The Optimal Monetary Policy Instrument, Inflation versus Asset Price Targeting, and Financial Stability

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Motivation

Features of the current crisis:
- Increased default in the U.S. mortgage market
- Contagion to securitized products and credit markets
- Interbank markets fail to act as a conduit for monetary policy
- Collapse of systemically important financial institutions

DSGE models are inappropriate for financial stability analysis.
- Representative agent models: no trade, no default
- Money is a veil
- No financial frictions: default risk, banks, contagion
- Limited scope for welfare improving economic policy: markets are complete
Our Model

Monetary General Equilibrium Model with Commercial Banks, Collateral, Securitisation and Default (MEBCSD)

- Non-trivial quantity theory of money
- Term structure of interest rates depends on aggregate liquidity and default risk
- Fisher effect
- Financial fragility is an equilibrium outcome
- Constrained inefficient equilibrium allocations
- Assessment of various policies for crisis management and prevention
Our Model

Extend the Goodhart, Sunirand and Tsomocos and Goodhart (2006), Tsomocos and Vardoulakis (2008) model to:

- Introduce an investment bank and a hedge fund, and allow for mortgage debt securitisation
- Separate the interbank from the repo market
- Model two types of default
  - Discontinuous default in mortgages (Geanakoplos, 2003)
  - Continuous default in credit markets (Shubik and Wilson, 1977 and Dubey et al., 2005)
Results

- Interest rate instrument is preferable to the monetary base instrument in times of financial distress.

- CPI should include an appropriate measure of housing prices.

- Central Banks’ Financial Stability objective is primarily achieved by regulating systemic financial agents.
Endowment economy

- 2 periods ($t \in T = \{0, 1\}$)
  - First period: a single state
  - Second period: $S$ possible states
  - $S^* = \{0\} \cup S = \{0, 1, 2\}$

- 2 goods:
  - Consumption goods basket (1)
  - Housing (2): a durable good, but infinitely divisible

- Agents
  - Households: $h \in H = \{\alpha, \theta\}$, CRRA preferences
  - Commercial Banks: $j \in J = \{\gamma, \delta\}$, quadratic preferences
  - Investment Banks: $\psi$, risk neutral
  - Hedge Fund: $\phi$, risk neutral
  - The Central Bank/Government/FSA: strategic dummies

- 10 Markets: goods, housing, mortgage, short term loans, consumer deposit, repo, interbank, MBS’s, CDO’s and wholesale money markets
Time Structure

- **t=0**
  1. OMO’s and government bond sales (CB and cb)
  2. Short term credit markets (cb and H)
  3. Mortgage markets meet (cb and H)
  4. Mortgage Backed Asset markets meet (cb and IB)
  5. Interbank market meets (cb)
  6. Wholesale Money market meets (cb and IB, HF)
  7. Deposit market meets (cb and H)

- **t=1**
  1. Trade between households (H)
  2. Trade in (cb and IB, HF)
  3. Short term loans settlement (CB, cb, H)
  4. Consumption (H)

  Nature chooses a state

  - **t=0**
    1. OMO’s and government bond sales (CB and cb)
    2. Short term credit markets (cb and H)
    3. Mortgage market settles (H and cb)
    4. CDO markets settles (IB and HF)
    5. Wholesale Money market settles (cb and IB, HF)
    6. Interbank market settles (cb)
    7. Deposit market settles (cb and H)

  - **t=1**
    1. Trade between households (H)
    2. Short term loans settlement (CB, cb, H)
    3. Consumption (H, IB, HF)
    4. Liquidation of Commercial banks (cb and CB)
Nominal Flows of the Economy

The straight lines and their direction represent lending flows. The dashed lines indicate trade.
Money and Collateral

Money
- Introduced by a cash-in-advance (liquidity) transaction technology
- Enters the system as *outside* or *inside* money

Collateral
- Household $\alpha$ pledges purchased housing as collateral when he takes out the mortgage
- If $\alpha$ defaults on the mortgage, the bank seizes the collateral and offers it for sale in the next period
Default

Two types of Default:

- **Discontinuous** mortgage default. Household $\alpha$ defaults on his mortgage if

\[
\left( \frac{p_{22} b_{02}^\alpha}{p_{02}} \right) \leq (\bar{\mu}^\alpha)
\]

(collateral’s worth) $\leq$ (mortgage debt)

- **Continuous** default in the interbank and wholesale money markets: agents choose a repayment rate satisfying the *On the Verge Condition* (for $k = \{\delta, \psi, \phi\}$):

\[
\left( \frac{\partial \Pi^k}{\partial \bar{v}_s^k} \right) = \tau_s^k
\]

(marginal utility of default) $= (bankruptcy\ penalty)$
Scarcity of collateral incentivizes agents to stretch it by using it many times.

- The investment bank ($\psi$) buys the mortgage from bank $\gamma$ at a price $p^\alpha$ in the MBS's market.
- The investment bank ($\psi$) structures a CDO by attaching a Credit Default Swap (CDS) to the MBS.
- The hedge fund ($\phi$) purchases the CDO at a price $\tilde{q}^\alpha$.
- CDO’s gross returns:

$$R^{CDO} = \left[ \frac{(1 + \bar{r}^\gamma) / \tilde{q}^\alpha}{1} \right]$$

- The investment bank bears the mortgage and CDS risk.
Household $\alpha$’s Optimisation Problem

\[ \max_{q_1^{\alpha*}, q_2^{\alpha*}, \mu^{\alpha*}, \bar{\mu}^{\alpha}} U^{\alpha} = u(e^{\alpha}_{01} - q^{\alpha}_{01}) + u\left(\frac{b^{\alpha}_{02}}{p_02}\right) + \sum_{s \in S} \omega_s u\left(e^{\alpha}_{s1} - q^{\alpha}_{s1}\right) \]

\[ + \sum_{s \in S_{1}^{\alpha}} \omega_s u\left(\frac{b^{\alpha}_{02}}{p_02} + \frac{b^{\alpha}_{s2}}{p_{s2}}\right) + \sum_{s \notin S_{1}^{\alpha}} \omega_s u\left(\frac{b^{\alpha}_{s2}}{p_{s2}}\right) \]

\[ s.t. \]

\[ b^{\alpha}_{02} \leq \frac{\bar{\mu}^{\alpha}}{(1 + \bar{r}^{\gamma \alpha})} + \frac{\mu^{\alpha}_0}{(1 + r^{\gamma}_0)} + e^{\alpha}_{m,0} \]

i.e. housing expenditure at $t=0 \leq$ mortgage loan + short-term borrowing + private monetary endowments at $t=0$

\[ \mu^{\alpha}_0 \leq p_{01} q^{\alpha}_{01} \]

i.e. short term loan repayment at $t=0 \leq$ goods sales revenues at $t=0$
Houshold $\alpha$’s Optimisation Problem

\[ b_{s2}^\alpha + \bar{\mu}_s^\alpha \leq \frac{\mu_s^\alpha}{(1 + r_s^\gamma)} + e_{m,s}^\alpha \quad \text{for} \quad s \in S_1^\alpha \]

i.e. housing expenditure at $s \in S_1^\alpha$ + mortgage repayment $\leq$ short-term borrowing + private monetary endowments at $s \in S_1^\alpha$

\[ b_{s2}^\alpha \leq \frac{\mu_s^\alpha}{(1 + r_s^\gamma)} + e_{m,s}^\alpha \quad \text{for} \quad s \notin S_1^\alpha \]

i.e. housing expenditure at $s \notin S_1^\alpha \leq$ short-term borrowing + private monetary endowments at $s \notin S_1^\alpha$

\[ \mu_s^\alpha \leq p_{s1} q_{s1}^\alpha \]

i.e. short term loan repayment $\leq$ goods sales revenues at $t=0$

\[ q_{s*1}^\alpha \leq e_{s*1}^\alpha \]

i.e. quantity of goods sold at $s \in S^* \leq$ goods endowments at $s \in S^*$
Houshold $\theta$’s Optimisation Problem

$$\max_{q_{s*2}, b_{s*1}, \mu_{s*}, \bar{d}} U^\theta = u\left(\frac{b_{01}}{p_{01}}\right) + u\left(e_{02}^\theta - q_{02}^\theta\right) + \sum_{s \in S} \omega_s u\left(\frac{b_{02}^s}{p_{02}}\right) + \sum_{s \in S} \omega_s u\left(e_{s2}^\theta - q_{s0}^\theta - q_{s2}^\theta\right)$$

s.t.

$$b_{01}^\theta + \bar{d}^\theta \leq \frac{\mu_0^\theta}{1 + r_0^\delta} + e_{m,0}^\theta$$

i.e. goods expenditure at $t=0$ + inter-period deposits $\leq$ short-term borrowing + private monetary endowments at $t=0$

$$\mu_0^\theta \leq p_{02} q_{02}^\theta$$

(i.e. short term loan repayment at $t=0 \leq$ housing sales revenues at $t=0$)
Houshold $\theta$’s Optimisation Problem

$$b_{s1}^\theta \leq \frac{\mu_{s}^\theta}{1 + r_{\delta}^s} + d^\theta (1 + r_{\gamma}^d) + e_{m,s}^\theta, \quad \text{for } s \in S$$

i.e. goods expenditure at $s \in S \leq$ short-term borrowing + deposits and interest payment + private monetary endowments at $s \in S$

$$\mu_{s}^\theta \leq p_{s2} q_{s2}^\theta$$

i.e. short term loan repayment at $s \in S \leq$ housing sales revenues at $s \in S$

$$q_{s*2}^\theta \leq e_{s2}^\theta - q_{02}^\theta$$

i.e. number of housing units sold at $s \in S \leq$ endowment of housing at $t=0$ - units of housing sold at $s \in S$
Bank $\gamma$’s Optimisation Problem

$$\max_{m_{s=2}, \bar{m}, \bar{d}, \bar{d}, \pi_s} \Pi = \sum_{s \in S} \omega_s \left( \pi_s - c(\pi_s)^2 \right)$$

s.t.

$$d_0^{G\gamma} + m_0^\gamma + \bar{m} + \bar{d} \leq e_0^\gamma + (\bar{\mu}_d/1 + \bar{r}_d)$$

i.e. deposits in the repo market + short-term lending + mortgage extension + interbank lending $\leq$ capital endowment at $t=0$ + consumer deposits

$$d_s^{G\gamma} + m_s^\gamma + \bar{\mu}_d + \leq e_s^\gamma + \pi_0^\gamma + \bar{R}_s^{d\gamma} \bar{d} (1 + \bar{\rho})$$

i.e. short-term lending + deposits in the repo market at $s \in S$ + deposits repayment $\leq$ capital endowment at $s \in S$ + accumulated profits + interbank loan repayments at $s \in S$

$$\pi_0^\gamma = m_0^\gamma (1 + r_0^\gamma) + d_0^{G\gamma} (1 + \rho_0^{CB}) + \rho^\alpha \bar{m}^\alpha$$

i.e. profits at $(t=0) = \text{short term loan repayment} + \text{repo deposits and interest payment at } t=0 + \text{MBS’s sales revenues}$

$$\pi_s^\gamma = m_s^\gamma (1 + r_s^\gamma) + d_s^{G\gamma} (1 + \rho_s^{CB})$$

i.e. profits at $s \in S = \text{short term loan repayment} + \text{repo deposits and interest payment at } s \in S$
Bank $\delta$'s Optimisation Problem

\[
\max_{m_\delta^*, \bar{m}, \bar{\mu}_G^\delta, \bar{\mu}^\delta, \bar{\nu}_s^\delta, \pi_s^\delta} \prod^\delta = \sum_{s \in S} \omega_s \left( \pi_s^\delta - c^\delta \left( \pi_s^\delta \right)^2 \right) - \sum_{s \in S} \omega_s \bar{\tau}_s^\delta \left[ \bar{D}_s^\delta \right]^+
\]

s.t.

\[
m_\delta^* + \bar{m} \leq e_0^\delta + \frac{\mu_0^{G\delta}}{1 + \rho_0^{CB}} + \frac{\bar{\mu}^\delta}{1 + \bar{\rho}}
\]

i.e. short-term lending at $t=0$ + wholesale money market credit extension $\leq$ capital endowment + short-term borrowing in the repo market at $t=0$ + interbank borrowing

\[
\mu_0^{G\delta} \leq m_0^\delta \left( 1 + r_0^\delta \right)
\]

i.e. repo loan repayment at $t=0 \leq$ short-term loan repayment at $t=0$

\[
m_s^\delta + \bar{\nu}_s^\delta \bar{\mu} \leq e_s^\delta + \frac{\mu_s^{G\delta}}{1 + \rho_s^{CB}} + \bar{R}_s \bar{m} (1 + \bar{r})
\]

i.e. short-term lending + interbank loan repayment at $s \in S \leq$ capital endowment + wholesale money market loan repayment short-term loan repayment at $s \in S$

\[
\pi_s^\delta = m_s^\delta \left( 1 + r_s^\delta \right) - \mu_s^{G\delta}
\]

i.e. profits at $s \in S = $ short term loan repayment - repo loan repayment at $s \in S$
Investment Bank \((\psi)\)'s Optimisation Problem

\[
\max_{\tilde{m}_s^\alpha, \tilde{\mu}_s^\psi, \tilde{v}_s^\psi} \Pi_s^\psi = \sum_{s \in S} \omega_s \pi_s^\psi - \sum_{s \in S} \omega_s \tilde{r}_s^\psi \left[ \tilde{D}_s^\psi \right]^+ 
\]

s.t.

\[
\tilde{m}_s^\alpha \leq e_s^\psi + \frac{\tilde{\mu}_s^\psi}{1 + \bar{r}} \quad \text{i.e. expenditure in MBS's} \leq \text{ capital endowments at } t=0 + \text{ wholesale money market borrowing}
\]

\[
\tilde{v}_s^\psi \tilde{\mu}_s^\psi \leq \frac{\tilde{m}_s^\alpha}{p_s^\alpha} \tilde{q}_s^\alpha \quad \text{for } s \in S_1^\alpha \quad \text{i.e. whole sale money market loan repayment at } s \in S_1^\alpha \leq \text{CDO's sales revenues + capital endowments at } s \in S_1^\alpha
\]

\[
\tilde{m}_s^\alpha \tilde{q}_s^\alpha + \tilde{v}_s^\psi \tilde{\mu}_s^\psi \leq e_s^\psi + \left( \tilde{q}_s^\alpha + \frac{b_{02}^\alpha p_{22}}{\tilde{m}_s^\alpha p_{22}} \right) \frac{\tilde{m}_s^\alpha}{p_s^\alpha} \quad \text{for } s \notin S_1^\alpha
\]

i.e. CDS settlement payment + wholesale money market loan repayment at \(s \notin S_1^\alpha \leq \text{ capital endowment at } s \notin S_1^\alpha + \text{CDO's sales revenues} + \text{collateral sales revenues}
Hedge Fund \((\phi)\)'s Optimisation Problem

\[
\max_{\bar{\mu}_\phi, \hat{m}_\alpha, \bar{v}_{s*}^\phi} \Pi^\phi = \sum_{s \in S} \omega_s \pi^\phi_s - \sum_{s \in S} \omega_s \bar{T}_s^\phi \left[ \bar{D}_s^\phi \right]^+ 
\]

\[s.t.
\]

\[
\hat{m}_\alpha \leq \frac{\bar{\mu}_\phi}{1 + \bar{r}}
\]

i.e. expenditure in the CDO's market \(\leq\) wholesale money market borrowing

\[
\bar{v}_s^\phi \bar{\mu}_\psi \leq \frac{\hat{m}_\alpha}{\bar{q}_\alpha} \left( 1 + \bar{r}^{\gamma_\alpha} \right) \quad \text{for} \quad s \in S_1^\alpha
\]

i.e. wholesale money market loan repayment \(\leq\) CDO's payoffs at \(s \in S_1^\alpha\)

\[
\bar{v}_s^\phi \bar{\mu}_\psi \leq \hat{m}_\alpha \quad \text{for} \quad s \notin S_1^\alpha
\]

i.e. wholesale money market loan repayment \(\leq\) CDO's payoffs at \(s \notin S_1^\alpha\)
Market Clearing Conditions

Goods Market

\[ p_{01} = \frac{b_{01}^\theta}{q_{01}^\alpha} \]
\[ p_{s1} = \frac{b_{s1}^\theta}{q_{s1}^\alpha} \quad \text{for} \quad s \in S \]

Housing Market

\[ p_{02} = \frac{b_{02}^\alpha}{q_{02}^\theta} \]
\[ p_{s2} = \frac{b_{s2}^\alpha}{q_{s2}^\theta} \quad \text{for} \quad s \in S_1^\alpha \]
\[ p_{s2} = \frac{b_{s2}^\alpha}{q_{s2}^\theta + b_{02}^\alpha / p_{02}} \quad \text{for} \quad s \notin S_1^\alpha \]
Market Clearing Conditions

Mortgage Market

\[(1 + \bar{r}^{\gamma\alpha}) = \frac{\bar{\mu}^{\alpha}}{\bar{m}^{\alpha}}\]

Clearing conditions for effective returns on mortgages

\[(1 + \bar{r}_{s}^{\gamma\alpha}) = \begin{cases} (1 + \bar{r}^{\gamma\alpha}) & \text{for } s \in S_1^{\alpha} \\ \left( \frac{p_{22} b_{02}^{\alpha}}{p_{02}} \right) \left( \frac{\bar{\mu}^{\alpha}}{1 + \bar{r}^{\gamma\alpha}} \right)^{-1} & \text{for } s \notin S_1^{\alpha} \end{cases}\]

Short-term Consumer Markets

\[(1 + r_{s}^{\gamma}) = \frac{\mu_{s}^{\alpha}}{m_{s}^{\gamma}}\]

\[(1 + r_{s}^{\delta}) = \frac{\mu_{s}^{\theta}}{m_{s}^{\delta}}\]
## Market Clearing Conditions

### Consumer Deposit Market

\[
(1 + \bar{r}_d) = \frac{\bar{\mu}_d}{d\theta}
\]

### Wholesale Money Market

\[
(1 + \bar{r}) = \frac{\bar{\mu}_\psi + \bar{\mu}_\phi}{\bar{m}}
\]

### Repo Market

\[
(1 + \rho_{s*}^{CB}) = \frac{\mu_{s*}^{G\delta}}{M_{s*}^{CB} + d_{s*}^{G\gamma}}
\]

### Interbank Market

\[
(1 + \bar{\rho}) = \frac{\bar{\mu}_\delta}{d\gamma}
\]

### MBS’s Market

\[
p^\alpha = \frac{\tilde{m}^\alpha}{\bar{m}^\alpha}
\]

### CDO’s Market

\[
\tilde{q}^\alpha = \frac{\hat{m}^\alpha}{\bar{m}^\alpha}
\]
Conditions on Expected Delivery Rates (Rational Expectations)

**Wholesale Money Market**

\[
\bar{R}_s = \begin{cases} 
\frac{\bar{V}_s^\psi \bar{\mu}^\psi + \bar{V}_s^\phi \bar{\mu}^\phi}{\bar{\mu}^\psi + \bar{\mu}^\phi} & \text{if } \bar{\mu}^\psi + \bar{\mu}^\phi > 0 \\
\text{arbitrary} & \text{if } \bar{\mu}^\psi + \bar{\mu}^\phi = 0
\end{cases} \quad \forall s \in S
\]

**Interbank Market**

\[
\bar{R}_s^\delta = \begin{cases} 
\frac{\bar{V}_s^\delta \bar{\mu}^\delta}{\bar{\mu}^\delta} = \bar{V}_s^\delta & \text{if } \bar{\mu}^\delta > 0 \\
\text{arbitrary} & \text{if } \bar{\mu}^\delta = 0
\end{cases} \quad \forall s \in S
\]
Definition of Equilibrium

Let

\[ \sigma^\alpha = (q_{s1}^\alpha, b_{s2}^\alpha, \mu_s^\alpha, \bar{\mu}^\alpha) \]
\[ \sigma^\theta = (q_{s1}^\alpha, b_{s2}^\alpha, \mu_s^\theta, \bar{d}^\theta) \]
\[ \sigma^\gamma = (\phi_s^\gamma, m_s^\gamma, d_s^G^\gamma, \bar{m}^\alpha, \bar{\mu}^\gamma, \bar{d}^\gamma) \]
\[ \sigma^\delta = (\phi_s^\delta, m_s^\delta, \mu_s^G^\gamma, \bar{v}_s^\delta, \bar{m}, \bar{\mu}^\delta) \]
\[ \sigma^\psi = (\bar{v}_s^\psi, \bar{\mu}^\psi, \bar{m}^\alpha) \]
\[ \sigma^\phi = (\bar{v}_s^\phi, \bar{\mu}^\phi, \bar{m}^\alpha) \]

\[ \eta = (p_{s1}, p_{s2}, \rho_s^\text{CB}, r_s^\gamma, r_s^\delta, \bar{r}^\gamma_\alpha, \bar{r}_d^\gamma, \bar{r}, \bar{\rho}, p^\alpha, \tilde{q}^\alpha) \]

Then \((\sigma^\alpha, \sigma^\theta, \sigma^\gamma, \sigma^\delta, \sigma^\psi, \sigma^\phi, \eta)\) is a MEBCSD iff:

1. All agents maximize given their budget sets
2. All markets clear.
3. Expectations are rational.
Credit Spreads

Proposition 1

At any MEBCSD, \( r_s^\delta, \rho_{s^*}^{CB} \geq 0, \quad r_s^\delta = \rho_{s^*}^{CB} \quad \forall s^* \in S^* \).
Credit Spreads

Proposition 1

At any MEBCSD, \( r_{s^*}^\delta, \rho_{s^*}^{CB} \geq 0, \ r_{s^*}^\delta = \rho_{s^*}^{CB} \ \forall s^* \in S^*. \)

Proposition 2

At any MEBCSD, \( r_{s^*}^\gamma, \rho_{s^*}^{CB} \geq 0, \ r_{s^*}^\gamma = \rho_{s^*}^{CB} \ \forall s^* \in S^*. \)
Credit Spreads

**Proposition 1**
At any MEBCSD, \( r^\delta_{s^*}, \rho^CB_{s^*} \geq 0 \), \( r^\delta_{s^*} = \rho^CB_{s^*} \) \( \forall s^* \in S^* \).

**Proposition 2**
At any MEBCSD, \( r^\gamma_{s^*}, \rho^CB_{s^*} \geq 0 \), \( r^\gamma_{s^*} = \rho^CB_{s^*} \) \( \forall s^* \in S^* \).

**Proposition 3**
At any MEBCSD, \( \bar{r}^\gamma_{d}, \bar{\rho}^CB_{0} \geq 0 \), \( \bar{r}^\gamma_{d} = \bar{\rho}^CB_{0} \).
Credit Spreads

Proposition 1
At any MEBCSD, \( r^\delta_{s^*}, \rho^\delta_{s^*} \geq 0, \quad r^\delta_{s^*} = \rho^\delta_{s^*} \quad \forall s^* \in S^* \).

Proposition 2
At any MEBCSD, \( r^\gamma_{s^*}, \rho^\gamma_{s^*} \geq 0, \quad r^\gamma_{s^*} = \rho^\gamma_{s^*} \quad \forall s^* \in S^* \).

Proposition 3
At any MEBCSD, \( \bar{r}^\gamma_{d}, \rho^\gamma_{0} \geq 0, \quad \bar{r}^\gamma_{d} = \rho^\gamma_{0} \).

Proposition 4
At any MEBCSD, \( p^\alpha_{\alpha}, \rho^\alpha_{0} \geq 0 \) and \( p^\alpha_{\alpha} = 1 + \rho^\alpha_{0} \).
Credit Spreads

**Proposition 1**
At any MEBCSD, \( r_{s^*}^{\delta}, \rho_{s^*}^{CB} \geq 0, \ r_{s^*}^{\delta} = \rho_{s^*}^{CB} \) \ \forall s^* \in S^*.

**Proposition 2**
At any MEBCSD, \( r_{s^*}^{\gamma}, \rho_{s^*}^{CB} \geq 0, \ r_{s^*}^{\gamma} = \rho_{s^*}^{CB} \) \ \forall s^* \in S^*.

**Proposition 3**
At any MEBCSD, \( \bar{r}_{d}^{\gamma}, \rho_{0}^{CB} \geq 0, \ \bar{r}_{d}^{\gamma} = \rho_{0}^{CB} \).

**Proposition 4**
At any MEBCSD, \( p^{\alpha}, \rho_{0}^{CB} \geq 0 \) and \( p^{\alpha} = 1 + \rho_{0}^{CB} \).

**Proposition 5**
At any MEBCSD, \( \bar{r}, \bar{\rho}, \bar{r}_{d}^{\gamma} \geq 0 \) and \( \bar{r} \geq \bar{\rho} \geq \bar{r}_{d}^{\gamma} \).
Proposition 6

At any MEBCSD for \( s \in S_1^\alpha \),

\[
\sum_{j \in J} \left( m^j_0 r^j_0 \right) + \rho^CB m^\alpha + \sum_{j \in J} \left( \pi^j_s \right) + \rho^CB M^CB_s + \rho^CB \bar{r}^\gamma m^\alpha = \\
\sum_{h \in H} \left( e^{h}_{m,0} + e^{h}_{m,s} \right) + \sum_{\tilde{k} = \{\gamma, \delta, \psi\}} \left( e^{k}_{0} + e^{k}_{s} \right) + \frac{r_0^\gamma}{1 + r_0^\gamma} \pi_0^\gamma
\]

For \( s \notin S_1^\alpha \),

\[
\sum_{j \in J} \left( m^j_0 r^j_0 \right) + \rho^CB m^\alpha + \sum_{j \in J} \left( \pi^j_s \right) + \rho^CB M^CB_s + \rho^CB \bar{r}^\gamma m^\alpha \left( \bar{q}^\alpha - (1 + \bar{r}_s^\gamma \alpha) \right) = \\
\sum_{h \in H} \left( e^{h}_{m,0} + e^{h}_{m,s} \right) + \sum_{\tilde{k} = \{\gamma, \delta, \psi\}} \left( e^{k}_{0} + e^{k}_{s} \right) + \frac{r_0^\gamma}{1 + r_0^\gamma} \pi_0^\gamma
\]

Put formally, \( \forall s \in S \) aggregate ex-post interest rate payments to commercial banks adjusted by default equal the economy's total amount of outside money plus interest payments of commercial banks' accumulated profits.
Proposition 6 (continued)

For $t = 0$

$$\sum_{j \in J} (m_j r_j^j) < \sum_{h \in H} (e_h^m, 0) + \sum_{\tilde{k} = \{\gamma, \delta, \psi\}} (e_{0}^k)$$

In the first period, where uncertainty induces commercial banks to accumulate profits and/or make indirect investments in the derivatives markets; thus aggregate interest payments will be less than or equal to aggregate initial monetary endowments.
Lemma 7

Assume agent $h$ borrows from bank $j$ in the short term credit market. Furthermore, let $\{\chi_{s^*,l}^h, \chi_{s^*,m}^h\}$ denote traded quantities of two distinct goods $\{l, m\}$, and suppose that $h$ purchases good $l$, and sells and has an endowment $(e_{s^*,m}^h)$ of good $m$ at $s^* \in S^*$. If $r_{s^*}^j > 0$, then

$$\frac{p_{s^*l} \left(1 + r_{s^*}^j\right)}{p_{s^*m}} = \frac{u'(\chi_{s^*l}^h)}{u'(e_{s^*m}^h - \chi_{s^*m}^h)}$$

i.e. there is a wedge between selling and purchasing prices.
Lemma 7

Assume agent $h$ borrows from bank $j$ in the short term credit market. Furthermore, let $\{\chi^h_{s^*l}, \chi^h_{s^*m}\}$ denote traded quantities of two distinct goods $\{l, m\}$, and suppose that $h$ purchases good $l$, and sells and has an endowment $(e^h_{s^*m})$ of good $m$ at $s^* \in S^*$. If $r^j_s > 0$, then

$$\frac{p^s_l (1 + r^j_s)}{p^s_m} = \frac{u'(\chi^h_{s^*l})}{u'(e^h_{s^*m} - \chi^h_{s^*m})}$$

i.e. there is a wedge between selling and purchasing prices.

Proposition 8

If nominal interest rates are positive, then monetary policy is non-neutral.
Proposition 9

In a MEBCSD, if $\rho_{s^*}^{CB} > 0$ for some $s^* \in S^*$, then at $s \in S_1^\alpha$

$$\sum_{h \in H, l} (p_{sl} q_{sl}^h) = \sum_{h \in H} e_m^h + \sum_{j \in J} e_j^i + M_{s}^{CB} + \pi_0^\gamma + \bar{R}_s \bar{m} (1 + \bar{r}) - \bar{m}^\alpha (1 + \bar{r}^\gamma)$$

Aggregate income at $s \in S_1^\alpha$ is equal to the stock of money at that period, namely the total amount of outside and inside money, plus commercial banks’ accumulated profits from the previous period, plus the banking financial sector's net payoffs from its indirect investments in the derivatives markets. When there is no default in the mortgage market, the mortgage’s repayment is forgone income to commercial banks and is used by the hedge fund to repay its wholesale money market obligation. (In stark constrast with Lucas and Real Business Cycle models)
Quantity Theory of Money Proposition

Proposition 9 (continued)

For \( s \notin S_1^\alpha \)

\[
\sum_{h \in H, l = \{1, 2\}} (p_{sl}q_{sl}^h) = \sum_{h \in H} e_{m,s}^h + \sum_{j \in J} e_j^s + M_s^{CB} + \pi_0^\gamma + \bar{R}_s \bar{m} (1 + \bar{r})
\]

When there is default in the mortgage market, the quantity theory of money holds as in the previous case but the banking financial sector’s loss due to default on the mortgage and derivatives markets is embedded in the expected repayment rates of wholesale money market loans.

Proposition 9 (continued)

For \( s = 0 \)

\[
\sum_{h \in H, l = \{1, 2\}} (p_{0l}q_{0l}^h) = \sum_{h \in H} e_{m,0}^h + \sum_{j \in J} e_j^0 + M_0^{CB} - \bar{m}
\]

National income is equal to the stock of money in the economy less indirect expenditures by commercial banks in the derivatives markets.
The Fisher Effect Proposition

Proposition 10

Suppose agent $\alpha$ chooses $b_{02}^\alpha, b_{12}^\alpha > 0$ and has money left over when the mortgage loan comes due, then at a MEBCSD the following equation must hold

$$
(1 + \bar{r}^{\gamma\alpha}) = \left(1 + \frac{u'(\chi_{02}^\alpha)}{u'(\chi_{02}^\alpha + \chi_{12}^\alpha)}\right) \left(\frac{p_{12}}{p_{02}}\right) \iff \bar{r}^{\gamma\alpha} \approx \frac{u'(\chi_{02}^\alpha)}{u'(\chi_{02}^\alpha + \chi_{12}^\alpha)} + \Pi_{12}
$$

Similarly, assume agent $\theta$ chooses $b_{s^*_2}^\theta > 0 \ \forall s^* \in S^*$, and has money left over when the consumer deposit market meets, then at a MEBCSD

$$
(1 + \bar{r}^\gamma) = \frac{u'(\chi_{01}^\theta)}{E_s\left\{u'(\chi_{01}^\theta) / p_{s1}\right\}} \iff \bar{r}^\gamma \approx \frac{u'(\chi_{01}^\theta)}{u'(\chi_{01}^\theta)} + \bar{\Pi}_{s1} + \log\left(\frac{\lambda_s}{\omega_s}\right)
$$

Hence, nominal long term interest rates are approximately equal to real interest rates plus expected inflation and a risk premium.
Discussion of the Equilibrium

- Economy experiences an adverse productivity shock: moderate at \( s = 1 \) and severe at \( s = 2 \)
- Central Bank reacts with expansionary monetary policy at \( s = 1 \) and contractionary monetary policy at \( s = 2 \)
- \( \alpha \) is poorer than \( \theta \) in monetary endowments at \( t = 0 \)
- Bank \( \gamma \) is more capitalized than bank \( \delta \) and the investment bank at all states
- The hedge fund has no capital
- Housing deflation and goods inflation
  - Negative productivity (supply) shock increases goods prices
  - House prices fall due to \( \alpha \)'s lower demand for housing
- Fall in relative house prices leads to
  - Lower trade in the housing and goods markets at \( s = 2 \)
  - Fall in the mortgage’s effective return at \( s = 2 \)
Discussion of the Equilibrium (continued)

- At $s = 1$ no mortgage default, hence there’s no default in wholesale money market
- At $s = 2$, $\alpha$ defaults on his mortgage
  - Significant losses in non-banking financial sector
  - CDS contract executed: $\phi$ delivers collateral to $\psi$ in exchange for initial investment value
  - $\psi$ assumes write down loss
- Economy becomes *financial unstable* at $s = 2$:
  - Default increases in wholesale and interbank markets
  - Banks’ profits fall
- Monetary policy
  - Partially offsets effects of adverse productivity shock at $s = 1$
  - Exacerbates effects of adverse productivity shock at $s = 2$
## Initial Equilibrium

<table>
<thead>
<tr>
<th>Prices</th>
<th><em>Housholds</em></th>
<th>Financial Sector</th>
<th>Trade and Spending</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lending</td>
<td>Lending</td>
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<tr>
<td></td>
<td>Borrowing</td>
<td>Repayment Rates</td>
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Comparative Statics: Crisis Catalysts

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<tr>
<th></th>
<th>Increase Money Supply $t = 0$</th>
<th>Increase $\theta$'s Housing Endowment $t = 0$</th>
<th>Increase Money Supply $t = 0$</th>
<th>Increase $\theta$'s Housing Endowment $t = 0$</th>
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<td>$\bar{p}_{22}$</td>
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<td>+</td>
<td>$\bar{U}^\alpha$</td>
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<tr>
<td>$d \gamma$</td>
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<td>+</td>
<td>$\pi_2^{\gamma}$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>$\approx$</td>
<td>-</td>
<td>$\pi_2^{\delta}$</td>
<td>$\approx$</td>
</tr>
</tbody>
</table>

Expansionary monetary policy at $t = 0$

- Improves households' welfare
- $\alpha$ and $\psi$ default more and $\downarrow \pi_2^{\gamma}$ as $\downarrow (\rho - \bar{r}_d^{\gamma})$
- Mortgage crisis exacerbated
- Leverage procyclicality
- $\uparrow$ Financial Fragility (FF)

Government Subsidies: The Transfer Paradox

- $\alpha$'s welfare increases at the expense of and $\theta$'s
- $\uparrow$ Mortgage default, and $\downarrow \bar{v}_2^{\delta}$, $\downarrow \bar{v}_2^{\psi}$ and $\downarrow \bar{v}_2^{\phi}$
- $\downarrow \pi_2^{\gamma}$ due to $\downarrow (\rho - \bar{r}_d^{\gamma})$ and $\downarrow \bar{v}_2^{\delta}$
- Leverage procyclicality
- $\uparrow$ FF

Greenspan Policy

Paulson Plan
Comparative Statics: Optimal Monetary Policy Instrument

<table>
<thead>
<tr>
<th>Increase Money Supply</th>
<th>Decrease Repo Rate</th>
<th>Increase Money Supply</th>
<th>Decrease Repo Rate</th>
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</thead>
<tbody>
<tr>
<td>$s = 2$</td>
<td>$s = 2$</td>
<td>$s = 2$</td>
<td>$s = 2$</td>
</tr>
</tbody>
</table>

| $p_{02}$             | +                  | +                     | $\bar{\mu}^{\psi}$ | -                  | -                     |
| $p_{22}$             | +                  | +                     | $\bar{\nu}_{2}^{\psi}$ | +                  | +                     |
| $\bar{r}^{\gamma \alpha}$ | -                | -                     | $\bar{\nu}_{2}^{\phi}$ | -                  | -                     |
| $\bar{r}$           | $\approx$         | -                     | $U^{\alpha}$        | $\approx$         | $\approx$             |
| $\bar{\rho}$        | $\approx$         | -                     | $U^{\theta}$        | -                  | +                     |
| $\bar{r}_{d}^{\gamma}$ | $\approx$     | $\approx$              | $\pi_{2}^{\gamma}$  | -                  | -                     |
| $d^{\gamma}$        | -                  | +                     | $\pi_{2}^{\delta}$  | -                  | -                     |
| $\bar{v}_{2}^{\delta}$ | -                | -                     |                       | -                  | -                     |

**Monetary Base Instrument**
- ↓ Households’ welfare (θ credit constrained)
- ↓ Default in mortgage, ↑ $\bar{v}_{2}^{\delta}$, ↑ $\bar{v}_{2}^{\psi}$ and ↑ $\bar{v}_{2}^{\phi}$
- ↓ Banks profits
- ‘Localized’ liquidity trap
- FF improves partially

**Interest Rate Instrument**
- ↑ Households’ welfare (no credit constraints)
- ↓ Mortgage default, and ↑ $\bar{v}_{2}^{\delta}$, ↑ $\bar{v}_{2}^{\psi}$ and ↑ $\bar{v}_{2}^{\phi}$
- ↓ Banks profits (insufficient ↑lending)
- Undistorted transmission mechanism of M.P.
- FF improves partially

Interest rate instrument is preferable to the monetary base instrument in times of financial distress
Comparative Statics: Regulatory Policies

<table>
<thead>
<tr>
<th>tighter $\psi$'s Default Penalty $s = 2$</th>
<th>Increase $\gamma$’s Risk Aversion Coefficient</th>
<th>tighter $\psi$’s Default Penalty $s = 2$</th>
<th>Increase $\gamma$’s Risk Aversion Coefficient</th>
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</thead>
<tbody>
<tr>
<td>$p_{02}$</td>
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<tr>
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<tr>
<td>$d^\gamma$</td>
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<td>$\pi^\gamma$</td>
<td>+</td>
</tr>
<tr>
<td>$\bar{v}_{\delta}^2$</td>
<td>$\approx$</td>
<td>$\pi^\delta_2$</td>
<td>-</td>
</tr>
</tbody>
</table>

Default Penalties for $\psi$

- Weak improvement of households’ welfare
- ↓ Default in mortgage, ↑ $\bar{v}_{\delta}^\delta$, ↑ $\bar{v}_{\psi}^\psi$ and ↑ $\bar{v}_{\phi}^\phi$
- ↑ Banks profits
- Countercyclical leverage
- ↑ FF

$\gamma$ becomes more prudent

- ↑ Households’ welfare (credit conditions ease)
- ↓ Mortgage default, and ↑ $\bar{v}_{\delta}^\delta$, ↑ $\bar{v}_{\psi}^\psi$ and ↑ $\bar{v}_{\phi}^\phi$
- ↑ Banks profits as ↑ $(\rho - \bar{r}_d^\gamma)$
- Countercyclical leverage
- ↑ FF

Central Banks’ Financial Stability objective should be primarily achieved by regulating systemic financial agents
Implications for Inflation Targeting

Central Banks are responsible for Price and Financial Stability

Initial Equilibrium

- House and goods prices move in opposite directions
- Central Bank reacts to stabilize goods inflation only: when goods inflation and relative prices are higher monetary policy is tightened (at $s = 2$)
- Tighter monetary policy contributes to default rates increase at $s = 2$ (Greenspan/Trichet/King)
Implications for Inflation Targeting

Comparative Statics

- Expansionary monetary policy at $t = 0$ increases default and reduces banks’ profits (Greenspan 2005-2007)
- Expansionary monetary policy at $s = 2$ (if effective) reduces default but fails to increase banks’ profits (Current Central Banks’ policy)
- Regulatory policies are more effective at reducing default and increasing banks’ profits (Price and Financial Stability cannot be achieved with a single instrument)

Hence, the Price Index should include the behavior of housing prices
Models to analyze Financial Stability should include
- Heterogenous agents
- Endogenous Default
- An essential role for money
- Incomplete financial markets

Collateral and securitisation features introduced also important for current juncture analysis

In our model
- Changes to the money supply feed into prices and quantities
- Monetary and regulatory policies are not neutral
- Fisher effect is incorporated
- Interest rates differentials respond to aggregate liquidity and default
In times of crisis, monetary policy conducted by means of the interest rate instrument is a more effective than using the monetary base instrument (See also Goodhart, Sunirand and Tsomocos, 2008)

- CPI should include an appropriate measure of housing prices
- Optimal regulatory policies should target systemic financial agents and induce them to behave more prudently before crises

THANK YOU