

Government Guarantees and Financial Stability

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Abstract

We analyze the desirability of government intervention in the banking sector in a context where both panic and fundamental crises are possible and banks' and depositors' withdrawal decision are endogenously determined. We show that different guarantee schemes differ in terms of their effectiveness in limiting the occurrence of runs, efficiency and costs. Any guarantee scheme leads to a moral hazard problem in that banks have an incentive to over-exploit the guarantee and take excessive risk. However, the severity of the moral hazard problem differs across schemes and depends on the time and size of the guarantee on offer as well as on the government's budget. When the government's budget is large, blanket guarantees removing all runs are more efficient than more limited form of guarantees. On the contrary, when the budget is limited, more moderate forms of guarantees limiting the moral hazard problem are more efficient even if panic and fundamental runs are not completely prevented.

1 Introduction

The 2008 financial crisis highlighted the inadequacy of the existing financial safety net in preventing the crisis and mitigating its negative effects. During the crisis, governments had to implement extraordinary emergency measures to preserve the stability of the financial system. The public intervention ranged from extensions of the coverage and scopes of the existing deposit insurance schemes to the introduction of new schemes and generalized guarantees. In

some cases, like in Ireland, the extension of the existing guarantee scheme had immediate negative effects on the solvency of the government. In general, it affected banks' incentives toward risk. The massive intervention during the crisis and its consequences re-opened the debate about the optimality of government intervention, its effects on sovereign solvency and on the risk-taking incentives of financial institutions.

The desirability of the guarantee schemes and other forms of interventions depends on the type of runs (panic or fundamental) considered. In the traditional literature on banking crises (e.g., Diamond and Dybvig, 1983), deposit insurance and other forms of guarantees can costlessly prevent instability since crises are a pure panic phenomenon and the government can fully commit to intervene. By contrast, when bank runs are linked to a deterioration of banks' assets, government intervention can entail substantial costs. The anticipation of the intervention worsens banks' incentives to behave prudently and limits market discipline as depositors don't have any longer an incentive to monitor their banks (see, e.g., Boot and Greenbaum, 1993). Consequently, risk is shifted onto the deposit insurer and there may be an actual disbursement for the government.

These contributions have two main limits. The first one is that the probability of occurrence of crises is exogenous. The second one is that they either consider panic or fundamental runs. In contrast to the previous literature, we analyze the desirability of government intervention in the banking sector in a context where both panic and fundamental crises are possible, and both banks' and depositors' withdrawal decision are endogenously determined.

We build on a standard banking model as developed in Goldstein and Pauzner (2005) in which depositors' withdrawal decisions are determined by using global game methodology. There are two periods. Banks raise funds from risk-averse consumers in the form of deposits and invest them in risky projects whose return depends on the fundamental of the economy. Depositors derive utility from con-

suming both a private and a public good. At the interim date, each depositor receives an imperfect signal regarding the fundamentals and decides when to withdraw based on the information received. In deciding whether to run or not, depositors compare the payoff they would get from going to the bank prematurely and waiting until maturity. These payoffs depend on the fundamentals and the proportion of depositors running. In this setting, bank runs result both from coordination failures among depositors (panic runs) or from real negative shocks (fundamental runs). The model has a unique equilibrium in which runs occur if and only if the fundamentals of the economy are below a certain threshold level. Thus, the probability of the occurrence of a run is uniquely determined and depends on the value of fundamentals and on the amount of risk chosen by the bank as represented by the deposit contract offered to depositors.

We first show that the decentralized solution is inefficient due to the coordination problem among depositors. To contain the occurrence of panic runs, banks offer a deposit contract that entail too little risk sharing relative to a social planner solution where only fundamental runs occur. Then, we consider the case in which the government prevents the occurrence of runs by guaranteeing (at least part of) the promised repayments through the transfer of resources from the public good to the banking sector.

We start by analyzing the case of a social planner that chooses both the amount of transfer to the banking sector and the deposit contract. The allocation is superior to the case without intervention, showing that intervening is optimal in the absence of moral hazard. Then, we analyze whether the efficient allocation can be implemented in a decentralized economy where the government can decide on the transfer but not on the deposit contract offered by banks. We consider different guarantee schemes ranging from standard deposit insurance schemes, to blanket guarantees, to a more moderate form of intervention where the amount of resources transferred to the banking sector and the

repayment that depositors receive in case of runs is conditional on the amount of resources that the bank has available at the time of the intervention. For each guarantee scheme we determine the probability of panic and fundamental runs, the optimal deposit contract offered by the bank and the cost of intervention as represented by a lower provision of the public good. We show that these interventions have very different implications in terms of probability and types of runs, risk sharing properties of the deposit contract and social welfare.

The introduction of any guarantee scheme generates a trade-off. On one hand, it reduces depositors' incentives to run and thus the probability of crises. On the other hand, as banks do not fully internalize the costs of the intervention, they are induced to take more risk in the form of a higher repayment to depositors withdrawing prematurely. This in turn increases the government's disbursement and, in case runs are not eliminated, also depositors' incentives to withdraw early. When the moral hazard problem is severe, the intervention can be counterproductive and lead to more runs and lower social welfare relative to the case without intervention.

The severity of the moral hazard problem and the welfare implications vary across the different types of guarantees depending on the time and the size of intervention as well as the amount of public resources available in the economy to finance it. In most cases, more moderate form of intervention are more efficient as the limitation in the coverage and scope of the guarantee increases the incentives of banks to behave prudently, thus reducing the probability of runs and the disbursement for the government. However, when the government's resources are abundant, a guarantee scheme, like the unlimited guarantees offered by the Irish government, can be better than a more moderate form of intervention as it removes all types of runs and offer a better risk-sharing to depositors.

The novelty of the paper is to analyze the effects of the introduction of guarantees in the banking sector in a context in which both fundamental and panic

crises are possible and both banks' and depositors' decisions are endogenously determined. In this sense, the paper is linked to various strands of the literature. A few papers have looked at the distortions entailed by deposit insurance and other forms of guarantees. Boot and Greenbaum (1993) and Cooper and Ross (2002) highlight that public guarantees eliminate runs but at the cost of reducing the incentive of depositors to monitor banks, thus increasing the occurrence of crises and the disbursement for the government. These findings are also supported by empirical findings in Demirgüç-Kunt and Detragiache (2001), Demirgüç-Kunt and Huizinga (2003) and Ioannidou and Penas (2010).

The closest paper to ours is Keister (2010), who analyzes the desirability of bailouts in a setting with limited commitment in which banks anticipate that self-fulfilling runs can occur with a certain exogenous probability. The paper shows that the decentralized economy without intervention is inefficient since banks invest excessively in short term assets as a form of private insurance against runs. In contrast, when bailouts protecting depositors in case of a bank's failure are possible, banks undertake an excessive maturity transformation as they invest excessively in the long-term asset. As in our framework, this suggests that government intervention leads to moral hazard. The main difference in our paper is that we are able to endogenize the probability with which both fundamental and panic runs can occur. This allows us to assess the effects of various governmental policies on the occurrence of crises, banks' risk-taking incentives, financial stability and solvency of the government.

The paper proceeds as follows. Section 2 describes the model without government intervention. Section 3 derives the decentralized solution. Section 4 looks at the allocation of a social planner that observes the fundamentals of the economy and can transfer resources to the banking sector. Section 5 derives the decentralized solution when different types of deposit insurance are introduced. Section 6 analyzes the government's choice of the optimal size of intervention

and characterizes the decentralized solution. Section 7 uses a parametric example to illustrate the properties of the model. Section 8 concludes.

2 The basic model

The basic model is based on Goldstein and Pauzner (2005). There are three dates ($t = 0, 1, 2$), one good and a continuum $[0, 1]$ of banks and consumers.

Banks raise one unit of funds from consumers in exchange for a deposit contract as specified below, and invest them in a risky project. For each unit invested at date 0, the project returns 1 if liquidated at date 1 and a stochastic return \tilde{R} at date 2 given by

$$\tilde{R} = \begin{cases} R > 1 & \text{w. p. } p(\theta) \\ 0 & \text{w. p. } (1 - p(\theta)), \end{cases}$$

with $\theta \sim U[0, 1]$ and $p'(\theta) > 0$. The variable θ represents the state of the economy and is unknown to agents before date 2. Also, $E_\theta[p(\theta)]R > 1$ must hold, so that the expected long-run return of the project is superior to the short-run return.

Consumers are endowed with 1 unit at date 0 and nothing thereafter. At date 0 they deposit their endowment at the bank in exchange for a promised consumption c_1 at date 1 or a risky return \tilde{c}_2 at date 2. Consumers are ex ante identical but of either of two types of ex post: each of them has a probability λ of being early and consuming at date 1, and $1 - \lambda$ of being late and consuming at either date 1 or 2. Consumers learn their type (which remains their private information) at date 1, and derive utility both from consuming at date 1 or 2 and from enjoying a public good g so that their utility is

$$U(c, g) = u(c) + v(g)$$

with $u'(c) > 0$, $v'(g) > 0$, $u''(c) < 0$, $v''(g) < 0$, $u(0) = v(0) = 0$ and $-cu''(c)/u'(c) > 1$, i.e., the relative risk averse coefficient is greater than one.

At the beginning of date 1, each consumer (also depositor thereafter) receives a private signal x_i regarding the fundamental of the economy θ of the form

$$x_i = \theta + \epsilon_i, \tag{1}$$

with $\epsilon_i \sim U[-\epsilon, +\epsilon]$ being i.i.d. across agents. Based on the signal, depositors update their beliefs about the fundamental θ and the actions of the other depositors. Early depositors always withdraw at date 1, whereas late depositors withdraw at date 1 if they receive a low enough signal or expect the other depositors withdrawing early. The bank satisfies consumers' withdrawal demands by liquidating the long term asset. If it liquidates all of it, depositors receive a pro-rata share of the bank resources.

The banking sector is perfectly competitive so that banks choose the deposit contract (c_1, \tilde{c}_2) to maximize depositors' expected utility.

3 The decentralized equilibrium

We solve the decentralized equilibrium by analyzing first depositors' withdrawal decisions at date 1 for a given deposit contract, and then by looking at the optimal choice of c_1 .

Early consumers always withdraw at date 1, while late consumers base their actions on the signal received. In particular, late depositors compare the expected payoffs from going to the bank at date 1 or at date 2. These ex post payoffs depend on the realization of θ as well as on the proportion n of depositors withdrawing at date 1. Since the signal x_i gives depositors information about both θ and n , it affects the calculation of their expected payoffs and thus their actions. As required to have a unique equilibrium in depositors' withdrawal decisions, we assume that there are two extreme regions in which fundamentals are extremely bad or extremely good. This implies that in these ranges the action of each late depositor is independent of his beliefs about the others'

behavior. We start with the lower. Note also that we focus on the case where ε is arbitrarily close to zero so that all depositors receive the same signal at the limit.

Lower Dominance Region. When the fundamentals are very bad (θ is very low), the expected utility from waiting until date 2 is lower than that from withdrawing at date 1, even if only the early types were to withdraw ($n = \lambda$). In this case, running is a dominant strategy for each late depositor. We then denote as $\underline{\theta}(c_1)$ the solution to

$$u(c_1) = p(\theta)u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right),$$

that is

$$\underline{\theta}(c_1) = p^{-1} \frac{u(c_1)}{u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right)}. \quad (2)$$

We refer to the interval $[0, \underline{\theta}(c_1)]$ as the lower dominance region, where fundamental runs occur. Note that $\underline{\theta}(c_1)$ is increasing in c_1 . Defining $c_{2\lambda} = \frac{1 - \lambda c_1}{1 - \lambda}R$, we have

$$\frac{\partial \underline{\theta}(c_1)}{\partial c_1} = \frac{p^{-1}}{\left[u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right)\right]^2} \left[u'(c_1)u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right) + \frac{u(c_1)\lambda R}{1 - \lambda} u'\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right) \right] > 0$$

since $u'(c_1) > 0$. Thus, as c_1 increases, the lower dominance region becomes bigger and fundamental runs are more likely to occur.

Upper Dominance Region. Similarly, the upper dominance region corresponds to the range of θ denoted as $(\bar{\theta}, 1]$ in which fundamentals are so good that no late depositors withdraw at date 1. As in Goldstein and Pauzner (2005) we assume that in this region the project is safe, i.e., $p(\theta) = 1$, and that it gives the same return R at dates 1 and 2 so that the liquidation cost is zero. These assumptions guarantee that no late depositor has an incentive to withdraw prematurely if he observes a signal in the interval $(\bar{\theta}, 1]$ since his payment at date 2 is guaranteed.

The Intermediate Region

The two dominance regions are just extreme ranges of fundamentals in which agents have a dominant strategy. In the range $(\underline{\theta}(c_1), \bar{\theta}]$, a depositor's decision to withdraw prematurely depends crucially on the signal x_i as it determines the beliefs about other agents' actions. To determine late depositors' withdrawal decisions in this region, we first look at their utility differential between withdrawing at date 2 and at date 1 as given by

$$v(\theta, n) = \begin{cases} p(\theta)u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) & \text{if } \lambda \leq n < n^* \\ 0 - u\left(\frac{1}{n}\right) & \text{if } n^* \leq n < 1, \end{cases} \quad (3)$$

where $n^* = 1/c_1$. The expression for $v(\theta, n)$ states that as long as the bank does not exhausts its resources at date 1, i.e., for $n < n^*$, depositors withdrawing early obtain c_1 and those waiting obtain the residual $\frac{1-nc_1}{1-n}R$ with probability $p(\theta)$. By contrast, for $n \geq n^*$ the bank exhausts its reserves at date 1. Depositors receive $1/n$ when withdrawing early and nothing otherwise. Figure 1 plots the function $v(\theta, n)$ as a function of n for a given state θ . As the figures shows, $v(\theta, n)$ is decreasing in n for $n < n^*$ and increases with n afterwards. However, as the function remains below the zero, the model exhibits the property of *one-sided strategic complementarity* as in Goldstein and Pauzner (2005). Thus, it is possible to derive a unique equilibrium in depositors' withdrawal decisions as follow.

Lemma 1 *The model has a unique equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1)$ and do not run above. At the limit, as $\epsilon_i \rightarrow 0$, the equilibrium threshold is*

$$\theta^*(c_1) = p^{-1} \frac{u(c_1) [1 - \lambda c_1 + c_1 \log(c_1)]}{c_1 \int_{n=\lambda}^{1/c_1} u\left(\frac{1-nc_1}{1-n}R\right) dn}. \quad (4)$$

The threshold $\theta^(c_1)$ is increasing in c_1 .*

Proof. See the appendix. ■

Although the realization of θ uniquely determines the number of late depositors running at the bank at date 1, runs also occur because of bad expectations. Due to strategic complementarity, depositors do not have a dominant action when the fundamentals are in the interval $(\underline{\theta}(c_1), \bar{\theta}]$ and panic-induced runs occur for θ in the range $(\underline{\theta}(c_1), \theta^*(c_1)]$. Importantly, the threshold $\theta^*(c_1)$ increases with the date 1 consumption c_1 . The higher c_1 the lower is the payment \tilde{c}_2 and thus the stronger is the incentive for the late depositors to withdraw early. This implies that the bank's choice of the optimal deposit contract has a direct impact on the probability of occurrence of runs at date 1.

Given depositors' withdrawal decisions, at date 0 each bank chooses c_1 to maximize

$$\int_0^{\theta^*(c_1)} u(1) d\theta + \int_{\theta^*(c_1)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta + v(g) \quad (5)$$

subject to

$$c_{2\lambda} \geq c_1. \quad (6)$$

The first term represents depositors' expected utility for $\theta < \theta^*(c_1)$, when all depositors run, the bank liquidate all units of the project and distributes 1 unit to each depositor. The second term is depositors' expected utility when, for $\theta > \theta^*(c_1)$, the bank continues till date 2. The λ early consumers still withdraw early and obtain c_1 , whereas the $(1 - \lambda)$ late depositors wait and receive the payment $c_{2\lambda} = \frac{1 - \lambda c_1}{1 - \lambda} R$ with probability $p(\theta)$. The last term is the utility that depositors receive from the consumption of the public good g . The amount of resources that the late depositors receive when only the early depositors withdraw must be higher than what the bank promises to early depositors as represented by the incentive compatibility constraint (6).

Proposition 2 *The optimal deposit contract $c_1^D > 1$ in the decentralized solution solves*

$$\begin{aligned} & \lambda \int_{\theta^*(c_1)}^1 [u'(c_1) - p(\theta^*(c_1))Ru'(c_{2\lambda})] d\theta + \\ & - \frac{\partial \theta^*(c_1)}{\partial c_1} [\lambda u(c_1) + (1 - \lambda)p(\theta^*(c_1))u(c_{2\lambda}) - u(1)] = 0. \end{aligned} \quad (7)$$

Proof. See the appendix. ■

The choice of c_1 trades off the positive effect of a higher c_1 on better risk sharing (as represented by the first term in (7)) with the negative effect in terms of a higher probability of runs (as represented by the second term in (7)). The result is that the decentralized solution entails some inefficiency due to the occurrence of panic runs. To see this, we next consider the case where only fundamental runs occurred. We consider this case as the solution to the problem of a social planner that can observe the realization of θ and decide whether to repay depositors withdrawing at date 1 conditional on the realization of the fundamentals.

Consider a social planner that observes θ and repays $n > \lambda$ depositors withdrawing at date 1 only if $\theta \leq \underline{\theta}(c_1)$. This implies that only fundamental runs can occur. The reason is that late consumers know that if they wait and $\theta > \underline{\theta}(c_1)$, they will receive the promised repayment $\frac{1-\lambda c_1}{1-\lambda}R$ with probability $p(\theta)$ independently of the number of depositors withdrawing at date 1. Thus, they will run at the bank at date 1 only if this is a dominant strategy for them, that is if $\theta \leq \underline{\theta}(c_1)$. The problem at date 0 is then given by

$$Max_{c_1} \int_0^{\underline{\theta}(c_1)} u(1) d\theta + \int_{\underline{\theta}(c_1)}^1 \left[\lambda u(c_1) + (1 - \lambda)p(\theta)u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right) \right] d\theta + v(g) \quad (8)$$

subject to

$$c_{2\lambda} \geq c_1.$$

The problem is the same as in (5) with the only difference that now only fundamental runs occur and thus the run threshold is $\underline{\theta}(c_1) < \theta^*(c_1)$, with $\underline{\theta}(c_1)$ being as in (2).

The following proposition summarizes the social planner's solution and compares it to the one in the decentralized economy.

Proposition 3 *A social planner that observes θ chooses $c_1^{SP} > c_1^D > 1$ as the solution to*

$$\begin{aligned} & \lambda \int_{\underline{\theta}(c_1)}^1 [u'(c_1) - p(\theta)Ru'(c_{2\lambda})] d\theta + \\ & - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} [\lambda u(c_1) + (1 - \lambda)p(\underline{\theta}(c_1))u(c_{2\lambda}) - u(1)] = 0. \end{aligned} \quad (9)$$

Proof. The proposition follows directly from differentiating (8) with respect to c_1 . Comparing (9) with (7) implies that $c_1^{SP} > c_1^D > 1$ since $\underline{\theta}(c_1) < \theta^*(c_1)$. \square

■

As before, the solution c_1^{SP} trades off the benefit of an increase in c_1 in terms of better risk sharing with the cost in terms of a higher probability of runs. However, as only fundamental runs occur now, the social planner chooses a greater level of c_1 relative to the decentralized solution. Thus, the occurrence of panic runs in the decentralized solution leads to an inefficiency in terms of too low consumption at date 1 and thus too little risk sharing.

4 Social planner with transfer of resources

The decentralized solution entails an inefficiency because of the occurrence of panic runs. To reduce the coordination problem among depositors, the bank chooses a level of c_1 which is too low relative to the one chosen by a social planner that can observe θ and prevent the occurrence of panic runs. One possibility to improve the decentralized solution is for the government to try and stop runs by guaranteeing (at least part of) the promised repayments by banks through the

transfer of resources from the public good g to the banking sector. The effect of such intervention in terms of depositors' withdrawal decisions and banks' optimal deposit contract depends on the precise timing of the intervention and the amount of the guarantee offered. We analyze various forms of intervention in the next section. We start here by analyzing the benchmark case where a social planner decides both the guarantees and the optimal deposit contract. As we shall see, this case represents the efficient intervention in that, by choosing also the consumption promised to depositors at date 1, the social planner internalizes the cost of providing guarantees and thus avoids any moral hazard problem on the side of the banks.

We start by considering a social planner that observes the realization of the fundamentals θ and guarantees c_1 to all depositors withdrawing at date 1 in the case that a run occurs –that is when $n > \lambda$ depositors withdraw prematurely– and $\theta \leq \underline{\theta}(c_1)$. As for the case without transfer, this implies that only fundamental runs occur. As they expect to obtain the promised repayment $\frac{1-\lambda c_1}{1-\lambda}R$ with probability $p(\theta)$ at date 2 when $\theta > \underline{\theta}(c_1)$, depositors run at date 1 only when this is their dominant strategy, that is when $\theta \leq \underline{\theta}(c_1)$. Differently from the case without transfer, however, depositors obtain c_1 when a run occurs instead of the pro-rata share of 1. This improves risk sharing and thus social welfare.

To implement the guarantee, the social planner transfers resources from the public good g to the banking sector after the first λ depositors withdraw so to limit banks' liquidation of the long term asset. In particular, the social planner provides a subsidy of $(n-\lambda)(c_1 - \frac{1-\lambda c_1}{1-\lambda})$ to each bank. This allows banks to pay c_1 to all the n depositors withdrawing early while at the same time still keeping sufficient resources to pay the promised repayment $\frac{1-\lambda c_1}{1-\lambda}R$ with probability $p(\theta)$ to the late depositors waiting till date 2. In the case where depositors' signal is precise, that is when $\varepsilon \rightarrow 0$, only complete runs occur and $n = 1$. The planner's

subsidy simplifies to $c_1 - 1$ and the date 0 problem is given by

$$\begin{aligned} & \underset{c_1}{Max} \int_0^{\underline{\theta}(c_1)} u(c_1) d\theta + \int_{\underline{\theta}(c_1)}^1 \left[\lambda u(c_1) + (1 - \lambda)p(\theta)u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right) \right] d\theta \\ & + \int_0^{\underline{\theta}(c_1)} v(g - (c_1 - 1)) d\theta + \int_{\underline{\theta}(c_1)}^1 v(g) d\theta \end{aligned} \quad (10)$$

subject to

$$g - (c_1 - 1) \geq 0 \quad (11)$$

$$c_{2\lambda} \geq c_1.$$

The problem is similar to the one in (8) in that there are only fundamental runs, but it differs in two important respects. When a run occurs, which happens for $\theta \leq \underline{\theta}(c_1)$, depositors obtain c_1 instead of 1. This guarantee is paid through a lower provision of the public good to $g - (c_1 - 1)$, as captured in the third term in (10). There is then a trade-off in the provision of the guarantee between better risk sharing but lower provision of g . Also, the amount $c_1 - 1$ of the guarantee cannot exceed the public good g , as required by the constraint (11).

The solution to the social planner's problem is summarized in the following proposition.

Proposition 4 *A social planner that can transfer resources to the banking sector chooses*

$$c_1^{SPI} = \min(\tilde{c}_1^{SPI}, g + 1),$$

where \tilde{c}_1^{SPI} is the solution to

$$\begin{aligned} & \int_0^{\underline{\theta}(c_1)} u'(c_1) d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 [u'(c_1) - p(\theta)Ru'(c_{2\lambda})] d\theta + \\ & - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} [v(g) - v(g - (c_1 - 1))] - \int_0^{\underline{\theta}(c_1)} v'(g - (c_1 - 1)) d\theta = 0. \end{aligned} \quad (12)$$

Proof. See the appendix. ■

The proposition shows that the social planner chooses an interior level of c_1 when the amount of public good is large, and a corner solution when g is small. The reason is that the amount $c_1 - 1$ of the guarantee chosen has to be feasible, that is cannot exceed the amount of public good available. Otherwise, the guarantee is not credible and the equilibrium is as in the decentralized solution. As already mentioned, by internalizing the cost of the provision of the guarantee, the solution of the social planner does not entail any form of moral hazard problem. We now contrast this with the case where a government provides a guarantee but the bank still chooses the optimal deposit contract.

5 Deposit insurance

Now that we have analyzed the intervention of the social planner, we turn to see whether this allocation can be implemented by a government that can provide a guarantee through a reduction of the provision of the public good, but that neither observes the fundamental of the economy nor chooses the deposit contract c_1 . This entails two important differences. First, as it does not observe θ , the government can affect depositors' withdrawal decisions only through the amount and the timing of the guarantee, but, differently from the social planner, it cannot condition depositors' repayments on the realization of θ . Second, the bank does not internalize the cost in terms of lower provision of the public good for the provision of the guarantee. Thus, as in any form of insurance which is not properly priced, there is now a problem of moral hazard in that the bank has an incentive to exploit the guarantee provided by the government when choosing c_1 . In this context, the precise type of guarantee that the government chooses to offer is crucial in determining its effect on the equilibrium allocation. We consider below various forms of government intervention ranging from deposit insurance to a more moderate type of intervention which aims specifically at

reducing the moral hazard problem. As we shall see, none of these intervention is able to achieve the same allocation as the social planner either because they still entail panic runs or because of the moral hazard problem. We start by considering deposit insurance.

When the return of the long-term project is deterministic and runs are pure panic phenomena as in Diamond and Dybvig (1983), introducing deposit insurance allows to eliminate panic runs and restore the efficient allocation. As the bank chooses the deposit contract without taking account of the occurrence of runs, the provision of insurance does not lead to any moral hazard problem. Also, it does not matter to specify the details of the insurance in terms of amount guaranteed and date at which the insurance is provided. Any insurance that guarantees depositors the promised amounts (at both date 1 and date 2, or only at one date) is enough to induce the late depositors to wait till the final date. In our context, however, this is not the case as the withdrawal decisions of the late depositors depend crucially on the form of insurance offered.

We consider three types of guarantees in turn. In the first one, the government guarantees to repay c_1 only to all depositors withdrawing at date 1. This guarantee ensures that panic runs are eliminated while fundamental runs still occur. In the second, the government guarantees c_1 to all depositors, either at date 1 or date 2 and irrespective of whether the bank is solvent at date 2. This implies that no runs occur anymore. The reason is that late depositors know that at date 2 they will receive the promised amount $c_{2\lambda}$ with probability $p(\theta)$ and the guaranteed c_1 with probability $1-p(\theta)$. Thus, their dominant strategy is to wait till date 2, irrespective of the number of depositors withdrawing at date 1. In the third type of intervention, the government guarantees the promised consumption c_1 to depositors withdrawing at date 1 and $c_{2\lambda} = \frac{1-\lambda c_1}{1-\lambda} R$ to depositors waiting till date 2, irrespective of the realized return of the long-term project. Clearly, also this intervention eliminates all types of runs, but it entails

a greater disbursement for the government than the second type of intervention since the promised guarantee is greater. All the three types of interventions entail a moral hazard problem in the choice of c_1 , but of different extent. In particular, as the three interventions have to be feasible, that is the disbursement needed to honor the guarantee cannot not exceed the amount of public good g , the ranking of the three interventions in terms of moral hazard and social welfare will crucially depend on the size of g as well as of the probability of date 2 insolvency $1 - p(\theta)$.

5.1 Guaranteeing promised consumption only to depositors withdrawing at date 1

The first type of deposit insurance we consider is a scheme that promises to repay c_1 to all depositors withdrawing at date 1. This means that in the case where more than λ depositors withdraw early, the government transfers resources from the public good g to the banking sector to ensure that the depositors withdrawing at date 1 receive c_1 while those waiting till date 2 obtain the promised $c_{2\lambda} = \frac{1-\lambda c_1}{1-\lambda} R$. This guarantee scheme removes all panic runs while leaving the fundamental runs for $\theta < \underline{\theta}(c_1)$, where $\underline{\theta}(c_1)$ is as in (2). Anticipating this, each bank's problem at date 0 is

$$\begin{aligned} \text{Max}_{c_1} \int_0^{\underline{\theta}(c_1)} u(c_1) d\theta + \int_{\underline{\theta}(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda)p(\theta)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) \right] d\theta + \\ + \int_0^{\underline{\theta}(c_1^*)} v(g - (c_1^* - 1)) d\theta + \int_{\underline{\theta}(c_1^*)}^1 v(g) d\theta \end{aligned} \quad (13)$$

subject to

$$g - (c_1^* - 1) \geq 0, \quad (14)$$

$$c_{2\lambda} \geq c_1,$$

where c_1^* denotes the equilibrium value of c_1 chosen by all banks. The problem is similar to the one in (10) with the difference that banks do not internalize

the cost of providing the insurance in terms of a lower provision of the public good g . The first term in (13) represents the expected utility of depositors withdrawing early and receiving c_1 when a run occurs. The second is depositors' expected utility when only the λ early depositors withdraw at date 1 and there is no run. The third term is the expected reduction in the provision of the public guarantee when a run occurs and the government transfer $c_1^* - 1$ to the banking sector. The last term represents the provision of the public good g when there is no run. As already mentioned, each single bank does not take account of the cost of providing the guarantee for the government. However, banks anticipate that the public good is reduced by the amount $c_1^* - 1$ when a run occurs, and also they take into account that the guarantee provided in equilibrium by the government has to be feasible, that is $g \geq (c_1^* - 1)$ must hold, as represented by the constraint (14).

The following proposition summarizes the bank's solution and compares it with that of the social planner that can transfer resources.

Proposition 5 *In the presence of a deposit insurance scheme that guarantees c_1 to all depositors withdrawing at date 1, each bank chooses*

$$c_1^{DW} = \min\{\tilde{c}_1^{DW}, g + 1\},$$

where $\tilde{c}_1^{DW} > \tilde{c}_1^{SPI}$ is the solution to

$$\int_0^{\underline{\theta}(c_1)} u'(c_1) d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 [u'(c_1) - p(\theta)Ru'(c_{2\lambda})] d\theta = 0. \quad (15)$$

Proof. See the appendix. ■

The proposition is very simple. Given the insurance scheme available, banks anticipate that only fundamental runs occur. This is positive in that it increases the level of c_1 relative to the decentralized solution. However, differently from the social planner, banks do not anticipate the cost of providing the insurance and thus choose a level of c_1 which is too high relative to the one chosen by

the social planner. This leads to a higher occurrence of (fundamental) runs in equilibrium and lower depositors' expected utility relative to the social planner allocation.

5.2 Guaranteeing consumption promised to early depositors at both dates 1 and 2

The deposit insurance scheme described above allows the government to eliminate panic runs but not fundamental. A possibility for the government to prevent both types of runs is to guarantee c_1 to all depositors, either at date 1 or date 2 and irrespective of whether the bank is solvent at date 2. Under this insurance scheme, the $1 - \lambda$ late depositors receive the promised repayment $c_{2\lambda}$ with probability $p(\theta)$ and c_1 with probability $1 - p(\theta)$. Thus, withdrawing at date 2 is a dominant strategy for the each late depositor and no runs occur any longer. Anticipating this, each bank's maximization problem at date 0 simplifies to

$$\begin{aligned} \underset{c_1}{Max} \int_0^1 & \left[\lambda u(c_1) + (1 - \lambda) \left(p(\theta) u \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - p(\theta)) u(c_1) \right) \right] d\theta + \\ & + \int_0^1 [p(\theta)v(g) + (1 - p(\theta))v(g - (1 - \lambda)c_1^*)] d\theta \end{aligned} \quad (16)$$

subject to

$$g - (1 - \lambda)c_1^* \geq 0, \quad (17)$$

$$c_{2\lambda} \geq c_1, \quad (18)$$

with c_1^* denoting the equilibrium value of c_1 chosen by all banks. The problem is now different in several respects from those described above. First, under this insurance scheme there are no longer runs and the government intervenes only at date 2 when the bank is insolvent, that is with probability $1 - p(\theta)$. Second the government disbursement is now different. By guaranteeing c_1 to

all depositors irrespective of when they withdraw, the actual disbursement is equal to $(1 - \lambda)c_1^*$ with probability $1 - p(\theta)$. As before, the optimal deposit contract has to be feasible in that the disbursement $(1 - \lambda)c_1^*$ cannot exceed the amount of public good g , as required by the constraint (17) and incentive compatible as required by the constraint (18). The last constraint is the usual incentive compatibility constraint.

The solution to the bank's problem and its comparison with the social planner's allocation is summarized in the following proposition.

Proposition 6 *In the presence of a deposit insurance scheme that guarantees c_1 to all depositors either at date 1 or 2, each bank chooses*

$$c_1^{D1} = \min\{\tilde{c}_1^{D1}, \frac{g}{1 - \lambda}\},$$

where \tilde{c}_1^{D1} is the solution to

$$\lambda \int_0^1 \left[u'(c_1) - p(\theta) R u' \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + (1 - \lambda) \int_0^1 u'(c_1) (1 - p(\theta)) d\theta = 0. \quad (19)$$

If $\underline{\theta}(c_1^{SPI}) < (1 - \lambda)(1 - E[p(\theta)])$, then $c_1^{D1} > c_1^{SPI}$.

Proof. See the appendix. ■

5.3 Guaranteeing promised consumption to early and late depositors

We now consider the last deposit insurance scheme in which the government guarantees the promised repayments $(c_1, c_{2\lambda})$ to all consumers. In other words, all depositors withdrawing at date 1 are guaranteed to receive c_1 while those who wait until date 2 receive $c_{2\lambda}$. As in the previous section, this guarantee scheme eliminates all runs and implies that the government only intervenes when the bank is insolvent (i.e., with probability $1 - p(\theta)$). This deposit insurance scheme resembles the blanket guarantees introduced in Ireland in October 2008 where

depositors were fully guaranteed in the case of bank failure. Analogously to the two other cases described above, the repayment c_1 that banks can offer to depositors is constrained by the amount of resources available g as given by the following constraint:

$$g - (1 - \lambda)c_{2\lambda}^* \geq 0 \quad (20)$$

where $c_{2\lambda}^*$ is the equilibrium repayment offered by the bank to late depositors. As the late consumers have no longer incentives to run, the government has an actual disbursement of $(1 - \lambda)c_{2\lambda}^*$ only in the case when the bank is insolvent at date 2, which happens with probability $(1 - p(\theta))$. At date 1 no guarantee is effectively provided, as only early consumers withdraw prematurely. It follows that the moral hazard problem is now very different relative to the two forms of deposit insurance analyzed so far. As the only guarantee actually provided entails the repayment $c_{2\lambda}$ to late consumers, the bank has an incentive to choose an excessively low c_1 so to increase the guaranteed repayment to late depositors at date 2. Thus, the constraint (20) requires now a minimum equilibrium value for c_1^* rather than a maximum. By substituting the expression $c_{2\lambda} = \frac{1 - \lambda c_1}{1 - \lambda} R$ into (20), we obtain the lower bound for c_1 as given by:

$$c_1^{min} = \frac{R - g}{\lambda R}. \quad (21)$$

In equilibrium, c_1 must be higher than c_1^{min} to ensure that (20) is satisfied and that the government's guarantee scheme is fully credible. Furthermore, as in the previous cases, the contract must be incentive compatible as otherwise late depositors have always an incentive to run. Anticipating that late depositors have no incentive to run, the bank's problem at date 0 is to

$$Max_{c_1} \int_0^1 \left[\lambda u(c_1) + (1 - \lambda) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta$$

subject to

$$g - (1 - \lambda)c_{2\lambda}^* \geq 0$$

$$c_{2\lambda} \geq c_1.$$

The solution to the bank's problem is summarized in the following proposition.

Proposition 7 *In an economy with a deposit insurance scheme that guarantees the promised repayment to all consumers, each bank chooses*

$$c_1^{D2} = \max\{c_1^{min}, \tilde{c}_1^{D2}\}$$

where c_1^{min} is given in (21) and \tilde{c}_1^{D2} is the solution to

$$u'(c_1) = Ru'\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right).$$

Proof. See the appendix. ■

6 Government intervention at date 1

The deposit insurance schemes analyzed in the previous sections allows the government to reduce the occurrence of runs but entail a moral hazard problem in the bank's choice of deposit contract. In contrast with Diamond and Dybvig (1983), the effect of the insurance scheme on depositors' withdrawals and the bank's choice depend on the precise details of the guarantee. Some guarantees eliminate runs completely, while others preserve the fundamental ones. Normally, guarantees lead to the bank setting a date 1 repayment for depositors which is too high relative to the one chosen by the social planner. Only the scheme promising $(c_1, c_{2\lambda})$ lead to date 1 repayment which is instead too low.

The problem with all the deposit insurance schemes analyzed so far is that the government cannot choose the size of the intervention more precisely so to ameliorate banks' moral hazard problem. In this section we analyze a more moderate type of intervention in which the government transfers some of the

resources g to the banking sector after the first $\alpha \geq \lambda$ depositors withdraw. Doing so the government stops the early liquidation of the long term project beyond that point, thus preserving the necessary units for the remaining late consumers at date 2. The amount of resources transferred by the government to the banking sector and the repayment that depositors are guaranteed to receive in case of a run depends on whether the government intervenes before or after the bank exhausts its resources. In the first case (i.e., $\alpha \leq \frac{1}{c_1}$), the government intervenes when the bank is paying c_1 to the withdrawing depositors so that it pays $(c_1 - 1)$ for each of the $n > \alpha$ depositors running. In the second case (i.e., $\alpha > \frac{1}{c_1}$), when the government intervenes, the bank has already liquidated all its resources so that depositors only receive the pro-rata share $\frac{1}{\alpha}$. In this case, the government pays $\frac{1}{\alpha}$ to the $n > \alpha$ withdrawing depositors. As the amount of the guarantee is now conditional on the resources left at the bank at the time of the intervention, the government can mitigate the moral hazard problem by choosing *when* to intervene (i.e., the size of α).

The timing of the model is similar to the previous cases. At date 0, the government chooses first the optimal value of α and then the bank chooses c_1 . At date 1, after receiving the private signal about the state of the fundamentals θ , depositors decide whether to withdraw early or wait till date 2. As before, we solve the model backward starting with depositors' withdrawal decisions. The lower and the upper dominance regions are the same as in the decentralized equilibrium, and the upper bound of the lower dominance region $\underline{\theta}(c_1)$ is still given by (2). The determination of the intermediate region is more complicated. Late depositors know c_1 and α when making their decisions, and thus they know whether the bank's resources at the time of the intervention. This implies that we need to distinguish two cases for the determination of the threshold $\theta^*(c_1)$, depending on whether $\alpha \gtrless \frac{1}{c_1}$.

We start with the case where $\alpha \leq \frac{1}{c_1}$. In this case, the utility differential for

a late consumer between withdrawing at date 2 versus date 1 is given by

$$v_1(\alpha, \theta, n) = \begin{cases} p(\theta)u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) & \text{if } \lambda \leq n < \alpha \\ p(\theta)u\left(\frac{1-\alpha c_1}{1-\alpha}R\right) - u(c_1) & \text{if } \alpha \leq n \leq 1. \end{cases} \quad (22)$$

The utility differential is a piecewise function because a late depositor's payoff from waiting depends on the number n of depositors withdrawing at date 1. If $\lambda \leq n < \alpha$, the government does not intervene and a waiting late depositor obtains $\frac{1-nc_1}{1-n}R$ at date 2 with probability $p(\theta)$. The date 2 payoff decreases in the number n of withdrawing depositors in the range $\lambda \leq n < \alpha$. Differently, in the range $\alpha \leq n \leq 1$, a late depositor obtains the constant repayment of $\frac{1-\alpha c_1}{1-\alpha}R$ with probability $p(\theta)$ if he waits and c_1 if he withdraws prematurely. The reason is that the government's intervention stops the liquidation of the bank's resources.

Differently, in the case where $\alpha > \frac{1}{c_1}$, the utility differential is equal to

$$v_2(\alpha, \theta, n) = \begin{cases} p(\theta)u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) & \text{if } \lambda \leq n < n^* \\ u(0) - u\left(\frac{1}{n}\right) & \text{if } n^* \leq n \leq \alpha \\ u(0) - u\left(\frac{1}{\alpha}\right) & \text{if } \alpha \leq n \leq 1. \end{cases} \quad (23)$$

The expression for $v_2(\alpha, \theta, n)$ states that as long as the bank does not exhausts its resources at date 1, i.e., for $\lambda \leq n < n^*$, depositors withdrawing early obtain c_1 and those waiting obtain the residual $\frac{1-nc_1}{1-n}R$ with probability $p(\theta)$. By contrast, for $n \geq n^*$ the bank exhausts its reserves at date 1. Depositors receive $1/n$ when withdrawing early and nothing otherwise until the government intervenes. After the first α depositors have withdrawn, the government transfers resources to the banking sector and depositors are guaranteed to receive $\frac{1}{\alpha}$ at date 1 while they still receive a payoff of 0 if they wait until date 2.

The functions $v_1(\alpha, \theta, n)$ and $v_2(\alpha, \theta, n)$ have different properties: $v_1(\alpha, \theta, n)$ is decreasing in n , while $v_2(\alpha, \theta, n)$ is first decreasing in n for $n < n^*$ and then increasing but it crosses zero only once as $v(\theta, n)$ in the decentralized economy. Thus, for $\alpha < \frac{1}{c_1}$ the model exhibits the property of *global strategic comple-*

mentarity while it only satisfies *one-sided global strategic complementarity* for $\alpha > \frac{1}{c_1}$. As in the case without government intervention, the model has a unique equilibrium in which depositors' withdrawal decision is defined as follows.

Lemma 8 *The model has a unique equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1)$ and do not run above. Two equilibrium thresholds exist depending on whether the government intervenes before or after the bank exhausts its resources. At the limit, as $\epsilon_i \rightarrow 0$, the equilibrium thresholds are*

$$\theta_1^*(c_1, \alpha) = p^{-1} \frac{(1 - \lambda)u(c_1)}{\int_{n=\lambda}^{\alpha} u\left(R \frac{1-nc_1}{1-n}\right) + \int_{n=\alpha}^1 u\left(R \frac{1-\alpha c_1}{1-\alpha}\right)} \quad (24)$$

if $\alpha \leq \frac{1}{c_1}$ and

$$\theta_2^*(c_1, \alpha) = p^{-1} \frac{\left(\frac{1}{c_1} - \lambda\right)u(c_1) + \int_{n=1/c_1}^{\alpha} u\left(\frac{1}{n}\right) + (1 - \alpha)u\left(\frac{1}{\alpha}\right)}{\int_{n=\lambda}^{1/c_1} u\left(R \frac{1-nc_1}{1-n}\right)} \quad (25)$$

when $\alpha > \frac{1}{c_1}$.

The threshold $\theta_1^*(c_1, \alpha)$ is increasing in both c_1 and α while $\theta_2^*(c_1, \alpha)$ is increasing in c_1 but decreasing in α .

Proof. See the appendix. ■

As in the decentralized economy runs occur because depositors have bad expectations and anticipate that other depositors have an incentive to run when the fundamentals are in the range $[0, \theta_i^*(c_1)]$. The government intervention reduces the panic in the range $\alpha \leq \frac{1}{c_1}$ as it removes the strategic complementarity between depositors' actions. In the other interval, $\alpha > \frac{1}{c_1}$, it has the opposite effect as α only affects the repayment at date 1. Thus, the more the government intervenes (i.e. the lower α is), the higher is the repayment that depositors obtain in case of a run. It is important to notice that when the government decides

not to intervene (i.e., $\alpha = 1$),

$$\theta_2^*(c_1, 1) = \theta^*(c_1)$$

holds so that depositors' withdrawal decisions are the same as in the decentralized economy without intervention. In the opposite case, when the government intervenes as soon as a late depositors withdraws (i.e., $\alpha = \lambda$),

$$\theta_1^*(c_1, \lambda) = \underline{\theta}(c_1)$$

holds so that only fundamental runs occur. Importantly, depositors' withdrawal decisions are continuous since for $\alpha = \frac{1}{c_1}$ it holds that

$$\theta_1^*(c_1, \alpha) = \theta_2^*(c_1, \alpha).$$

Given the depositors' withdrawal decisions, at date 0 each bank chooses c_1 to maximize

$$Max_{c_1} \begin{cases} \int_0^{\theta_1^*(c_1, \alpha)} u(c_1) d\theta + \int_{\theta_1^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta + & \text{if } \alpha \leq \frac{1}{c_1} \\ \quad + \int_0^{\theta_1^*(c_1^*, \alpha)} v(g - (c_1^* - 1)) d\theta + \int_{\theta_1^*(c_1^*, \alpha)}^1 v(g) d\theta & \\ \int_0^{\theta_2^*(c_1, \alpha)} u\left(\frac{1}{\alpha}\right) d\theta + \int_{\theta_2^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta + & \text{if } \alpha > \frac{1}{c_1}, \\ \quad + \int_0^{\theta_2^*(c_1^*, \alpha)} v\left(g - \frac{1 - \alpha}{\alpha}\right) d\theta + \int_{\theta_2^*(c_1^*, \alpha)}^1 v(g) d\theta & \end{cases} \quad (26)$$

subject to

$$g - (c_1^* - 1) \geq 0, \quad (27)$$

$$c_2 \lambda \geq c_1, \quad (28)$$

where c_1^* denotes the equilibrium value of c_1 chosen by all banks. For a given α , each bank chooses the deposit contract c_1 to offer to depositors anticipating that depositors' withdrawal decisions will depend on whether $\alpha \lesseqgtr \frac{1}{c_1}$. The first term, in both ranges of α , represents depositors' expected utility in case of a

run. When $\alpha \leq \frac{1}{c_1}$, depositors receive c_1 , while in the range $\alpha > \frac{1}{c_1}$, depositors are guaranteed to receive a payment $1 \leq \frac{1}{\alpha} \leq c_1$. Irrespective of the value of α , the second term represents depositors' expected utility when only the λ early consumers withdraw at date 1 and there is no run. The last two terms in each expression represent depositors' expected utility from the public good. As with the deposit insurance, the bank does not internalize the cost in terms of a lower provision of the public good for the provision of the guarantee when they choose the optimal deposit contract. This is due to the fact that each bank is atomistic and thus the actual provision of public good depends on the equilibrium consumption. However, as for the previous forms of intervention, the optimal deposit contract must not exceed the amount of resources available as the government has to be feasible. This means that the constraint (27) must hold. Finally, the deposit contract must be incentive compatible as represented by the constraint (28). The solution to the bank's problem is summarized in the following proposition.

Proposition 9 : *The optimal deposit contract c_1 depends on the size of the government intervention α as follows;*

i) *When $\lambda \leq \alpha \leq \alpha_1$, $c_1^* \rightarrow \bar{c}_{11}$ where \bar{c}_{11} is defined by the following equation*

$$\lim_{c_1 \rightarrow \bar{c}_{11}} \theta_1^*(c_1, \alpha) = \bar{\theta}.$$

ii) *When $\alpha_1 < \alpha \leq \alpha_2$, $c_1^* \leq 1/\alpha$ is the solution to*

$$\begin{aligned} & \int_0^{\theta_1^*(c_1, \alpha)} u'(c_1) d\theta + \lambda \int_{\theta_1^*(c_1, \alpha)}^1 \left[u'(c_1) - Rp(\theta)u' \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\ & - \frac{\partial \theta_1^*(c_1, \alpha)}{\partial c_1} (1 - \lambda) \left[p(\theta_1^*(c_1, \alpha)) u \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(c_1) \right] \\ & = 0. \end{aligned} \tag{29}$$

iii) When $\alpha_2 < \alpha \leq 1$, $c_1^* \geq 1 \setminus \alpha$ is the solution to

$$\begin{aligned} & \lambda \int_{\theta_2^*(c_1, \alpha)}^1 \left[u'(c_1) - Rp(\theta)u' \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\ & - \frac{\partial \theta_2^*(c_1, \alpha)}{\partial c_1} \left[(1 - \lambda) p(\theta_2^*(c_1, \alpha)) u \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) + \lambda u(c_1) - u \left(\frac{1}{\alpha} \right) \right] \\ = & 0. \end{aligned} \tag{30}$$

The expressions for the thresholds α_1 and α_2 are provided in the appendix.

Proof. See the appendix. ■

The proposition shows that the bank's choice of c_1 depends crucially on the size of the government guarantees α announced by the government. In choosing c_1 banks trade off the benefits of an increase in c_1 in terms of better risk sharing and of a larger repayment in the case of a run with the cost in terms of increased probability of runs. When the government intervenes a lot (i.e., $\lambda \leq \alpha \leq \alpha_1$), banks choose a very high level of c_1 . The reason is twofold. On the one hand, when α is small, a higher c_1 has a relatively small impact on the probability of a run, thus giving the bank incentives to increase the repayment to consumers. On the other hand, by choosing a higher c_1 the bank guarantees a higher repayment to consumers in case of a run. Together these forces lead to the bank choosing $c_1 \rightarrow \bar{c}_{11}$, where \bar{c}_{11} is the level of consumption at which runs always occur except when θ lies in the upper dominance region. As α increases the likelihood of a run increases for a given c_1 . This induces banks to reduce the promised repayment to consumers to limit the occurrence of runs. As α increases further (i.e., $\alpha_2 < \alpha \leq 1$), banks reduce c_1 even more. As the government intervenes so little, even with a lower c_1 the bank exhausts its resources before the government intervention with the consequence that, in case of a run, depositors only obtain the pro-rata share $\frac{1}{\alpha}$.

Having characterized the bank's problem, we now turn to analyze the government's choice of α . Given banks' optimal choice of c_1 , at date 0 the government

chooses α to maximize depositors' total expected utility. The government's problem is given by

$$Max_{\alpha} \begin{cases} Eu_1(\bar{c}_{11}, \alpha) + \int_0^{\theta_1^*(\bar{c}_{11}, \alpha)} v(g - (\bar{c}_{11} - 1)) d\theta + \int_{\theta_1^*(\bar{c}_{11}, \alpha)}^1 v(g) d\theta & \text{if } \lambda \leq \alpha \leq \alpha_1 \\ Eu_1(c_1^*(\alpha), \alpha) + \int_0^{\theta_1^*(c_1^*(\alpha), \alpha)} v(g - (c_1^*(\alpha) - 1)) d\theta + \int_{\theta_1^*(c_1^*(\alpha), \alpha)}^1 v(g) d\theta & \text{if } \alpha_1 < \alpha \leq \alpha_2 \\ Eu_2(c_1^*(\alpha), \alpha) + \int_0^{\theta_2^*(c_1^*(\alpha), \alpha)} v(g - \frac{1-\alpha}{\alpha}) d\theta + \int_{\theta_2^*(c_1^*(\alpha), \alpha)}^1 v(g) d\theta & \text{if } \alpha_2 < \alpha \leq 1, \end{cases} \quad (31)$$

where $Eu_1(c_1, \alpha) = \int_0^{\theta_1^*(c_1, \alpha)} u(c_1) d\theta + \int_{\theta_1^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda)p(\theta)u\left(\frac{1 - \lambda c_1}{1 - \lambda}R\right) \right] d\theta$ and $c_1^*(\alpha)$ is the solution to the bank's problem as in (26). The first term in (31) is depositors' indirect utility function for the private good. The last two terms represents the expected utility from the consumption of the private good g . We have the following result.

Proposition 10 : *The optimal size of intervention α depends on the amount of resources available for the consumption of the public good g .*

- i) If $g \geq g_2$, $\alpha^* \in [\lambda, \alpha_1]$
- ii) If $g_1 \leq g \leq g_2$, $\alpha^* \in (\alpha_1, \alpha_2]$
- iii) If $g < g_1$, $\alpha^* \in (\alpha_2, 1]$.

The threshold g_2 is the solution to $u'(\frac{1}{\alpha}) - v'(g - \frac{1-\alpha}{\alpha}) = 0$. The threshold g_1 is the solution to $\frac{\partial Eu_2(c_1^*(\alpha), \alpha)}{\partial \alpha} |_{\alpha=\alpha_2} = 0$.

Proof. To be added. ■

The proposition has a simple interpretation. The government chooses the optimal size of intervention α to maximize depositors' expected utility. In choosing α , the government has to anticipate the effect that this choice will have on banks' and depositors' behaviour. When the government's budget is tight, the government intervention generates a large disutility due to a low provision of the public good. Given the concavity of the function $v(g)$, when g is small, even a small reduction in the provision of the public good leads to high costs in

terms of a lower expected utility. In this case, the government finds it optimal to choose a large α so to induce the bank to choose a low level of c_1 and thus reduce its disbursement. When g is large, the government intervention has a smaller effect on depositors' expected utility. Thus, the government optimally chooses a lower value of α even if this leads to a more severe moral hazard problem on the bank's side.

7 A numerical example

In this section we illustrate the properties of the model and in particular the comparison across the various government interventions with the use of a numerical example. We consider the following:

$$u(c) + v(g) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma} + \frac{(g+f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma},$$

$p(\theta) = \theta$, $\sigma = 3$; $R = 5$; $\lambda = 0.3$ and $f = 4$. The upper dominance region corresponds to $\theta = 1$. We consider different amountx of the public good g to show how the optimal government intervention and banks' choice change depending on the amount of available resources in the economy. Specifically, we report the results for the following cases: $g = 1.03$, $g = 2$ and $g = 2.18$. We consider $g = 2$, unless it is differently stated.

The decentralized economy without government intervention and comparison with social planner

In this section we report the results relative to the decentralized economy and we compare it with that of a social planner that observes θ and repays $n > \lambda$ depositors withdrawing at date 1 only if $\theta \leq \underline{\theta}(c_1)$. We distinguish two cases depending on whether we allow or not the social planner to transfer resources to the banking sector in case of a run.

Table 1 about here

The comparison between the decentralized solution and the social planner highlights that the latter removes some of the inefficiencies of the decentralized solution due to the occurrence of panic runs. The social planner offers a higher level of c_1 than the decentralized solution, thus allowing a better risk-sharing. The highest promised repayment corresponds to the case in which the social planner can transfer resources to the banking sector in case of a run. The intuition behind this results is simple. As depositors are guaranteed to receive c_1 in case of runs, the social planner finds it optimal to offer a higher repayment so to increase depositors' utility. It is not surprising that depositors' expected utility is the highest when the social planner is allowed to transfer resources to the banking sector even if the choice of a higher c_1 induces more runs. The reason is that the social planner can optimally choose how to allocate resources between the public and the banking sector and runs are only fundamental ones. This suggests that runs are not *per se* bad as they can be efficient when they are not driven by panics.

The economy with government intervention

We now turn to analyze the various forms of government intervention analyzed above. The results in the tables show that the introduction of the deposit insurance schemes leads to a severe moral hazard problem while the moderate form of intervention reduces the bank's incentive to take excessive risk and over-exploit the guarantee.

Table 2 about here

All types of intervention entail a moral hazard problem in the choice of c_1 , but of different extent. They differ in terms of the repayment offered to consumers, depositors' expected utility and occurrence of runs.

The ranking of the different guarantee schemes in terms of moral hazard and social welfare crucially depends on the size of g as well as of the probabilities of

a run θ^* or of date 2 insolvency $1 - p(\theta)$. To see this, we illustrate in Tables 3 and 4 the cases for $g = 2.18$ and $g = 1.03$.

Tables 3 and 4 about here

The deposit insurance scheme DW in which depositors are guaranteed to receive c_1 only if they withdraw at date 1 entails a severe more hazard problem. In this case, the optimal deposit contract is almost always pinned down by one of the constraints of the problem. The incentive compatibility constraint $c_2 \geq c_1$ always binds first except when $g = 1.03$ in which case (14) binds first. The promised repayment is higher in this case than in the case $D1$ when c_1 is guaranteed either at date 1 or 2 irrespective of whether the bank is solvent at date 2. This is due to the fact that the probability of a fundamental run $\underline{\theta}(c_1)$ is higher than the expected probability of date 2 insolvency $E[1 - p(\theta)] = \frac{1}{2}$. This also implies that the government's disbursement is larger and depositors' expected utility from both the private and the public good is lower.

The government intervention in which the government chooses when to intervene (i.e., α) is effective in limiting banks' moral hazard problem and leads to the highest depositors' expected utility in two out of the three cases analyzed. The level of consumption offered by the bank is always lower and the government's budget constraint (27) is never binding as it is instead the case in the other forms of intervention. However, when the government budget is not tight, as when $g = 2.18$, the deposit insurance offering c_1 to depositors irrespective of when they withdraw is better than the intervention in which the government chooses α . This is due to the fact that, when the amount of resources g is big, the government is less concerned about limiting the moral hazard problem on the side of the bank. By comparing Table 2 with 3 and 4, it emerges that, as g increases, the government finds it optimal to reduce α and thus to intervene

more. A reduction of α makes runs more likely. On the one hand, a larger size of the intervention induces banks to choose a higher c_1 , while on the other, it affects directly the equilibrium threshold making relatively more profitable for depositors to withdraw at date 1. When $g = 2.18$, the government optimally chooses to intervene a lot and the allocation is very different from that of the decentralized economy without intervention as illustrated in Table 1. Runs occur almost in all feasible range of values of θ , except in the upper dominance region with negative consequences on depositors' expected utility as they suffer a loss in terms of risk-sharing. On the contrary, the deposit insurance scheme guaranteeing c_1 always, by still offering a liquidity insurance to depositors leads to a higher expected utility, even if it is more costly in terms of a lower provision of the public good.

8 Concluding Remarks

In this paper we have developed a simple model where both panic and fundamental runs are possible and both banks' and depositors' decisions are endogenously determined. We have shown that government intervention is needed in order to correct the inefficiency of the decentralized economy due to the coordination problem among consumers. However, government's guarantees induce banks to take excessive risk, thus leading sometimes to inefficient outcomes. We have shown that guarantee schemes have very different implications in terms of probability and types of runs, efficiency of the deposit contract and social welfare. The severity of the moral hazard problem generated by the introduction of the guarantee and consequently the disbursement for the government vary across the types of intervention depending on the amount of resources available to finance the scheme. In economies in which the government has a tight budget, the consequences of the moral hazard problem are severe. In this case, the most

efficient form of intervention is a very limited guarantees which limits moral hazard by leaving panic runs. On the contrary, when government has a large amount of resources to transfer to the banking sector, blanket guarantees, which removes all types of runs, are more efficient than other more moderate form of intervention.

The paper offers an ideal framework to evaluate the effect of government intervention as the probability of the occurrence of a crisis is completely endogenous. The likelihood of runs and whether they are fundamental or panic-driven depends crucially on the guarantee offered in two ways. It depends *directly* on the type and features of the intervention as depositors anticipate that they will receive their promised repayment (or part of it). The features of the intervention also affects the probability and type of runs *indirectly* as they affect banks' choice of the deposit contract and thus banks' exposure to liquidity risk.

The model highlights the existence of a link between government guarantees and financial stability and offers insights for future research. One potentially interesting extension would be the analysis of the feedback effect between government guarantees and financial stability. When the introduction of a guarantee scheme entails an actual disbursement for the government, it can threaten the solvency of the country and thus undermines the credibility of the guarantees themselves. The threat of sovereign default represents a new source of risk that has been completely overlooked in the literature on government intervention so far, but, as the recent European sovereign crisis has shown, it is a very relevant drawback of the massive government intervention which took place during the crisis.

Another possible extension would be removing the assumption of full commitment so that the government only intervenes only if it is ex post optimal. The credibility of the intervention will then be conditional on its ex post optimality which endogenously depends on the features of the guarantee introduced. There

is a growing literature analyzing different form of interventions in a context of limited commitment (e.g., Ennis and Keister, 2009 and 2010 and Cooper and Kempf, 2011), but all those contributions consider an exogenous probability of runs.

A Proofs

Proof of Lemma 1: The proof follows Goldstein and Pauzner (2005). The arguments in their proof establish that there is a unique equilibrium in which depositors run if and only if the signal they receive is below a common signal $x^*(c_1)$. The number n of depositors withdrawing at date 1 is equal to the probability of receiving a signal x_i below $x^*(c_1)$ and, given that the posterior distribution of θ is uniform over the interval $[x^*(c_1) - \epsilon, x^*(c_1) + \epsilon]$, it is given by:

$$n(\theta, x^*(c_1)) = \begin{cases} 1 & \text{if } \theta \leq x^*(c_1) - \epsilon \\ \lambda + (1 - \lambda) \left(\frac{x^*(c_1) - \theta + \epsilon}{2\epsilon} \right) & \text{if } x^*(c_1) - \epsilon \leq \theta \leq x^*(c_1) + \epsilon \\ \lambda & \text{if } \theta \geq x^*(c_1) + \epsilon \end{cases} \quad (32)$$

The posterior distribution of n is uniform over the range $[\lambda, 1]$. When θ is below $x^*(c_1) - \epsilon$, all patient depositors receive a signal below $x^*(c_1)$ and run. When θ is above $x^*(c_1) + \epsilon$, all late depositors wait until date 2 and only the λ early consumers withdraw early. In the intermediate interval, when θ is between $x^*(c_1) - \epsilon$ and $x^*(c_1) + \epsilon$, there is a partial run as some of the late depositors run. The proportion of late consumers withdrawing decreases linearly with θ as fewer agents observe a signal below the threshold.

Having characterized the proportion of agents withdrawing for any possible value of the fundamentals θ , we can now compute the threshold signal $x^*(c_1)$. A patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1)$ can be then found as the solution to

$$f(\theta, c_1) = \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta(n)) u \left(\frac{1 - nc_1}{1 - n} R \right) - u(c_1) \right] + \int_{n=\frac{1}{c_1}}^1 \left[u(0) - u \left(\frac{1}{n} \right) \right] = 0, \quad (33)$$

where, from (32), $\theta(n) = x^*(c_1) + \epsilon - 2\epsilon \frac{(n-\lambda)}{1-\lambda}$. Equation (33) follows from (3) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits till date 2. Note that at the limit, when $\epsilon \rightarrow 0$, $\theta(n) \rightarrow x^*(c_1)$, and we denote it as $\theta^*(c_1)$. Solving (33) with respect to $\theta^*(c_1)$ gives the threshold as in the proposition.

To prove that $\theta^*(c_1)$ is increasing in c_1 , we use the implicit function theorem and obtain

$$\frac{\partial \theta^*(c_1)}{\partial c_1} = -\frac{\frac{\partial f(\theta^*, c_1)}{\partial c_1}}{\frac{\partial f(\theta^*, c_1)}{\partial \theta^*}}.$$

It is easy to see that $\frac{\partial f(\theta^*, c_1)}{\partial \theta^*} > 0$. Thus, the sign of $\frac{\partial \theta^*(c_1)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^*, c_1)}{\partial c_1}$, where

$$\begin{aligned} \frac{\partial f(\theta^*, c_1)}{\partial c_1} &= -p(\theta^*) \frac{1}{c_1^2} u\left(\frac{1 - \frac{1}{c_1} c_1}{1 - \frac{1}{c_1}} R\right) - \int_{n=\lambda}^{\frac{1}{c_1}} p(\theta^*) \left(\frac{nR}{1-n}\right) u'\left(\frac{1 - nc_1}{1-n} R\right) \\ &\quad + \frac{1}{c_1^2} u(c_1) - \int_{n=\lambda}^{\frac{1}{c_1}} u'(c_1) - \frac{1}{c_1^2} u(c_1) \\ &= - \int_{n=\lambda}^{\frac{1}{c_1}} p(\theta^*) \left(\frac{nR}{1-n}\right) u'\left(R \frac{1 - nc_1}{1-n}\right) - \int_{n=\lambda}^{\frac{1}{c_1}} u'(c_1) < 0. \end{aligned}$$

This implies $\frac{\partial \theta^*(c_1)}{\partial c_1} > 0$. \square

Proof of Proposition 2: Differentiating (5) with respect to c_1 gives the optimal deposit contract c_1^D as the solution to (7).

To show that $c_1^D > 1$, we evaluate (7) at $c_1 = 1$. From (4), at $c_1 = 1$ the threshold $\theta^*(c_1)$ simplifies to

$$\theta^*(1) = p^{-1} \frac{(1-\lambda)u(c_1)}{(1-\lambda)u(R)},$$

and, from (2), it is then

$$\theta^*(1) = \underline{\theta}(1).$$

Thus, when $c_1 = 1$, (7) can be rewritten as follows:

$$\lambda \int_{\underline{\theta}(1)}^1 [u'(1) - p(\theta)Ru'(R)] - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} \Big|_{c_1=1} (1-\lambda) [p(\underline{\theta}(1))u(R) - u(1)]$$

The second term is equal to zero because of the definition of $\underline{\theta}(c_1)$ in (2), and thus the expression simplifies to

$$\lambda \int_{\underline{\theta}(1)}^1 [u'(1) - p(\theta)Ru'(R)].$$

Since the relative risk aversion coefficient is bigger than 1, it holds

$$1 \cdot u'(1) > Ru'(R),$$

so that $\lambda \int_{\underline{\theta}(1)}^1 [u'(1) - p(\theta)Ru'(R)] > 0$ and thus $c_1^D > 1$. \square

Proof of Proposition 4: Suppose first that (11) is not binding. Then, differentiating (10) with respect to c_1 gives

$$\begin{aligned} & \int_0^{\underline{\theta}(c_1)} u'(c_1) d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 [u'(c_1) - p(\theta)Ru'(c_{2\lambda})] d\theta + \\ & - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} (1-\lambda) [p(\underline{\theta}(c_1)) u(c_{2\lambda}) - u(c_1)] - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} [v(g) - v(g - (c_1 - 1))] + \\ & - \int_0^{\underline{\theta}(c_1)} v'(g - (c_1 - 1)) d\theta \end{aligned} \quad (34)$$

$$= 0.$$

Given the definition of $\underline{\theta}(c_1)$ in (2), the term $\frac{\partial \underline{\theta}(c_1)}{\partial c_1} (1-\lambda) [p(\underline{\theta}(c_1)) u(c_{2\lambda}) - u(c_1)]$ is equal to zero and the expression above simplifies to (12) in the proposition.

To see when the constraint (11) is binding, we substitute $c_1 = g + 1$ in (12) and obtain

$$\begin{aligned} & \int_0^{\underline{\theta}(g+1)} u'(g+1) d\theta + \lambda \int_{\underline{\theta}(g+1)}^1 \left[u'(g+1) - p(\theta)Ru' \left(\frac{1-\lambda(g+1)}{1-\lambda} R \right) \right] d\theta + \\ & - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} \Big|_{c_1=g+1} v(g) - \int_0^{\underline{\theta}(g+1)} v'(0) d\theta. \end{aligned} \quad (35)$$

Denote as \bar{g}^{SPI} the amount of public good for which (35) is equal to zero. Given the concavity of the function (12), (35) is negative for $g > \bar{g}^{SPI}$ and positive for $g < \bar{g}^{SPI}$. This implies that the constraint (11) is not binding for $g > \bar{g}^{SPI}$, while it is for $g < \bar{g}^{SPI}$. The proposition follows. \square

Proof of Proposition 5: Suppose first that (14) is not binding. Then, differentiating (13) with respect to c_1 gives

$$\int_0^{\underline{\theta}(c_1)} u'(c_1) d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 [u'(c_1) - p(\theta)Ru'(c_{2\lambda})] d\theta - \frac{\partial \underline{\theta}(c_1)}{\partial c_1} (1-\lambda) [p(\underline{\theta}(c_1)) u(c_{2\lambda}) - u(c_1)] = 0. \quad (36)$$

Given the definition of $\underline{\theta}(c_1)$ in (2), the term $\frac{\partial \underline{\theta}(c_1)}{\partial c_1} (1-\lambda) [p(\underline{\theta}(c_1)) u(c_{2\lambda}) - u(c_1)]$ is equal to zero and the expression simplifies to (15) as in the proposition. By comparing (15) with (12) it is easy to see that $\tilde{c}_1^{DW} > \tilde{c}_1^{SPI}$.

To see when the constraint (14) is binding, we substitute $c_1^* = g + 1$ into (15) and obtain

$$\int_0^{\underline{\theta}(g+1)} u'(g+1)d\theta + \lambda \int_{\underline{\theta}(g+1)}^1 \left[u'(g+1) - p(\theta)Ru' \left(\frac{1 - \lambda(g+1)}{1 - \lambda} R \right) \right] d\theta. \quad (37)$$

Denote as \bar{g}^{DW} the amount of public good for which (37) is equal to zero. Given the concavity of the objective function (13), the expression (37) is negative for $g > \bar{g}^{DW}$ and positive otherwise. This implies that in the range $g > \bar{g}^{DW}$, the constraint (14) is not binding and the optimal deposit contract chosen by the bank is $\tilde{c}_1^{DW} < g + 1$. By contrast, when $g < \bar{g}^{DW}$, the constraint (14) is binding and the optimal deposit contract is given by the corner solution $c_1^{DW} = g + 1$. It is easy to see by comparing (35) with (37) that $\bar{g}^{DW} > \bar{g}^{SPI}$. The proposition follows. \square

Proof of Proposition 6: Suppose first that (17) is not binding. Then, differentiating (16) with respect to c_1 gives \tilde{c}_1^{D1} as the solution to (19).

To see when the constraint (17) is binding, we evaluate (19) at $c_1 = \frac{g}{1-\lambda}$ and obtain:

$$\lambda \int_0^1 \left[u' \left(\frac{g}{1-\lambda} \right) - Rp(\theta)u' \left(\frac{1 - \lambda \frac{g}{1-\lambda}}{1 - \lambda} R \right) \right] d\theta + (1-\lambda) \int_0^1 u' \left(\frac{g}{1-\lambda} \right) (1-p(\theta)) d\theta. \quad (38)$$

Given the concavity of the objective function (16), there exists a level of public good g , denoted as \bar{g}^{D1} , for which (38) is zero. Thus, (38) is positive and the constraint (17) is binding for $g < \bar{g}^{D1}$, implying that $c_1^{D1} = \frac{g}{1-\lambda}$. By contrast, (38) is negative and the constraint (17) is not binding for $g > \bar{g}^{D1}$ so that the solution is $c_1^{D1} = \tilde{c}_1^{D1}$. Furthermore, by comparing (19) and (12), it is easy to see that $\lambda \int_0^1 [u'(c_1) - Rp(\theta)u'(c_{2\lambda})] d\theta > \lambda \int_{\underline{\theta}(c_1)}^1 [u'(c_1) - p(\theta)Ru'(c_{2\lambda})] d\theta$. Thus, a sufficient condition for $\tilde{c}_1^{D1} > \tilde{c}_1^{SPI}$ is that

$$(1 - \lambda) \int_0^1 u'(c_1)(1 - p(\theta))d\theta > \int_0^{\underline{\theta}(c_1)} u'(c_1) d\theta. \quad (39)$$

The proposition follows. \square

Proof of Lemma 8: The proof is analogous to the one of Lemma 1 with the difference that now we have to compute two equilibrium thresholds depending on the value of α relatively to $\frac{1}{c_1}$. In both cases (i.e. $\alpha \leq \frac{1}{c_1}$), a patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2.

We start from the case $\alpha \leq \frac{1}{c_1}$. The threshold $x_1^*(c_1)$ can be then found as the solution to

$$f_1(\theta, c_1) = \int_{n=\lambda}^{\alpha} \left[p(\theta(n))u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) \right] + \int_{n=\alpha}^1 \left[p(\theta(n))u\left(\frac{1-\alpha c_1}{1-\alpha}R\right) - u(c_1) \right] = 0, \quad (40)$$

where, similarly to (32), $\theta(n) = x_1^*(c_1) + \epsilon - 2\epsilon \frac{(n-\lambda)}{1-\lambda}$. Equation (40) follows from (22) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits till date 2. At the limit, when $\epsilon \rightarrow 0$, $\theta(n) \rightarrow x_1^*(c_1)$, and we denote it as $\theta_1^*(c_1)$. Solving (40) with respect to $\theta_1^*(c_1)$ gives the threshold as in the proposition.

We now consider the case $\alpha > \frac{1}{c_1}$. Using the same arguments as above, the threshold $x_2^*(c_1)$ is the solution to

$$\begin{aligned} f_2(\theta, c_1) &= \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta(n))u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) \right] + \int_{n=\frac{1}{c_1}}^{\alpha} \left[u(0) - u\left(\frac{1}{n}\right) \right] + \int_{n=\alpha}^1 \left[u(0) - u\left(\frac{1}{\alpha}\right) \right] = \\ &= \int_{n=\lambda}^{\frac{1}{c_1}} \left[p(\theta(n))u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1) \right] - \int_{n=\frac{1}{c_1}}^{\alpha} u\left(\frac{1}{n}\right) - \int_{n=\alpha}^1 u\left(\frac{1}{\alpha}\right) = 0. \end{aligned} \quad (41)$$

Thus, at the limit when $\epsilon \rightarrow 0$, we denote as $\theta_2^*(c_1)$ the solution to (41).

To prove that $\theta_1^*(c_1)$ and $\theta_2^*(c_1)$ are increasing in c_1 , we use the implicit function theorem and obtain

$$\frac{\partial \theta_i^*(c_1)}{\partial c_1} = - \frac{\frac{\partial f_i(\theta_i^*, c_1)}{\partial c_1}}{\frac{\partial f_i(\theta_i^*, c_1)}{\partial \theta_i^*}}$$

with $i = 1, 2$.

It is easy to see that $\frac{\partial f_i(\theta_i^*, c_1)}{\partial \theta_i^*} > 0$ for any $i = 1, 2$. Thus, the sign of $\frac{\partial \theta_i^*(c_1)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f_i(\theta_i^*, c_1)}{\partial c_1}$.

For $\theta_1^*(c_1)$, we have

$$\begin{aligned} \frac{\partial f_1(\theta_1^*, c_1)}{\partial c_1} &= - \int_{n=\lambda}^{\alpha} p(\theta_1^*) \left(\frac{nR}{1-n} \right) u' \left(\frac{1-nc_1}{1-n}R \right) + \\ &\quad - \int_{n=\alpha}^1 p(\theta_1^*) \left(\frac{\alpha R}{1-\alpha} \right) u' \left(\frac{1-\alpha c_1}{1-\alpha}R \right) - \int_{n=\lambda}^1 u'(c_1) < 0. \end{aligned}$$

This implies $\frac{\partial \theta_1^*(c_1)}{\partial c_1} > 0$.

For $\theta_2^*(c_1)$, we have

$$\begin{aligned} \frac{\partial f_2(\theta_2^*, c_1)}{\partial c_1} &= -p(\theta_2^*) \frac{1}{c_1^2} u\left(\frac{1 - \frac{1}{c_1} c_1}{1 - \frac{1}{c_1}} R\right) - \int_{n=\lambda}^{\frac{1}{c_1}} p(\theta_2^*) \left(\frac{nR}{1-n}\right) u'\left(\frac{1-nc_1}{1-n} R\right) \\ &\quad + \frac{1}{c_1^2} u(c_1) - \int_{n=\lambda}^{\frac{1}{c_1}} u'(c_1) - \frac{1}{c_1^2} u(c_1) \\ &= - \int_{n=\lambda}^{\frac{1}{c_1}} p(\theta_2^*) \left(\frac{nR}{1-n}\right) u'\left(R \frac{1-nc_1}{1-n}\right) - \int_{n=\lambda}^{\frac{1}{c_1}} u'(c_1) < 0. \end{aligned}$$

This implies $\frac{\partial \theta_2^*(c_1)}{\partial c_1} > 0$.

To prove that $\theta_1^*(c_1)$ is increasing in α while $\theta_2^*(c_1)$ is decreasing in α , we use again the implicit function theorem and obtain

$$\frac{\partial \theta_i^*(c_1)}{\partial \alpha} = - \frac{\frac{\partial_i(\theta_i^*, c_1)}{\partial \alpha}}{\frac{\partial f_i(\theta_i^*, c_1)}{\partial \theta_i^*}}$$

with $i = 1, 2$.

Being $\frac{\partial_i(\theta_i^*, c_1)}{\partial \theta_i^*} > 0$ for any $i = 1, 2$, the sign of $\frac{\partial \theta_i^*(c_1)}{\partial \alpha}$ is given by the opposite sign of $\frac{\partial_i(\theta_i^*, c_1)}{\partial \alpha}$.

For $\theta_1^*(c_1)$, we have

$$\frac{\partial f_1(\theta_1^*, c_1)}{\partial \alpha} = - \int_{n=\alpha}^1 p(\theta_1^*) \left[\frac{c_1 - 1}{(1 - \alpha)^2} \right] u'\left(\frac{1 - \alpha c_1}{1 - \alpha} R\right) - \int_{n=\lambda}^1 u'(c_1) < 0.$$

This implies $\frac{\partial \theta_1^*(c_1)}{\partial \alpha} > 0$.

For $\theta_2^*(c_1)$, we have

$$\begin{aligned} \frac{\partial f_2(\theta_2^*, c_1)}{\partial \alpha} &= -u\left(\frac{1}{\alpha}\right) + u\left(\frac{1}{\alpha}\right) + \frac{1}{\alpha^2} \int_{n=\alpha}^1 u'\left(\frac{1}{\alpha}\right) = \\ &= \frac{1}{\alpha^2} \int_{n=\alpha}^1 u'\left(\frac{1}{\alpha}\right) > 0. \end{aligned}$$

This implies $\frac{\partial \theta_2^*(c_1)}{\partial \alpha} < 0$.

Thus, the proposition follows. \square

Proof of Proposition 9: Denote as \bar{c}_{11} and \bar{c}_{12} the maximum level of consumption that the bank can choose. The two upper bounds are defined by the following equations, respectively

$$\lim_{c_1 \rightarrow \bar{c}_{11}} \theta_1^*(c_1, \alpha) = \bar{\theta} \quad (42)$$

and

$$\lim_{c_1 \rightarrow \bar{c}_{12}} \theta_2^*(c_1, \alpha) = \bar{\theta}. \quad (43)$$

As $\theta_1^*(c_1, \alpha)$ is increasing in α , \bar{c}_{11} is decreasing in α . On the contrary, given that $\theta_2^*(c_1, \alpha)$ is decreasing in α , \bar{c}_{12} is increasing in α .

Recall that, when $c_1 = \frac{1}{\alpha}$, the two equilibrium thresholds $\theta_1^*(c_1, \alpha)$ and $\theta_2^*(c_1, \alpha)$ are equal. Assume that there exists an $\hat{\alpha} \in [\lambda, 1]$ such that

$$\lim_{\alpha \rightarrow \hat{\alpha}} \theta_1^*\left(\frac{1}{\alpha}, \alpha\right) = \lim_{\alpha \rightarrow \hat{\alpha}} \theta_2^*\left(\frac{1}{\alpha}, \alpha\right) = \bar{\theta}.$$

In other words this means that, when $\alpha = \hat{\alpha}$, \bar{c}_{11} and \bar{c}_{12} are equal to $\frac{1}{\hat{\alpha}}$. Increasing α above $\hat{\alpha}$, it must be that $\bar{c}_{11} < \frac{1}{\alpha}$ for (42) to hold and $\bar{c}_{12} > \frac{1}{\alpha}$ to satisfy (43). Symmetrically, if we consider the range $\alpha < \hat{\alpha}$, $\bar{c}_{11} > \frac{1}{\alpha}$ and $\bar{c}_{12} < \frac{1}{\alpha}$. Thus, two intervals can be distinguished

- $\alpha < \hat{\alpha}$ where $\bar{c}_{12} < \frac{1}{\alpha} < \bar{c}_{11}$,
- $\alpha > \hat{\alpha}$ where $\bar{c}_{11} < \frac{1}{\alpha} < \bar{c}_{12}$.

We analyze bank's optimal choice of c_1 in each interval in turn. We start from the range $\alpha < \hat{\alpha}$.

As $\bar{c}_{12} < \frac{1}{\alpha}$, we only have to consider the interval $\alpha \leq \frac{1}{c_1}$ in the bank's problem defined in (26). We start by evaluating (29) at $\alpha = \lambda$ and $\alpha = \hat{\alpha}$. When $\alpha = \lambda$, (29) simplifies to

$$\int_0^{\underline{\theta}(c_1)} u'(c_1) d\theta + \lambda \int_{\underline{\theta}(c_1)}^1 \left[u'(c_1) - Rp(\theta)u' \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta > 0.$$

We assume that at $\alpha = \hat{\alpha}$ (29) is negative. Given the concavity of the objective function, this implies that at $\alpha = \lambda$ the bank chooses a corner solution $c_1^* = \bar{c}_{11}$, while at $\alpha = \hat{\alpha}$ the bank chooses $c_1^* < \bar{c}_{11} < \frac{1}{\alpha}$. As α increases from λ to $\hat{\alpha}$, the solution to bank's problem switches from $c_1^* = \bar{c}_{11}$ to $c_1^* < \bar{c}_{11}$. We

denote as α_1 the exact value of α at which the bank is indifferent between the corner and the interior solution. Thus, α_1 is defined as

$$\begin{aligned} & \lim_{c_1 \rightarrow \bar{c}_{11}} \int_0^{\theta_1^*(c_1, \alpha)} u(c_1) d\theta + \int_{\theta_1^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta \\ = & \int_0^{\theta_1^*(c_1^*, \alpha)} u(c_1^*) d\theta + \int_{\theta_1^*(c_1^*, \alpha)}^1 \left[\lambda u(c_1^*) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1^*}{1 - \lambda} R\right) \right] d\theta \end{aligned}$$

where $c_1^* = \arg \max \int_0^{\theta_1^*(c_1, \alpha)} u(c_1) d\theta + \int_{\theta_1^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta$.

Consider now the range $\alpha > \hat{\alpha}$ where $\bar{c}_{11} < \frac{1}{\alpha} < \bar{c}_{12}$. In this interval, it can also be $c_1^* > \frac{1}{\alpha}$ with c_1^* being the solution to **(30)**. By continuity with the previous case, for α sufficiently close to $\hat{\alpha}$, the optimum is given by

$$c_1^* = \arg \max \int_0^{\theta_1^*(c_1, \alpha)} u(c_1) d\theta + \int_{\theta_1^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta.$$

At $\alpha = 1$, bank's problem is the same as the problem of a bank in the decentralized economy without government intervention. As shown in Proposition **2**, $c_{1D} \equiv c_1^* > 1$. It is easy to show that in this case, $c_1^* < \bar{c}_{12}$ as **(30)** evaluated at the limit for $c_1 \rightarrow \bar{c}_{12}$ simplifies to

$$\lim_{c_1 \rightarrow \bar{c}_{12}} \frac{\partial \theta_2^*(c_1, \alpha)}{\partial c_1} \left[(1 - \lambda) p(\theta_2^*(c_1, \alpha)) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) + \lambda u(c_1) - u\left(\frac{1}{\alpha}\right) \right]$$

and it is negative as $\frac{\partial \theta_2^*(c_1, \alpha)}{\partial c_1} > 0$ and $\lim_{c_1 \rightarrow \bar{c}_{12}} p(\theta_2^*(c_1, \alpha)) = p(\bar{\theta}) = 1$ and $\bar{c}_{12} > \frac{1}{\alpha}$.

By continuity, for an α sufficiently close to 1, we have $\frac{1}{\alpha} \leq c_1^* < \bar{c}_{12}$. We denote as α_2 the value of α at which $c_1^* = \frac{1}{\alpha}$ as given by

$$\left. \frac{\partial E u_2(c_1, \alpha)}{\partial c_1} \right|_{\substack{c_1 = 1/\alpha \\ \alpha = \alpha_2}} = 0$$

where $E u_2(c_1, \alpha) = \int_0^{\theta_2^*(c_1, \alpha)} u\left(\frac{1}{\alpha}\right) d\theta + \int_{\theta_2^*(c_1, \alpha)}^1 \left[\lambda u(c_1) + (1 - \lambda) p(\theta) u\left(\frac{1 - \lambda c_1}{1 - \lambda} R\right) \right] d\theta$.

Thus, the proposition follows. \square

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TABLE 1

	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E[u(c_1, c_2)] \\ E[v(g)] \end{bmatrix}$	$[SW(c_1, c_2, g)]$	$\left[\frac{\theta}{\theta^*} \right]$
Decentralized economy	1.0076 4.98372	0.0139202 0.0173611	0.0312813	0.451436 0.463162
Social planner with fund. runs and no transfers	1.10762 4.7694	0.013945 0.0173611	0.0313061	0.488273 None
Social planner with fund. runs and tranfers	1.32918 4.29462	0.0152042 0.0164171	0.0316213	0.56893 None

TABLE 2: $g = 2$

	α	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E[u(c_1, c_2)] \\ E[v(g)] \end{bmatrix}$	$[SW(c_1, c_2, g)]$	$\left[\frac{\theta}{\theta^*} \right]$
D.I. guaranteeing c_1 at $t = 1$	None	2.27273	0.0185426	0.0274183	$\underline{\theta} \rightarrow 1$
		2.27273	0.00887574		None
D.I. guaranteeing c_1 at $t = 1$ and $t = 2$	None	2.17671	0.0185618	0.0303906	None
		2.47848	0.0118288		None
D.I. guaranteeing the promised repayments	None	2	0.0196398	0.0283203	None
		2.85714	0.00868056		None
Government intervention at $t = 1$	0.905	1.09054	0.0142161	0.0313325	0.48202
		4.80598	0.0171164		0.570596

TABLE 3:g = 2.18

	α	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E[u(c_1, c_2)] \\ E[v(g)] \end{bmatrix}$	$[SW(c_1, c_2, g)]$	$\begin{bmatrix} \underline{\theta} \\ \theta^* \end{bmatrix}$
D.I. guaranteeing c_1 at $t = 1$	None	2.27273 2.27273	0.0185426 0.010487	0.0290296	$\underline{\theta} \rightarrow 1$ None
D.I. guaranteeing c_1 at $t = 1$ and $t = 2$	None	2.17671 2.47848	0.0185618 0.0131734	0.0317353	None None
D.I. guaranteeing the promised repayments	None	1.88 3.11429	0.0199963 0.0090792	0.0290755	None None
Government intervention at $t = 1$	0.628931	1.59 3.73571	0.015249 0.015249	0.0304981	0.666055 $\theta^+ \rightarrow 1$

TABLE 4: $g = 1.03$

	α	$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$	$\begin{bmatrix} E[u(c_1, c_2)] \\ E[v(g)] \end{bmatrix}$	$[SW(c_1, c_2, g)]$	$\begin{bmatrix} \theta \\ \theta^* \end{bmatrix}$
D.I. guaranteeing c_1 at $t = 1$	None	2.03	0.0176447	0.0192851	0.8572
		2.79286	0.00164047		None
D.I. guaranteeing c_1 at $t = 1$ and $t = 2$	None	1.47143	0.0176523	0.0233963	None
		3.9898	0.0118207		None
D.I. guaranteeing the promised repayments	None	1.47143	0.014548	0.0202919	None
		1.47143	0.00574393		None
Government intervention at $t = 1$	0.9917	1.00967	0.0139512	0.0254083	0.452206
		4.97928	0.0114571		0.466807