.

We consider the model under no deposit insurance, as well as the case where a "government" sets-up a deposit insurance scheme that is resource-feasible and partially or totally insures the principal of depositors' investment in a bank. Although there is admittedly no explicit rationale for deposit insurance in our model, as there is none in all partial

³ This feature is absent in general equilibrium constructs where either the distribution of initial resources or the partition of agents in banks or depositors is exogenous (see e.g. Holmstrom and Tirole, 1997, or Morrison and White, 2005).

equilibrium models just mentioned, we wish to assess whether the presence of an arguably realistic deposit insurance scheme affects the welfare ranking of competitive conditions.⁴

The key result of this paper is that perfect bank competition is constrained Pareto optimal. This result holds both without and with deposit insurance. Notably, this result holds even though competitive banks exhibit a level of risk of failure higher than banks enjoying monopoly rents. However, this result clearly indicates that a particular ranking of banks' risk of failure obtained in partial equilibrium set-ups is neither necessary nor sufficient for optimality. Most importantly, this result holds even in the presence of social costs consistent with the existence of bank intermediation that are not internalized by banks and are typically associated with bank failures. Therefore, perfect bank competition supports a constrained efficient allocation and results in an optimal level of banks' risk of failure. Thus, a general equilibrium economy with investment choices subject to moral hazard delivers implications qualitatively identical to those obtained by Allen and Gale (2004) and Boyd, De Nicolò and Smith (2004) in general equilibrium set-ups that lack these features.

The mechanism delivering the constrained Pareto optimality of perfect bank competition is simple. An increase in bank competition is welfare-improving owing to a resource re-allocation mechanism we term the *general equilibrium effect* of bank competition. As bank competition for funds increase, the relative return of deposits versus that of shares of bank ownership increases, prompting a larger (smaller) fraction of agents to become depositors (bankers). This shift depicts stylistically an economy-wide shift of resources from investment in costly bank intermediation to investment in productive assets intermediated by banks. The resulting increase in economy-wide investment in productive assets generates an increase in expected output (net of monitoring and production costs) large enough to offset any reduction in the expected return due to the comparatively higher risk of failure of banks operating under more intense competition.

We obtain a further result that is of independent interest. The introduction of deposit insurance increases the banks' risk of failure in the competitive sector, while it decreases it in

⁴ Most partial equilibrium models assume the existence of deposit insurance either for the sake of realism, or under the implicit assumption that deposit insurance corrects some not explicitly modeled coordination failures, such as the occurrence of runs.

the monopolistic sector. Thus, the introduction of deposit insurance is not neutral to rankings of bank failures according to different degree of competition. However, as noted, different degrees of deposit insurance coverage do not affect the welfare-maximizing property of perfect bank competition.

The remainder of the paper is composed of three sections. Section II describes the model. Section III the bank problems, and Section IV equilibrium and welfare. Section V concludes. Proofs are in the Appendix.

II. THE MODEL

There are two dates, 0, and 1, and a continuum of risk neutral agents on [0, A]. Agents are endowed with 1 unit of the date 0 good and with effort, they derive disutility from effort, and have preferences over final date consumption. All agents have access to a safe (risk-free) technology which yields ρ per unit invested. At date 0 agents decide either to become bankers or depositors.

Banks

If an agent chooses to become a banker, she forgoes her initial endowment in exchange of the ability to form coalitions, called banks, which operate a risky project. The set of risky projects banks choose from is indexed by the probability of success $P \in [\underline{P}, 1]$. A risky project yields X per unit invested with probability P, and 0 otherwise. We also assume $\underline{PX} = \rho$, so that the expected return of any risky project with $P \in (\underline{P}, 1]$ is higher than that of the safe technology.

Banks choose P and operating capacity z (or demand for funds) at an effort cost. The transformation of effort into a probability of project success $P \in [\underline{P}, 1]$ is interpreted as representing an *intermediation technology* that embeds banks' project screening and/or monitoring. Denoting with z total investment in the bank, the bank effort cost function is given by $C(P) = \frac{1}{2\alpha}P^2z$. Therefore, the bank intermediation technology exhibits constant returns to scale, as the effort cost to implement *P* is linearly related to *z*.⁵ The transformation of effort into productive capacity is a standard *production technology*. The effort cost of setting up operating capacity *z* is $c(z) = \frac{1}{2\beta}z^2$

Competition

To introduce different degrees of competition for funds, we assume that the agents who have chosen to be bankers can move at no cost to one of two unconnected locations, labeled M and C.

In location M, bankers are either unrestricted to communicate and choosing to behave cooperatively, or are endowed with the power to set up local monopolies. Thus, each bank in M acts as a monopolist, choosing project risk and deposit rates so as to maximize expected profits subject to depositors' participation constraints. Location M represents the monopolistic banking sector.

In location C, bankers do not communicate and compete for depositors' funds a la Bertrand. They set up competitive banks that choose project risk to maximize expected profits and deposit rates that maximize depositors' expected returns. Location C represents the competitive banking sector.

As bankers do not incur any cost in moving to either location, there is free entry in the monopolistic and competitive banking sectors, and banks in both locations distribute profits to bankers in equal shares.

For simplicity, we further assume that project risks are independent *across* locations, but perfectly correlated *within* locations. Denote with P_c and P_M the risk choices in the competitive and monopolistic sectors respectively. Then, projects are successful in both sectors with probability $P_c P_M$, successful only in the competitive sector with probability

⁵ The assumption of constant returns to scale in monitoring is fairly standard in the banking literature (see e.g. Besanko and Kanatas, 1993, Boot and Greenbaum, 1993, Boot and Thakor, 2000, Dell'Ariccia and Marquez, 2006, and Allen et al., 2011).

 $P_C(1-P_M)$, successful only in the monopolistic sector with probability $(1-P_C)P_M$, and fail in both sectors with probability $(1-P_C)(1-P_M)$.

Depositors

If an agent chooses to be a depositor, he will move to location C with probability σ , and to location M with probability $1-\sigma$. Since the remuneration of deposits in the monopolistic sector will be lower than in the competitive banking sector, agents are exposed to the risk of depositing in the monopolistic banking sector. Parameter σ can be viewed as indexing depositors' switching costs to move to the competitive banking sector, where they can get a better return.⁶ Thus, higher values of parameter σ index an increase in funding market competition. We assume that relocation risks are independent, so that σ is also the fraction of depositors moving to location C.⁷

Deposit insurance

Deposit insurance (DI) is pre-funded by taxation of initial resources A. The tax revenues are invested in the safe technology that yields ρ . Let τ denote the tax rate. The total "end-of-period" assets of the deposit insurance fund (DIF) are equal to $\tau A \rho$.

Denote with Z_c and Z_M total investment (deposits) in the competitive and monopolistic banking sectors respectively. A guarantee per unit of deposits $g \in (0,1]$ implies that the DIF will have contingent liabilities as follows. It will pay depositors nothing with probability P_cP_M , gZ_M with probability $P_c(1-P_M)$, gZ_c with probability $P_M(1-P_c)$, and $g(Z_c + Z_M)$ with probability $(1-P_M)(1-P_c)$. Note that if g = 0 there is *no* deposit insurance, if $g \in (0,1)$ there is a *partial* guarantee that covers a fraction g of the principal, whereas g = 1 is a full guarantee that repays the entire principal. Whatever is left in the DIF after payments to depositors is distributed lump-sum to all agents in equal shares.

⁶ For a model of funding competition through switching costs, see Park and Pennacchi (2009).

⁷ Introducing agents' heterogeneity with respect to switching costs yields the same results, but the algebra becomes more cumbersome without any additional insight. Assuming an exogenous probability of relocation for depositors is thus without loss of generality.

The DIF must have total assets whose value covers the worst-case payment outlays. Clearly, the DIF will not invest more than what is necessary to honor insurance in the worstcase outcome where all banks fail: doing that would be inefficient since the safe technology is dominated in rate of return by the risky technology. Hence, deposit insurance is feasible if the DIF can credibly guarantee payments in every contingency. This requires that total DIF assets equal total payments in the worst case outcome, that is, $\tau A\rho = g(Z_C + Z_M)$.

Contracts and sequence of decisions

Depositors finance the bank with simple debt contracts that pay a fixed amount R per unit invested if the outcome of the investment is successful and 0 otherwise. *Moral hazard* is introduced by assuming that bank choices of P are not observable by depositors. However, depositors take bank's optimal choice of P into account in their decision to accept the deposit terms offered by the bank.

Denote with x the *fraction* of bankers in C, with A_B the measure of bankers, with n_i the number of banks, with z_i bank size (capacity), and with R_i the deposit rates, for $i \in \{C, M\}$. Table 1 summarizes the sequence of decisions and the variable determined in the model.

Time	Agents' sequence of decisions	Determined variables
t=0	If $g \in (0,1]$, the deposit insurance fund (DIF) is established by taxing initial resources Agents choose to become bankers or depositors Bankers choose to locate in M or in C Depositors' locate in M or C according to their location draw The number of banks and debt equilibrium are determined	x(1-x) : fraction of bankers in C (M) A_B : measure of bankers $A-A_B$: measure of depositors σ : fraction of depositors in C n_C, n_M : number of banks in C and M Z_C, Z_M : total supply of funds (deposits) in the competitive and monopolistic sectors
t=1	Banks choose capacity (fund demand) Debt contract terms between the bank and depositors are determined. Banks choose risk.	z_C, z_M R_C, R_M : deposit rates in the competitive and monopolistic sectors Z_C, Z_M : total investment in the competitive

Table 1. Sequence of decisions and determined variables

Projects' output is realized and agents'	and monopolistic sectors
consumption follows.	P_{C}, P_{M} : risk choices in the competitive and
The DIF pays out depositors (if	monopolistic sectors
necessary) and distributes remaining	monoponstie sectors
funds in equal shares to all agents	

III. BANK PROBLEMS

We solve backward, starting with the competitive and monopolistic bank problems.

Competitive banks

The representative competitive bank chooses P_c to maximize

$$P_{C}(X-R_{C})z_{C} - \frac{1}{2\alpha}P_{C}^{2}z_{C} - \frac{1}{2\beta}z_{C}^{2} \quad (1)$$

The optimal interior solution is given by:

$$P_C^* = \alpha (X - R_C) \qquad (2)$$

We focus on interior solutions assuming the following sufficient condition for $P_c^* \in (\underline{P}, 1)$:

(A1) $\underline{P} < \alpha X \leq 1$,

Bertrand competition implies that R_c maximizes depositors' expected return, with depositors taking into account the optimal bank risk decision given by (2). Depositors' expected return is therefore:

$$P_{C}^{*}R_{C} + (1 - P_{C}^{*})g = \alpha(X - R_{C})(R_{C} - g) + g \quad (3).$$

This expected return is a strictly concave function of the deposit rate, with the maximum reached at:

$$R_{c}^{*} = \frac{X+g}{2}$$
 (4)

Substituting (4) in (2), the optimal risk choice of the competitive bank is:

$$P_C^* = \alpha(\frac{X-g}{2}) \qquad (5)$$

Using (4) and (5), the competitive bank expected profits are:

$$\Pi^{C}(z_{C}) \equiv \alpha \frac{(X-g)^{2}}{8} z_{C} - \frac{1}{2\beta} z_{C}^{2} \qquad (6)$$

Let $\pi_c \equiv \alpha \frac{(X-g)^2}{8}$. The optimal bank choice of capacity (or fund demand) z_c is $z_c = \beta \pi_c$ (7),

The expected per-unit bank profits are given by:

$$\frac{\Pi^C}{z_C} = \frac{\pi_C}{2} \quad (8)$$

Monopolistic banks

The representative monopolistic bank chooses (P_M, R_M) to maximize expected profits:

$$\Pi^{M} \equiv (P_{M}(X - R_{M}) - \frac{1}{2\alpha} P_{M}^{2}) z_{M} - \frac{1}{2\beta} z_{M}^{2} \qquad (9)$$

subject to the depositors' participation constraint

$$P_{M}^{*}R_{M} + (1 - P_{M}^{*})g \ge \rho \qquad (10),$$

where $P_M^* \in \arg \max \Pi^M$, and it is given by:

$$P_M^* = \alpha (X - R_M) \quad (11),$$

Since the monopolistic bank profit function is strictly decreasing in the deposit rate, constraint (10) is satisfied at equality, which can be written as:

$$(X - R_M)R_M - g(X - R_M) + \alpha^{-1}(g - \rho) = 0$$
 (12),

Equation (12) is equivalent to the quadratic equation:

$$R_{M}^{2} - (X+g)R_{M} + (gX - \alpha^{-1}(g-\rho)) = 0 \quad (13)$$

The smaller root of this equation, if it exists, is the solution of the monopolistic bank deposit rate. This root is given by:

$$R_{M}^{*} = \frac{X + g - \sqrt{X^{2} - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}}{2} \quad (14)$$

We introduce the following parametric assumptions to ensure well-defined deposit rates and existence of equilibriums with monopolistic banks. First, a sufficient condition for a non negative determinant of the solution to the quadratic equation for all $g \in [0,1]$ is

(A2)
$$X^2 - 4\alpha^{-1}\rho > 0$$
,

since $g(g+4\alpha^{-1}-2X) \ge 0$ by assumption (A1). Combining (A1) and (A2), the parameter α lies in the interval $[4\rho X^{-2}, X^{-1}]$. This interval is non-empty assuming

(A3)
$$X > 4\rho$$
.

Finally, a necessary condition for existence of equilibriums with banks is a strictly positive deposit rate, which holds if $X + g > \sqrt{X^2 - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}$. We assume this inequality holds, which can be easily shown to be equivalent to

(A4)
$$\rho > g$$
.

The optimal risk choice of the monopolistic bank is thus:

$$P_{M}^{*} = \alpha (X - R_{M}^{*}) = \alpha \frac{X - g + \sqrt{X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})}}{2}$$
(15)

Using (14) and (15), the expected profits of the monopolistic bank are:

$$\Pi^{M} \equiv \alpha \frac{(X - g + \sqrt{X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})})^{2}}{8} z_{M} - \frac{1}{2\beta} z_{M}^{2}$$
(16)

Let $\pi_M \equiv \alpha \frac{(X - g + \sqrt{X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})})^2}{8}$. The optimal bank choice of

capacity (or fund demand) z_M is

$$z_M = \beta \pi_M \quad (17),$$

The bank expected per-unit profits are given by:

$$\frac{\Pi^M}{z_M} = \frac{\pi_M}{2} \quad (18)$$

Comparing bank optimal choices

Recall that the risk of failure of the competitive and the monopolist banks are respectively $P_c^* = \alpha(X - R_c^*)$ and $P_M^* = \alpha(X - R_M^*)$. From equations (4) and (14), we see that:

$$R_{M}^{*} \equiv R_{C}^{*} - \frac{\sqrt{X^{2} - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}}{2}$$
(19)

where the term $\frac{\sqrt{X^2 - 4\alpha^{-1}\rho + g(g + 4\alpha^{-1} - 2X)}}{2}$ captures *monopoly rents*. Thus, we obtain

Lemma 1 For all $g \in [0,1]$, $P_C^* < P_M^*$

The risk of failure of the competitive bank is always strictly higher than that of the monopoly bank. This is the standard result implied by risk-shifting. Note that this result holds both under no deposit insurance (g = 0), and with deposit insurance ($g \in (0,1]$).

However, the relationship between deposit insurance coverage and bank risk differs for competitive and monopolistic banks. From Equation (5) we obtain:

Lemma 2 The risk of failure of the competitive bank increases monotonically with deposit insurance coverage.

By contrast, as it is evident from Equation (15), the risk of failure of the monopolistic bank has the first term decreasing in g, while the second term—which represents monopoly rents—increases in g, since $g + 4\alpha^{-1} - 2X > 0$ by assumption (A1). It turns out that the net effect of an increase in deposit insurance coverage on bank risk is negative, as shown in:

Lemma 3 The risk of failure of the monopolistic bank declines monotonically with deposit insurance coverage.

Proof: See Appendix

As a result of Lemmas 2 and 3, the difference in bank risk of failures of competitive and monopolistic banks increases with deposit insurance coverage. Yet, as we show below, different levels of deposit insurance coverage do not affect the welfare ranking of competitive conditions.

IV. EQUILIBRIUM AND WELFARE

For any given competition parameter $\sigma \in [0,1]$, and given banks' optimal choices of risk, deposit rates and their demand for funds $(P_i^*, R_i^*, z_i^*)_{i \in (C,M)}$ we have just determined, we complete the characterization of equilibriums by determining the seven-tuple $(\tau, x, A_B, (n_i, Z_i)_{i \in (C,M)})$, using correspondingly seven equilibrium conditions.

The first two of equilibrium conditions establish equality between the demand for funds of all banks in each sector to the supply of funds:

$$n_i z_i = Z_i \qquad \text{for} \quad i \in \{C, M\} \qquad (20i)$$

The third equilibrium condition establishes free-entry by bankers in the competitive and monopolistic sectors. Free entry implies that the returns of shares of bank ownership in the two sectors are equalized:

$$\frac{n_C \Pi^C}{x A_R} = \frac{n_M \Pi^M}{(1-x) A_R} \tag{21}$$

Specifically, Equation (21) says that the return of owning a bank share in the competitive sector, given by total profit $n_C \Pi^C$ divided by the number of bankers in that sector xA_B , equals the return of owning a bank share in the monopolist sector, given by total profit $n_M \Pi^M$ divided by the number of bankers in that sector $(1-x)A_B$.

The fourth equilibrium condition establishes the fraction of agents who decide to become bankers. This condition is determined by equalization of the return of shares of bank ownership with the expected return of deposits:

$$\frac{n_{c}\Pi^{c}}{xA_{B}} = \sigma(P_{c}R_{c} + (1 - P_{c})g) + (1 - \sigma)(P_{M}R_{M} + (1 - P_{M})g) =$$

$$\sigma P_{c}R_{c} + (1 - \sigma)\rho + g[\sigma(1 - P_{c}) + (1 - \sigma)(1 - P_{M})] \equiv r(\sigma, g)$$
(22)

Note that in (22), the term $r(\sigma, g)$ denotes the expected return on deposits of an agent who has chosen to be a depositor *prior to relocation* to the C or M locations.

The next two equilibrium conditions establish the supply of funds in the two sectors:

$$Z_{c} = \sigma(A(1-\tau) - A_{B}) \qquad (23)$$
$$Z_{M} = (1-\sigma)(A(1-\tau) - A_{B}) \qquad (24)$$

Finally, the seventh equilibrium condition determines the tax rate charged to set up the deposit insurance fund (DIF):

$$\rho\tau A = g(Z_C + Z_M) \Leftrightarrow \tau = \frac{gZ}{\rho A} \Longrightarrow 1 - \tau = \frac{\rho A - gZ}{\rho A}$$
(25)

The seven equations (20i)-(25) form a linear system that can be easily solved by substitution. Inserting (20i) in (21), and using (8) and (18), we obtain the equilibrium fraction of bankers who choose to operate in the competitive sector:

$$x = \frac{\pi_C Z_C}{\pi_C Z_C + \pi_M Z_M} \quad (26)$$

Equation (26) has a natural interpretation, as it says that the fraction of bankers choosing to operate as owners of a competitive bank is increasing in the ratio of total bank profits in the competitive sector $\pi_C Z_C$ to total bank profits $\pi_C Z_C + \pi_M Z_M$.

Inserting (26) in (22) yields the equilibrium measure of bankers:

$$A_{B} = \frac{\pi_{C}Z_{C} + \pi_{M}Z_{M}}{2r(\sigma,g)} \quad (27),$$

The measure of bankers is an increasing function of total bank profits $\pi_C Z_C + \pi_M Z_M$, and a decreasing function of the expected return of becoming a depositor.

Inserting (27) in (23) and (24), we obtain:

$$Z_{c} = \sigma(A(1-\tau) - \frac{\pi_{c}Z_{c} + \pi_{M}Z_{M}}{2r(\sigma,g)})$$
(28)
$$Z_{M} = (1-\sigma)(A(1-\tau) - \frac{\pi_{c}Z_{c} + \pi_{M}Z_{M}}{2r(\sigma,g)})$$
(29)

The total supply of deposits (investment) in the banking sectors is $Z \equiv Z_C + Z_M$, where $Z_C = \sigma Z$ and $Z_M = (1 - \sigma)Z$. Summing (28) and (29), using $Z_C = \sigma Z$ and $Z_M = (1 - \sigma)Z$, and solving for Z, we obtain:

$$Z(\sigma,g) = \frac{2r(\sigma,g)\rho}{2r(\sigma,g)(\rho+g) + (\pi_c\sigma + \pi_M(1-\sigma))\rho} A \quad (30)$$

Total investment in the banking sector can be expressed as a fraction of total available resources A, where the coefficient of proportionality depends on linear combinations of depositors' returns and profits in the competitive and monopolistic banking sectors.

An important implication of the model is summarized by the following

Lemma 4 For all $g \in [0,1], \frac{\partial Z}{\partial \sigma} > 0$

Proof: See Appendix

Lemma 4 says that an increase in bank competition increases intermediated investment (total deposits), at the same time reducing the amount of resources used in setting up banks. This occurs because an increase in the remuneration of bank-intermediated investment (deposits) resulting from an increase in competition (i.e. a decrease in the cost to access the competitive sector) prompts a larger fraction of agents to prefer to become depositors rather than bankers. As detailed momentarily, this mechanism is key for the determination of the general equilibrium effect of bank competition on welfare.

Welfare

As all agents are risk neutral, the welfare metric is expected total output net of total effort costs. Note that a welfare function evaluated at equilibrium values embeds the incentive compatibility constraint for banks due to their unobservable risk choices. For this reason, a maximum of the welfare function will measure *constrained* Pareto optimal outcomes.

The welfare function depends on the competition parameter σ and the level of deposit insurance coverage g, and is defined by:

$$Y(\sigma, g) \equiv P_{c}P_{M}X(Z_{c} + Z_{M}) + P_{c}(1 - P_{M})XZ_{c} + (1 - P_{c})P_{M}XZ_{M}$$

- $(\frac{1}{2\alpha}P_{c}^{2}Z_{c} + \frac{1}{2\alpha}P_{M}^{2}Z_{M}) - (n_{c}c(z_{c}) + n_{M}c(z_{M})) + \rho\tau A$ (31)

The first term is expected output in the competitive and banking sectors, the second and third terms are the sum of monitoring and capacity costs in the two sectors respectively, and the fourth one is the investment of tax receipts of the DIF. Using the equilibrium conditions (20i)-(25), the welfare function can be written as:

$$Y(\sigma,g) = [(P_C X - \frac{1}{2\alpha}P_C^2 - \frac{1}{2}\pi_C)\sigma + (P_M X - \frac{1}{2\alpha}P_M^2 - \frac{1}{2}\pi_M)(1-\sigma) + g]Z(\sigma,g)$$
(32)

The terms $P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2} \pi_C$ and $P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2} \pi_M$ are the expected outputs *net* of monitoring and production costs per unit of investment of the competitive and the monopolistic banking sectors respectively. Observe that:

$$\frac{\partial Y}{\partial \sigma} = \left[(P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2} \pi_C) - (P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2} \pi_M) \right] Z(\sigma, g) + \\ \left[(P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2} \pi_C) \sigma + (P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2} \pi_M) (1 - \sigma) + g \right] \frac{\partial Z}{\partial \sigma}$$
(33)

Clearly, if $P_C X - \frac{1}{2\alpha} P_C^2 - \frac{1}{2} \pi_C > P_M X - \frac{1}{2\alpha} P_M^2 - \frac{1}{2} \pi_M$, then the welfare function would be

strictly increasing in the competition parameter by Lemma 4. However, it is easy to generate numerical examples for which the above inequality is reversed. When this occurs, the expected outputs *net* of monitoring and production costs per unit of investment of the monopolistic sector may be *higher* than that in the competitive banking sector. Nevertheless, independently of the direction of the above inequality, we obtain the following

Proposition 1

For all
$$g \in [0,1]$$
, $\frac{\partial Y}{\partial \sigma} > 0$: perfect competition ($\sigma = 1$) is (constrained) Pareto optimal *Proof:* See Appendix

Proposition 1 says that even though banks risk of failure under perfect competition is higher than under imperfect competition, and even in the case the expected net output under competition might be lower than under monopoly, still the implied level of bank risk of failure and the overall resource allocation under perfect bank competition are optimal.

The quantitative dominance of the general equilibrium effect of bank competition—, which is captured by the increase in intermediated investment prompted by more competition illustrated in Lemma 4—drives this result. Specifically, an increase in the expected returns on deposits due to an increase in competition increases the measure of agents that choose to be depositors and correspondingly decreases the measures of agents choosing to be bankers. The increase in the supply of funds and the decrease in resources employed in setting up banks results in higher *total* expected output net of monitoring and production costs. In other words, there is a shift in the allocation of investment from bank intermediation to intermediated investment, which is generated endogenously by agents' optimal occupational choices and free entry into the banking sectors.

Social costs of bank failures

Restrictions on competition, as well as several regulations in banking, are typically justified by the existence of social costs associated with bank failures that are not internalized by banks. Would the welfare ranking of competitive conditions we obtained change by introducing social costs?

Assume that social costs are an increasing and convex function of intermediated investment as follows: they are 0 with probability $P_C P_M$, CZ_M^{γ} with probability $P_C(1-P_M)$, CZ_C^{γ} with probability $P_M(1-P_C)$, and $C(Z_C^{\gamma}+Z_M^{\gamma})$ with probability $(1-P_M)(1-P_C)$, with $\gamma \ge 1$ and C > 0.

Therefore, expected social costs from bank failures are given by:

$$SC(\sigma,g) \equiv P_{C}(1-P_{M})CZ_{M}^{\gamma} + (1-P_{C})P_{M}CZ_{C}^{\gamma} + (1-P_{M})(1-P_{C})C(Z_{C}^{\gamma} + Z_{M}^{\gamma}) =$$

$$C[(1-P_{M})(1-\sigma)^{\gamma} + (1-P_{C})\sigma^{\gamma}]Z(\sigma,g)^{\gamma}$$
(34)

A welfare function augmented with social cost is defined by:

$$W(\sigma, g) \equiv Y(\sigma, g) - SC(\sigma, g) \tag{35}$$

However, social costs cannot be assumed arbitrarily large, since they need to be consistent with the existence of bank intermediation. In our model, we must require that social costs are not greater than what an economy could achieve by just investing in the safe asset. Without this requirement, it might be optimal to invest all resources in the safe asset, making bank intermediation inessential. This requirement implies an upper bound on C, which must hold for all competitive conditions and all levels of deposit insurance coverage. This upper bound on C therefore becomes a function of these parameters, and is implicitly defined by the following inequality:

$$W(\sigma, g) \equiv Y(\sigma, g) - SC(\sigma, g) \ge \rho A \text{ for all } \sigma \in [0, 1] \text{ and } g \in [0, 1]$$
(36)

A social cost function that satisfies (35) is called *admissible*.

The following result establishes the constrained optimality of perfect bank competition also in the presence of social costs of bank failures:

Proposition 2

For any admissible social cost function that is increasing and convex in investment

(deposits), for all $g \in [0,1]$, and if bank intermediation is essential, then $\frac{\partial W}{\partial \sigma} > 0$: perfect competition ($\sigma = 1$) is (constrained) Pareto optimal. *Proof:* See Appendix

V. CONCLUSION

We studied a general equilibrium model in which banks make their investment and financing decisions under moral hazard. The model exhibits all features of a large partial equilibrium banking literature which obtains contrasting results with respect to the ranking of bank's risk of failure according to competitive conditions, but does not address the key normative issue of whether there exists a trade-off between bank competition and financial stability.

We showed that perfect competition in banking is constrained Pareto optimal, even though the risk of failure of a competitive bank may be higher than that of a bank operating in imperfect competition, and even when social costs are taken into account. Welfare implications derived from partial equilibrium modeling are likely to result in unwarranted normative prescriptions.

A general equilibrium perspective on desirable banking systems' structures and welfare-improving bank regulation has only slowly entered the current policy discourse, with theoretical explorations still limited in numbers. While capturing the essential features of several set-ups studied in a large partial equilibrium banking literature, our model is still highly stylized and richer versions of it undoubtedly deserve further study. Yet, the recent financial crisis provides a stark example of the dichotomy between a general and a partial equilibrium view of the world, where in the latter one banks are evaluated as individual entities and not as part of a system. General equilibrium modeling of intermediation appear an essential tools to throw light on the desirable level of systemic risk in the economy, and how it could be attained.

APPENDIX

Lemma 3 The risk of failure of the monopolistic bank declines monotonically with deposit insurance coverage.

Proof: Differentiating Equation (14) with respect to g we get:

$$\frac{dR_{M}^{*}}{dg} = \frac{1}{2} \left(1 - \frac{1}{2} \left(X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})\right)^{-1/2} \left(2g - 2X + 4\alpha^{-1}\right)\right)$$

Therefore, $sign\{\frac{dR_{M}^{*}}{dg}\} = sign\{1 - \frac{1}{2}(X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{-1/2}(2g - 2X + 4\alpha^{-1})\}$

 $\frac{dR_{M}}{dg} < 0 \text{ is equivalent to the following inequalities:}$ $1 < \frac{1}{2} (X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{-1/2} (2g - 2X + 4\alpha^{-1}) \Leftrightarrow$ $2(X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}))^{1/2} < 2g - 2X + 4\alpha^{-1} \Leftrightarrow$ $4(X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})) < (2(g - X) + 4\alpha^{-1})^{2} \Leftrightarrow$ $4X^{2} - 16\alpha^{-1}\rho + 4g(g - 2X + 4\alpha^{-1}) < 4(g - X)^{2} + 16\alpha^{-2} + 16(g - X)\alpha^{-1} \Leftrightarrow$ $4X^{2} - 16\alpha^{-1}\rho + 4g^{2} - 8gX + 16g\alpha^{-1} < 4g^{2} + 4X^{2} - 8gX + 16\alpha^{-2} + 16\alpha^{-1}g - 16X\alpha^{-1} \Leftrightarrow$ $-16\alpha^{-1}\rho < 16\alpha^{-2} - 16X\alpha^{-1} \Leftrightarrow$ $-\rho < \alpha^{-1} - X$ By (A1), $\alpha^{-1} - X \ge 0$. Therefore, $\frac{dR_{M}}{d\rho}^{*} < 0$, which implies $\frac{dP_{M}}{d\rho}^{*} > 0$ by Equation (15).

Lemma 4 For all $g \in [0,1]$, $\frac{\partial Z}{\partial \sigma} > 0$

Proof:

$$\frac{\partial Z}{\partial \sigma} = \frac{2A}{(.)^2} \left(\frac{\partial r}{\partial \sigma} \rho [2r(\sigma, g)(\rho + g) + (\pi_c \sigma + \pi_M (1 - \sigma))\rho] - r(\sigma, g)\rho [2\frac{\partial r}{\partial \sigma}(\rho + g) + \rho(\pi_c - \pi_M) \Leftrightarrow \frac{2A\rho}{(.)^2} \left(\frac{\partial r}{\partial \sigma} (\pi_c \sigma + \pi_M (1 - \sigma)) - r(\sigma, g)\rho(\pi_c - \pi_M) \right)$$

QED

The term
$$\frac{\partial r}{\partial \sigma}(\pi_c \sigma + \pi_M(1 - \sigma)) - r(\sigma, g)\rho(\pi_c - \pi_M)$$
 is strictly positive for all $g \in [0, 1]$, since
 $\frac{\partial r}{\partial \sigma} = P_c R_c - \rho + g(P_M - P_c) > 0$ and $\pi_c < \pi_M$. Thus, $\frac{\partial Z}{\partial \sigma} > 0$.
OED

Proposition 1

For all $g \in [0,1]$, $\frac{\partial Y}{\partial \sigma} > 0$: perfect competition is (constrained) Pareto optimal

Proof:

Using the bank profit functions in the two sectors, we can write:

$$P_{C}X - \frac{1}{2\alpha}P_{C}^{2} = \pi^{C} + P_{C}R_{C} \text{ (a)}$$
$$P_{M}X - \frac{1}{2\alpha}P_{M}^{2} = \pi^{M} + \rho \text{ (b)}$$

Hence, expected output net of monitoring and production costs in the two sectors are::

$$P_{C}X - \frac{1}{2\alpha}P_{C}^{2} - \frac{1}{2}\pi_{C} = P_{C}R_{C} + \frac{1}{2}\pi_{C} \text{ (c)}$$
$$P_{M}X - \frac{1}{2\alpha}P_{M}^{2} - \frac{1}{2}\pi_{M} = \pi^{M} + \rho - \frac{1}{2}\pi_{M} = \rho + \frac{1}{2}\pi_{M} \text{ (d)}$$

Substituting (c) and (d) in (32), and using (30), we can write:

$$Y(\sigma,g) \equiv [(P_{c}X - \frac{1}{2\alpha}P_{c}^{2} - \frac{1}{2}\pi_{c})\sigma + (P_{M}X - \frac{1}{2\alpha}P_{M}^{2} - \frac{1}{2}\pi_{M})(1-\sigma) + g]Z(\sigma,g) = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma) + g]r(\sigma,g)\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + (1-\sigma)\rho) + \pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + \pi_{M}(1-\sigma)) + \pi_{c}\sigma + \pi_{M}(1-\sigma)]}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + \pi_{M}(1-\sigma)) + \pi_{c}\sigma + \pi_{M}(1-\sigma)]}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + \pi_{M}(1-\sigma)) + \pi_{m}(1-\sigma)]}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + \pi_{M}(1-\sigma)) + \pi_{m}(1-\sigma)]}{2r(\sigma,g)(\rho+g) + (\pi_{c}\sigma + \pi_{M}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + \pi_{M}(1-\sigma)]}{2r(\sigma,g)(\rho+g) + (\pi_{m}(1-\sigma))\rho}A = \frac{[2(P_{c}R_{c}\sigma + \pi_{M}(1-\sigma)]}{2r(\sigma+g)(1-\sigma)}A = \frac{[2(P_{c$$

Let g = 0. Then

$$Y(\sigma,0) = \frac{\left[2(P_C R_C \sigma + (1-\sigma)\rho) + \pi_C \sigma + \pi_M (1-\sigma)\right]r(\sigma,0)}{2r(\sigma,0) + (\pi_C \sigma + \pi_M (1-\sigma))}A$$

Since $r(\sigma, 0) = \sigma P_C R_C + (1 - \sigma)\rho$, $Y(\sigma, 0)$ can be written as:

$$Y(\sigma, 0) = \frac{[2(\sigma P_{c} R_{c} + (1 - \sigma)\rho) + \pi_{c} \sigma + \pi_{M})(1 - \sigma)]r(\sigma, 0)}{2(\sigma P_{c} R_{c} + (1 - \sigma)\rho) + (\pi_{c} \sigma + \pi_{M}(1 - \sigma))} A = r(\sigma, 0)A$$

Thus,
$$\frac{\partial Y}{\partial \sigma} = \frac{\partial r}{\partial \sigma} (\sigma, 0) A = (P_c R_c - \rho) A > 0$$
, since $P_c R_c > \rho$.

Let $g \in (0,1]$ and re-write:

$$Y(\sigma, g) = h(\sigma) f(\sigma) A$$
.

where

$$f(\sigma) = \frac{r(\sigma, g)\rho}{2r(\sigma, g)(\rho + g) + (\pi_c \sigma + \pi_M (1 - \sigma))\rho}$$
$$h(\sigma) = 2(P_c R_c \sigma + (1 - \sigma)\rho) + \pi_c \sigma + \pi_M (1 - \sigma) + g$$

Next, we show that both functions $f(\sigma)$ and $h(\sigma)$ are monotonically increasing in σ . Consider

$$f'(\sigma) = \frac{1}{(2r(\sigma,g)(\rho+g) + (\pi_c \sigma + \pi_M (1-\sigma))\rho)^2} x$$
$$\frac{\partial r}{\partial \sigma} \rho 2r(\sigma,g)(\rho+g) + \frac{\partial r}{\partial \sigma} \rho (\pi_c \sigma + \pi_M (1-\sigma))\rho)$$
$$-r(\sigma,g)\rho 2r'(\sigma,g)(\rho+g) - r(\sigma,g)\rho (\pi_c - \pi_M)\rho =$$
$$\frac{(\frac{\partial r}{\partial \sigma} \rho (\pi_c \sigma + \pi_M (1-\sigma))\rho) - r(\sigma,g)\rho (\pi_c - \pi_M)\rho)}{(2r(\sigma,g)(\rho+g) + (\pi_c \sigma + \pi_M (1-\sigma))\rho)^2}$$

By Lemma 3, $\frac{\partial r}{\partial \sigma} \rho(\pi_c \sigma + \pi_M (1 - \sigma))\rho) - r(\sigma, g)\rho(\pi_c - \pi_M)\rho) > 0$, hence $f'(\sigma) > 0$.

Now consider:

$$h'(\sigma) \equiv 2(P_C R_C - \rho) + \pi_C - \pi_M$$

Using equilibrium values, this derivative can be written as:

$$h'(\sigma) = 2(P_{C}R_{C} - \rho) + \pi_{C} - \pi_{M} = 2\alpha(\frac{X-g}{2})\frac{X+g}{2} + (\alpha\frac{(X-g)^{2}}{8} - \alpha\frac{(X-g+\sqrt{X^{2}-4\alpha^{-1}\rho+g(g-2X+4\alpha^{-1})})^{2}}{8})^{(\mathbf{B})}$$

Therefore:

$$h'(\sigma) \Leftrightarrow 2\alpha(\frac{X-g}{2})\frac{X+g}{2} > \frac{X^2 - 4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 2(X-g)\sqrt{X^2 - 4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1})}}{8} + 2\rho^{(e)}$$

By (A4):

$$\sqrt{X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})} < X + g \Leftrightarrow X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}) < (X + g)^{2}$$

Therefore:

$$\alpha \frac{X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}) + 2(X - g)\sqrt{X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1})}}{8} < \frac{X^{2} - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}) + 2(X - g)(X + g)}{8}$$

Hence, inequality (e) is satisfied if:

$$2\alpha(\frac{X-g}{2})\frac{X+g}{2} > \frac{X^2 - 4\alpha^{-1}\rho + g(g - 2X + 4\alpha^{-1}) + 2(X-g)(X+g)}{8}) + 2\rho$$
(f)

Note that if (f) holds, we can write it as:

$$2\alpha(\frac{X-g}{2})\frac{X+g}{2} > \frac{X^{2}-4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 2(X-g)(X+g)}{8}) \Leftrightarrow$$

$$4(X^{2}-g^{2}) > X^{2}-4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 2(X^{2}-g^{2}) + 16\rho \Leftrightarrow$$

$$2X^{2}-2g^{2} > X^{2}-4\alpha^{-1}\rho + g(g-2X+4\alpha^{-1}) + 16\rho \Leftrightarrow$$

$$X^{2} > -4\alpha^{-1}\rho + 2g^{2} + g(g-2X+4\alpha^{-1}) + 16\rho \Leftrightarrow$$

$$X^{2} + 4\alpha^{-1}(\rho-g) + g(2X-g) > 2g^{2} + 16\rho \Leftrightarrow$$

$$X^{2} + 4\alpha^{-1}(\rho-g) + g2X > 3g^{2} + 16\rho$$

By (A3) and (A4), $g2X > 3g^2$, since $X > 4\rho > \frac{3}{2}g$, and by (A3) $X^2 > 16\rho$, which implies that inequality (f) holds since $X^2 + 4\alpha^{-1}(\rho - g) + g2X > 3g^2 + 16\rho$. Hence, $h'(\sigma) > 0$. In conclusion, $Y(\sigma, g) = h(\sigma)f(\sigma)A$ is strictly increasing in σ since both component functions are increasing in σ .

QED

Proposition 2

For any admissible social cost function that is increasing and convex in investment

(deposits), for all $g \in [0,1]$, and if bank intermediation is essential, then $\frac{\partial W}{\partial \sigma} > 0$: perfect

competition ($\sigma = 1$) is (constrained) Pareto optimal.

Proof:

The welfare function (35) can be written as:

$$W(\sigma, g) = Z(\sigma, g)[(P_{c}R_{c}\sigma + \rho(1-\sigma) + \frac{1}{2}\pi_{c}\sigma + \frac{1}{2}\pi_{M}(1-\sigma) + g] + -C[(1-P_{M})(1-\sigma)^{\gamma} + (1-P_{c})\sigma^{\gamma}]Z(\sigma, g)^{\gamma}$$

Thus, the upper bound established by inequality (36) for all $\sigma \in [0,1]$ and $g \in [0,1]$ is:

$$W(\sigma,g) = Z(\sigma,g)[(P_{c}R_{c}\sigma + \rho(1-\sigma) + \frac{1}{2}\pi_{c}\sigma + \frac{1}{2}\pi_{M}(1-\sigma) + g] + -C[(1-P_{M})(1-\sigma)^{\gamma} + (1-P_{c})\sigma^{\gamma}]Z(\sigma,g)^{\gamma} \ge \rho A \Longrightarrow$$
$$C \le \frac{Z(\sigma,g)[(P_{c}R_{c}\sigma + \rho(1-\sigma) + \frac{1}{2}\pi_{c}\sigma + \frac{1}{2}\pi_{M}(1-\sigma) + g] - \rho A}{[(1-P_{M})(1-\sigma)^{\gamma} + (1-P_{c})\sigma^{\gamma}]Z(\sigma,g)^{\gamma}} \equiv \overline{C}(\sigma,g)$$

A lower bound to any welfare function can be defined as:

$$\underline{W}(\sigma,g) = Z(\sigma,g)[(P_{c}R_{c}\sigma + \rho(1-\sigma) + \frac{1}{2}\pi_{c}\sigma + \frac{1}{2}\pi_{M}(1-\sigma) + g] + -\overline{C}(\sigma,g)[(1-P_{M})(1-\sigma)^{\gamma} + (1-P_{c})\sigma^{\gamma}]Z(\sigma,g)^{\gamma} = Z(\sigma,g)g + \rho A$$

Since $W(\sigma, g) \ge \underline{W}(\sigma, g)$, then $\frac{\partial W}{\partial \sigma} \ge \frac{\partial \underline{W}}{\partial \sigma} = 0$, therefore $W(\sigma, g)$ is monotonically

increasing in σ . If $\frac{\partial W}{\partial \sigma} = 0$ for some pair (σ, g) , then $W(\sigma, g)$ is a constant. If $W(\sigma, g)$ is a constant, then $W(\sigma, g) = \rho A$. In this case all resources are invested in the safe asset and bank intermediation is inessential. Thus, if bank intermediation is essential, then $\frac{\partial W}{\partial \sigma} > 0$: perfect bank competition ($\sigma = 1$) is constrained Pareto optimal. QED

REFERENCES

- Allen, Franklin, Elena Carletti and Robert Marquez, 2011, "Credit competition and capital regulation", *Review of Financial Studies*, 24(4), 983-1018.
- Allen, Franklin, and Douglas Gale, 2000, "Comparing Financial Systems" (MIT Press, Cambridge, Massachusetts)
- Allen, Franklin, and Douglas Gale, 2004a, "Competition and Financial Stability", *Journal of Money, Credit and Banking*, 36(2), 453-480.
- Allen, Franklin, and Douglas Gale, 2004b, "Financial Intermediaries and Markets", *Econometrica*, Vol. 72, 4, 1023-1061.
- Besanko, David, and Kanatas, George, 1993, Credit market equilibrium with bank monitoring and moral hazard, *Review of Financial Studies*, Vol. 6, N.1: 213-232.
- Boot, Arnoud W., and Greenbaum, Stuart (1993) "Bank regulation, reputation, and rents: Theory and policy implications". In: Mayer, C., and Vives, X. (eds), Capital markets and financial intermediation. Cambridge, UK: Cambridge University Press, 292-318.
- Boyd, John H., and Gianni De Nicolò, 2005, "The Theory of Bank Risk Taking and Competition Revisited", *Journal of Finance*, 60, 3, 1329-1343.
- Boyd, John H., Gianni De Nicolò and Bruce D. Smith, 2004, "Crises in Competitive versus Monopolistic Banking Systems", *Journal of Money Credit and Banking* 36, 3, 487-506.
- Cordella Tito, and Levi-Yeyati, 2002, "Financial Opening, Deposit Insurance, and Risk in A Model of Banking Competition", *European Economic Review*, 46, 693-733.
- Dell'Ariccia, Giovanni, and Marquez, Robert, 2006, "Competition among regulators and credit market integration", *Journal of Financial Economics*, 79, :401-430.
- Hellmann, Thomas, Kevin Murdock and Joseph Stiglitz, 2000, "Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough?" *American Economic Review* 90(1), 147–165.
- Holmstrom, Bengt, and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds, and the Real Sector" *The Quarterly Journal of Economics*, CXII, 3: 663-691.
- Keeley, Michael, 1990, "Deposit Insurance, Risk and Market Power in Banking", *American Economic Review*, 80, 1183–1200.
- Klemperer, P., 1995, "Competition when Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics and International Trade," *Review of Economic Studies*, 62, 515–539.
- Matutes, Carmen, and Vives, Xavier, 1996, "Competition for Deposits, Fragility, and Insurance", *Journal of Financial Intermediation* 5, 186-216.

- Martinez-Miera, David, and Repullo, Raphael, 2010, "Does Competition Reduce the Risk of Bank Failure?", *Review of Financial Studies* 23 (10), 3638-3664.
- Morrison, Alan D., and Lucy White, 2005, Crises and Capital Requirements in Banking,: *American Economic Review*, Vol. 95, No. 5 :1548-1572.
- Park, Kwangwoo and George Pennacchi, 2009, "Harming Depositors and Helping Borrowers: The Disparate Impact of Bank Consolidation", *Review of Financial Studies*, 22: 1 1-40
- Repullo, Raphael, 2004, "Capital Requirements, Market Power, and Risk-Taking in Banking", *Journal of Financial Intermediation*, Vol. 13: 156-182.