Expectations, Deflation Traps and Macroeconomic Policy

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Abstract

We examine global economic dynamics under infinite-horizon learning in a New Keynesian model in which the interest-rate rule is subject to the zero lower bound. As in Evans, Guse and Honkapohja (2008), we find that under normal monetary and fiscal policy the intended steady state is locally but not globally stable. Unstable deflationary paths can arise after large pessimistic shocks to expectations. For large expectation shocks pushing interest rates to zero lower bound, temporary increases in government spending can be used to insulate the economy from deflation traps.

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1 Introduction

The experiences of 2008 and 2009 clearly point to the possibility that the zero lower bound on nominal interest rates has the potential to generate a "liquidity trap" with the potential of the economy becoming stuck in a deflationary situation with declining or persistently low levels of output. The theoretical plausibility of the economy becoming trapped in a deflationary state, and the macroeconomic policies that might be able to avoid or extricate the economy from a liquidity trap, have been examined predominantly from the rational expectations (RE) perspective in the recent literature.[1] Our own view, reflected in Evans and Honkapohja (2005) and Evans, Guse, and Honkapohja (2008), is that the evolution of expectations plays a key role in the dynamics of the economy and that the tools from learning theory are needed for a realistic analysis of these issues.

Under learning private agents are assumed to form expectations using an adaptive forecasting rule, which they update over time in accordance with standard statistical procedures. The analysis of Evans, Guse, and Honkapohja (2008) was conducted in a standard New Keynesian model with sticky prices using the assumption that the decisions of private agents are based on short-horizon decision rules, inherited from their Euler equations, under subjective expectations with forecasts based on standard adaptive learning. This framework of "Euler equation learning" yielded important results about formulating robust policies to combat deflationary outcomes, but the assumed short decision horizon means that one cannot study the implications for current behavior of explicit commitment to future policies. An example of the literature is the commitment to low interest rates for a sustained period in the future suggested by Krugman (1998) and Eggertsson and Woodford (2003).

In this paper we replace Euler equation learning with the assumption that agents have infinite-horizon decision rules derived from intertemporal optimization under given paths of expectations of aggregate economic variables. This type of formulation has recently been emphasized by Preston (2005) and Preston (2006).[2] In general, the individual consumers need to forecast both future interest rates, inflation, income and taxes. As a benchmark, we assume in this paper that the consumers are fully Ricardian and incorporate the government's intertemporal budget constraint into their own lifetime budget constraint. This last assumption means that the consumption function depends on expected future real interest rates and incomes net of government spending. In this formulation the mix of government financing does not influence private consumption behavior.

The possibility of deflation traps under a forward-looking global Taylor rule, emerges as a serious concern. Although the targeted steady state is

[1]See Krugman (1998) for a seminal discussion and Adam and Billi (2007), Coenen, Orphanides, and Wieland (2004), and Eggertsson and Woodford (2003) and for representative recent analyses and further references.

[2]The formulation has earlier been used in Marcet and Sargent (1989) and Evans, Honkapohja, and Romer (1998). Other recent papers include Evans, Honkapohja, and Mitra (2007) and Eusepi and Preston (2007).

locally stable under learning, a large pessimistic shock to expectations can result, under learning, in a self-reinforcing deflationary process accompanied by declining and output. We reconsider the policies recommended in Evans, Guse, and Honkapohja (2008) to insulate the economy from this possibility.

One case is aggressive monetary easing in which the Taylor rule is overridden by dropping the interest rate to (very near) zero when expected inflation falls below a specified threshold. It turns out that this policy, although it does offer some protection, is not sufficient if the negative expectations shock is very large. However, if the preceding policy is augmented by adding aggressive fiscal easing when required to keep inflation at or above the threshold, we find that we are able to always eliminate the possibility of deflationary spirals and ensure global stability of the targeted steady state.

2 The Model

We start with the same economic framework as in Evans, Guse, and Honkapohja (2008). There is a continuum of household-firms, which produce a differentiated consumption good under monopolistic competition and price-adjustment costs. There is also a government which uses both monetary and fiscal policy and can issue public debt as described below.

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint:

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s} \left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1 \right) \quad (1)$$

$$st.\ c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,s} + \frac{P_{t,s}}{P_t} y_{t,s}, \quad (2)$$

where $c_{t,s}$ is the Dixit-Stiglitz consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, $b_{t,s}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t, $\Upsilon_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,s}$ is the price of consumption good s, $y_{t,s}$ is output of good s, P_t is the aggregate price level and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β. The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,s}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,s}}{P_{t-1,s}} - 1 \right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982). The household decision problem is also subject to the usual "no Ponzi game" condition.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^{\alpha}$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward-sloping demand curve given by

$$P_{t,s} = \left(\frac{y_{t,s}}{Y_t}\right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. Y_t is aggregate output, which is exogenous to the firm.

The government's flow budget constraint is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1} + \eta_t, \tag{5}$$

where η_t is a white noise shock and where $\beta^{-1} - 1 < \kappa < 1$. The restriction on κ means that fiscal policy is "passive" in the terminology of Leeper (1991), and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal. In a companion paper we investigate the implications of "active" fiscal policy in which $0 \leq \kappa < \beta^{-1} - 1$.

We assume that g_t is stochastic

$$g_t = \bar{g} + u_t, \tag{6}$$

where u_t is an observable stationary AR(1) mean zero shock. From market clearing we have

$$c_t + g_t = y_t \tag{7}$$

Monetary policy is assumed to follow a global interest rate rule

$$R_t - 1 = \theta_t f\left(\pi^e_{t+1}\right). \tag{8}$$

The function $f(\pi)$ is taken to be positive and non-decreasing, while θ_t is an exogenous, observable stationary AR(1) positive random shock with mean 1 representing random shifts in the behavior of the monetary policy-maker. The rule (8) is a nonlinear forward-looking Taylor rule, in which dependence on output expectations is suppressed for simplicity.[3] We assume the existence of π^*, R^* such that $R^* = \beta^{-1}\pi^*$ and $f(\pi^*) = R^* - 1$. π^* can be viewed as the inflation target of the Central Bank, and we will assume that $\pi^* \geq 1$. In the numerical analysis we will use the functional form

$$f(\pi) = (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^*-1)},$$

which implies the existence of a nonstochastic steady state at π^*. Note that $f'(\pi^*) = AR^*$, which we assume is bigger than β^{-1}. Equations (4), (5) and (8) constitute "normal policy".

2.1 Optimal decisions for private sector

As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$\begin{aligned} 0 &= -h^{\varepsilon}_{t,s} + \frac{\alpha\gamma}{\nu}(\pi_{t,s} - 1)\pi_{t,s}\frac{1}{h_{t,s}} \\ &\quad +\alpha\left(1 - \frac{1}{\nu}\right)Y_t^{1/\nu}\frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}}c_{t,s}^{-\sigma_1} + -\frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,s}}E_{t,s}(\pi_{t+1,s} - 1)\pi_{t+1,s}. \end{aligned}$$

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s}\left(\pi_{t+1}^{-1}c_{t+1,s}^{-\sigma_1}\right)$$

and

$$m_{t,s} = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_{t,s}^{-\sigma_1}}{E_t\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2}.$$

We now make use of the representative agent assumption. In the representative-agent economy all agents s have the same utility functions, initial money and

[3] The main results below would also hold in the case of a contemporaneous-data Taylor rule, which is used in Evans, Guse, and Honkapohja (2008).

debt holdings and prices. We assume also that they make the same forecasts $E_{t,s}c_{t+1,s}$ $E_{t,s}\pi_{t+1,s}$, $E_{t,s}\pi_{t+1}$, as well as forecasts of other variables that will become relevant below. Under these assumptions all agents make the same decisions at each point in time, so that $h_{t,s} = h_t$, $y_{t,s} = y_t$, $c_{t,s} = c_t$ and $\pi_{t,s} = \pi_t$, and they make the same forecasts $E_{t,s}\pi_{t+1,s} = E_{t,s}\pi_{t+1} = E_t\pi_{t+1}$ and $E_{t,s}c_{t+1,s} = E_{t,s}c_{t+1} = E_tc_{t+1}$. Imposing also the equilibrium condition $Y_t = y_t = h_t^\alpha$ we obtain the equations

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = h_t\left(h_t^\varepsilon - \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha-1}c_t^{-\sigma_1}\right) + \beta\frac{\alpha\gamma}{\nu}E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\right],$$

$$c_t^{-\sigma_1} = \beta R_t E_t\left(\pi_{t+1}^{-1}c_{t+1}^{-\sigma_1}\right),$$

$$m_t = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1 - R_t^{-1}\right)c_t^{-\sigma_1}}{E_t\pi_{t+1}^{\sigma_2 - 1}}\right)^{-1/\sigma_2},$$

In the analysis of this paper we will focus on "steady-state" learning and we can therefore proceed without the random shocks. We thus set $\theta_t = \eta_t = u_t = 0$. For convenience, we also make the assumptions $\sigma_1 = \sigma_2 = 1$, i.e. utility of consumption and of money is logarithmic, and that agents have point expectations, so that their decisions depend only on the mean of their subjective forecasts. This allows us the write the system as

$$m_t = \chi\beta(1 - R_t^{-1})^{-1}c_t, \tag{9}$$

$$c_t^{-1} = \beta r_{t+1}^e(c_{t+1}^e)^{-1}, \text{ where } r_{t+1}^e = R_t/\pi_{t+1}^e, \text{ and} \tag{10}$$

$$\frac{\alpha\gamma}{\nu}(\pi_t - 1)\pi_t = h_t\left(h_t^\varepsilon - \alpha\left(1 - \frac{1}{\nu}\right)h_t^{\alpha-1}c_t^{-1}\right) + \beta\frac{\alpha\gamma}{\nu}\left[\left(\pi_{t+1}^e - 1\right)\pi_{t+1}^e\right]. \tag{11}$$

We now proceed to rewrite the decision rules for c_t and π_t so that they depend on forecasts of key variables over the infinite horizon.

2.2 The infinite-horizon Phillips curve

We start with an infinite-horizon version of the Phillips curve (11). Let

$$Q_t = (\pi_t - 1)\pi_t. \tag{12}$$

The appropriate root for given Q is $\pi \geq \frac{1}{2}$ and so we need to impose $Q \geq -\frac{1}{4}$ to have a meaningful model. Making use of the aggregate relationships

$h_t = y_t^{1/\alpha}$ and $c_t = y_t - g_t$ we can rewrite (11) as

$$Q_t = \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} y_t^{\alpha} (y_t - g_t)^{-1} + \beta Q_{t+1}^e.$$

Solving this forward we obtain

$$Q_t = \frac{\nu}{\gamma} \sum_{j=0}^{\infty} \alpha^{-1} \beta^j \left(y_{t+j}^e \right)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \sum_{j=0}^{\infty} \beta^j \left(\frac{y_{t+j}^e}{x_{t+j}^e} \right). \tag{13}$$

Here x_{t+j}^e denotes expected net output, which equals expectations of $y_{t+j} - g_{t+j}$. The expectations are formed at time t and it is assumed variables at time t are assumed to be in the information set of the agents.

We will treat (13), together with (12), as the temporary equilibrium equations that determine π_t given expectations $\{y_{t+j}^e, x_{t+j}^e\}_{j=1}^{\infty}$. A key implicit assumption that we are making is that the pricing decisions of firms depend simply on the trajectory of expected future aggregate gross and net output. That is, these variables are adequate sufficient statistics for the optimal pricing decisions of a representative firm. This will be true under rational expectations, but we assume that (13) also describes the decisions of the representative firm for given expectations that will be generated according to an adaptive learning rule.

2.3 The consumption function

To derive the consumption function from (10) we use the flow budget constraint and the NPG (no Ponzi game) to obtain an intertemporal budget constraint. Write

$$b_t = r_t b_{t-1} + \Phi_t,$$

where $r_t = R_{t-1}/\pi_t$ and

$$\Phi_t = y_t + m_{t-1}\pi_t^{-1} - c_t - m_t - \Upsilon_t. \tag{14}$$

Note that we assume $(P_{jt}/P_t) y_{jt} = y_t$, i.e. the representative agent assumption is being invoked. Iterating (14) forward and imposing

$$\lim_{j\to\infty} (D_{t,t+j}^e)^{-1} b_{t+j} = 0, \tag{15}$$

we obtain the life-time budget constraint of the household

$$0 = r_t b_{t-1} + \Phi_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Phi^e_{t+j}, \tag{16}$$

where

$$D^e_{t,t+j} = \prod_{i=1}^{j} r^e_{t+i},$$

where $r^e_{t+j} = R_{t+j-1}/\pi^e_{t+j}$ and

$$\Phi^e_{t+j} = y^e_{t+j} + m^e_{t+j-1}(\pi^e_{t+j})^{-1} - c^e_{t+j} - m^e_{t+j} - \Upsilon^e_{t+j}. \tag{17}$$

Here all expectations are formed in period t, which is indicated in the notation for $D^e_{t,t+j}$ but is omitted from the other expectational variables.

The consumption Euler equation (10) implies that

$$c^e_{t+j} = c_t \beta^j D^e_{t,t+j}.$$

Substituting this expression for c^e_{t+j} in (17) for Φ^e_{t+j} that $c^e_{t+j} = c_t \beta^j D^e_{t,t+j}$ it follows that

$$0 = r_t b_{t-1} - \sum_{j=0}^{\infty} c_t \beta^j + \phi_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \phi^e_{t+j}, \tag{18}$$

where

$$\begin{aligned} \phi_t &= y_t + m_{t-1}\pi_t^{-1} - m_t - \Upsilon_t, \\ \phi^e_{t+j} &= y^e_{t+j} + m^e_{t+j-1}(\pi^e_{t+j})^{-1} - m^e_{t+j} - \Upsilon^e_{t+j}. \end{aligned}$$

A crucial issue is how households form expectations of future taxes. In this paper we make the strong Ricardian equivalence assumption that households understand that the government's intertemporal budget constraint will be satisfied. First we write down the latter constraint. From (4) we have

$$\begin{aligned} b_t + m_t + \Upsilon_t &= g_t + m_{t-1}\pi_t^{-1} + r_t b_{t-1} \text{ or} \\ b_t &= \Delta_t + r_t b_{t-1} \text{ where} \\ \Delta_t &= g_t - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}. \end{aligned}$$

By forward substitution, and assuming $\lim_{T\to\infty} D_{t,t+T} b_{t+T} = 0$ we have

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=0}^{\infty} D_{t,t+j}^{-1} \Delta_{t+j}. \tag{19}$$

Note that Δ_{t+j} is the primary government deficit in $t+j$, measured as government purchases less lump-sum taxes and less seigniorage. Under the Ricardian Equivalence assumption, we assume that agents at each time t expect this constraint to be satisfied, i.e.

$$\begin{aligned} 0 &= r_t b_{t-1} + \Delta_t + \sum_{j=0}^{\infty} (D_{t,t+j}^e)^{-1} \Delta_{t+j}^e, \text{ where} \\ \Delta_{t+j}^e &= g_{t+j}^e - \Upsilon_{t+j}^e - m_{t+j}^e + m_{t+j-1}^e (\pi_{t+j}^e)^{-1} \text{ for } j = 1, 2, 3, \ldots. \end{aligned}$$

Substituting out $r_t b_{t-1}$ from (18) and rearranging we get

$$(1-\beta)^{-1} c_t = (\phi_t - \Delta_t) + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} \left(\phi_{t+j}^e - \Delta_{t+j}^e \right),$$

or

$$c_t = (1-\beta) \left(y_t - g_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} x_{t+j}^e \right). \tag{20}$$

We view (20) as the temporary equilibrium equation that, under Ricardian Equivalence, determines consumption, given expectations. As for the inflation equation we assume that households form $\{x_{t+j}^e\}_{j=1}^{\infty}$ using an adaptive learning rule that treats these aggregates as an exogenously given process. For the consumption function we need also to specify how private agents form the subjective discount factors $D_{t,t+j}^e = \prod_{i=1}^{j} r_{t+i}^e$. Various assumptions are natural, but we will focus on the assumption that r_{t+i}^e is obtained from separate forecasts of inflation and interest rates, making use of the monetary policy rule to forecast the latter.[4] In this case, combining $r_{t+j}^e(t) = R_{t+j-1}^e / \pi_{t+j}^e$ and $R_t = 1 + f\left(\pi_{t+1}^e\right)$ we have

$$D_{t,t+j}^e = \prod_{i=1}^{j} (1 + f(\pi_{t+j}^e)) / \pi_{t+j}^e. \tag{21}$$

[4] Thus, monetary policy is both transparent and credible in that agents incorporate the interest rate rule in their expectations formation for all future periods.

We remark that our consumption function (20) exhibits Ricardian Equivalence in the following sense:

Proposition 1 *Household consumption depends on the sequence of expected government spending but not in any way on how it is financed.*

This temporary equilibrium result for arbitrary subjective expectations generalizes the results of Wallace (1981) and Eggertsson and Woodford (2003), which presume that the RE hypothesis holds. The assumption of Ricardian consumers has, in particular, the implication that an open market operation altering the initial composition of wealth between money, and bonds has no effect on consumption, given subsequent interest rate policy and the sequence of government spending. In addition, the standard result about the neutrality of changes in lump-sum taxes holds in our setting.

3 Learning and Stability

We now look at the system under learning and examine the stability of the steady states of the model. Throughout this section we assume that government spending is fixed, i.e., $g_t = \bar{g}$ and that agent know this and set $x^e_{t+j} = y^e_{t+j} - \bar{g}$. For any steady state π, equation (10) implies that the nominal interest rate factor satisfies the Fisher equation

$$R = \beta^{-1}\pi. \tag{22}$$

As emphasized by Benhabib, Schmitt-Grohe, and Uribe (2001), because $f(.)$ is nonnegative, continuous (and differentiable) and has a steady state π^* with $f'(\pi^*) > \beta^{-1}$, there must be a second steady state $\pi_L < \pi^*$ with $f'(\pi_L) < \beta^{-1}$. For our parametrization of $f(\cdot)$, there are no steady states other than the intended steady state π^* and the unintended low-inflation steady state π_L. We remark that it follows from Evans, Guse, and Honkapohja (2008) that π^* is locally determinate and π_L is locally indeterminate under rational expectations.

The other steady state equations are given by

$$c = h^\alpha - \bar{g}, \tag{23}$$

$$-h^{1+\varepsilon} + \frac{\alpha\gamma}{\nu}(1-\beta)(\pi - 1)\pi + \alpha\left(1 - \frac{1}{\nu}\right)h^\alpha c^{-\sigma_1} = 0 \tag{24}$$

and a steady-state version of (9). For a given steady state $\pi \geq 1$, it is shown in the Appendix of Evans, Guse, and Honkapohja (2008) that there is a corresponding unique interior steady state $c > 0$ and $h > 0$.

3.1 Temporary equilibrium

Collecting the preceding, the following equations define the temporary equilibrium under normal policy:

1) Phillips curve

$$\begin{aligned} Q_t &= \frac{\nu}{\gamma}\sum_{j=0}^{\infty}\alpha^{-1}\beta^j\left(y_{t+j}^e\right)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma}\sum_{j=0}^{\infty}\beta^j\left(\frac{y_{t+j}^e}{y_{t+j}^e-\bar{g}}\right), \\ Q_t &= (\pi_t - 1)\pi_t. \end{aligned} \tag{25}$$

2) Consumption function

$$\begin{aligned} c_t &= (1-\beta)\left(y_t - g_t + \sum_{j=1}^{\infty}(D_{t,t+j}^e)^{-1}(y_{t+j}^e - \bar{g})\right), \\ D_{t,t+j}^e &= \prod_{i=1}^{j}(1+f(\pi_{t+j}^e))/\pi_{t+j}^e. \end{aligned}$$

3) Money demand

$$m_t = \chi\beta(1-R_t^{-1})^{-1}c_t.$$

4) Government budget constraint

$$b_t + m_t + \kappa_0 + \kappa b_{t-1} = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}.$$

5) The interest-rate rule

$$R_t - 1 = f\left(\pi_{t+1}^e\right),$$

where

$$f(\pi) = (R^* - 1)\left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^*-1)}.$$

6) Market clearing

$$y_t = c_t + g_t,$$

where we have assumed that $g_t = \bar{g}$.

Given expectations $\{y^e_{t+j}, \pi^e_{t+j}\}_{j=1}^{\infty}$, the above six equations define the temporary equilibrium in $c_t, \pi_t, y_t, R_t, m_t, b_t$. The model dynamics are then completed by specifying how expectations evolve over time. Here the idea of adaptive learning is employed in place of the hypothesis that RE prevails in all periods. It is assumed that private agents make forecasts using a reduced form econometric model of the relevant variables and that the parameters of this model are estimated using past data. The forecasts are input to agent's decision rules and in each period the economy attains a temporary equilibrium, i.e., an equilibrium for the current period variables given the forecasts of the agents. See e.g. Sargent (1993) and Evans and Honkapohja (2001) for general discussions of adaptive learning.

The temporary equilibrium provides a new data point, which in the next period leads to re-estimation of the parameters and updating of the forecasts and, in turn, to a new temporary equilibrium. The sequence of temporary equilibria may generate parameter estimates that converge to a fixed point corresponding to a rational expectations equilibrium for the economy. When the convergence takes place, we say that the RE equilibrium is stable under learning.

A simple set-up to investigate analytically is steady state learning with point expectations. Under steady state learning we assume that

$$y^e_{t+j} = y^e_t \text{ and } \pi^e_{t+j} = \pi^e_t \text{ for all } j \geq 1.$$

and that

$$z^e_t = z^e_{t-1} + \omega_t(z_{t-1} - z^e_{t-1}) \tag{26}$$

for $z = y, \pi$. Here $\omega_t = t^{-1}$ under "decreasing gain" learning for $\omega_t = \omega$, for $0 < \omega \leq 1$ for "constant gain" learning. In the latter case we usually assume that ω is sufficiently small. We now examine the stability of the steady states under these simple learning rules.[5]

The dynamics of learning can be conveniently described by using the close connection between the possible convergence of least squares learning to an RE equilibrium and a stability condition, known as E-stability. E-stability of an equilibrium is based on a mapping from the perceived law of motion

[5]It would be straightforward to analyze learning in the setting with random shocks, which were included in the initial discussion above. With *iid* shocks the simple formulation just given continues to hold. If the shocks were autocorrelated, agents would naturally forecast using regressions with observable shocks as explanatory variables. See Evans, Guse, and Honkapohja (2008) for more discussion.

that private agents are estimating and using to make forecasts to the implied actual law of motion generating the data (i.e. the temporary equilibrium) under these perceptions. E-stability is defined in terms of local stability, at an RE equilibrium, of a differential equation based on this map. For a general discussion of adaptive learning and the E-stability principle see Evans and Honkapohja (2001).

Before turning to the E-stability results, we briefly discuss the issue of the transversality conditions in our temporary equilibrium set-up. Under steady-state learning, $\pi^e_{t+j} = \pi^e_t$ for all $j \geq 1$ implies $D^e_{t,t+j} = ((1+f(\pi^e_t))/\pi^e_t)^j = (r^e_t)^j$, where $r^e_t = (1+f(\pi^e_t))/\pi^e_t$ is the expected real interest factor, and the consumption function takes the form

$$c_t = (1-\beta)\left(y_t - g_t + \frac{1}{r^e_t - 1}(y^e_t - \bar{g})\right).$$

provided $r^e_t > 1$.[6] The consumption function gives the time t choice of consumption based on information and forecasts at time t, and can be viewed as the first step of an infinite-horizon dynamic plan. From the consumption Euler equation it follows that the expected path of future consumption (with $\sigma_1 = 1$) is given by

$$c^{-1}_{t+j} = (r^e_t)^{-j}\beta^j c^{-1}_t, \text{ for } j = 1, 2, 3, \ldots,$$

where here c^{-1}_{t+j} is the expected marginal utility of money at $t+j$. The relevant transversality condition for the household is that

$$\lim_{j\to\infty} c^{-1}_{t+j}\beta^j b_{t+j} = 0 \tag{27}$$

holds along the planned path of consumption and bonds. Because the consumption function is derived using the intertemporal budget constraint obtained using the NPG condition, we know that the condition

$$\lim_{j\to\infty}(D^e_{t,t+j})^{-1}b_{t+j} = \lim_{j\to\infty}(r^e_t)^{-j}b_{t+j} = 0$$

is satisfied. Since, using the consumption Euler equation, we have $c^{-1}_{t+j}\beta^j b_{t+j} = (r^e_t)^{-j}c^{-1}_t b_{t+j}$, it follows that (27) is satisfied along the planned path.[7] Thus, at each point in time, the transversality condition is met for the households' planned path of consumption and wealth.

[6] See the discussion below for our treatment of the case $r^e_t \leq 1$.

[7] Using the money demand equation it follows that $\lim_{j\to\infty} m^{-1}_{t+j}\beta^j b_{t+j} = 0$ also holds along the planned path.

3.2 E-Stability

The theoretical results for learning below are based on E-stability analysis of the system under the learning rules (26). An equilibrium is said to be stable (or unstable) under learning if it is E-stable. In fact, convergence of least-squares learning to the equilibrium point assumes that the learning rules employ a decreasing gain. Under constant gain, when an equilibrium is E-stable there is local convergence of learning in a weaker sense to a random variable that is centered near the equilibrium.[8]

We now proceed to the analysis of E-stability of the two possible steady states when the global interest rate rule (8) describes monetary policy. We have

$$D^e_{t,t+j} = \prod_{i=1}^{j} (1 + f(\pi^e_t))/\pi^e_t, \tag{28}$$

so that

$$\begin{aligned} y_t &= \bar{g} + (\beta^{-1} - 1)(y^e_t - \bar{g}) \left(\frac{\pi^e_t}{1 + f(\pi^e_t) - \pi^e_t} \right) \\ &\equiv G_1(y^e_t, \pi^e_t). \end{aligned} \tag{29}$$

Temporary equilibrium is given by equations (29) and

$$\pi_t = Q^{-1}[K(G_1(y^e_t, \pi^e_t), y^e_t)] \equiv G_2(y^e_t, \pi^e_t),$$

where

$$Q(\pi_t) \equiv (\pi_t - 1)\, \pi_t \tag{30}$$

$$\begin{aligned} K(y_t, y^e_t) &\equiv \frac{\nu}{\gamma} \left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right) \frac{y_t}{(y_t - \bar{g})} \right) \\ &+ \frac{\nu}{\gamma} \left(\beta(1-\beta)^{-1} \left(\alpha^{-1} (y^e_t)^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right) \frac{y^e_t}{(y^e_t - \bar{g})} \right) \right). \end{aligned} \tag{31}$$

The E-stability equations are[9]

$$\begin{aligned} \frac{dy^e}{d\tau} &= G_1(y^e, \pi^e) - y^e \\ \frac{d\pi^e}{d\tau} &= G_2(y^e, \pi^e) - \pi^e. \end{aligned} \tag{32}$$

[8] See Section 7.4 of Evans and Honkapohja (2001).

[9] Here τ denotes notional time that is related to the discrete real time at specific points of the latter $t_n = \sum_{i=1}^{n} \omega_i$.

By construction, the steady state are fixed points of this system of differential equations. It should be noted that the differential equations operate in notional or virtual time. It can be shown that for large values of the (discrete) real time t, the continuous time paths $(y^e(\tau), \pi^e(\tau))$ of (32) are approximately related to the discrete-time trajectories (y_t^e, π_t^e) of (26) at specific points of real time: $(y^e(t_n), \pi^e(t_n)) \approx (y_n^e, \pi_n^e)$ for $t_n = \sum_{i=1}^{n} \omega_i$.

To examine local stability of a steady state, we need to calculate the Jacobian

$$DG = \begin{pmatrix} D_{y^e}G_1 - 1 & D_{\pi^e}G_1 \\ D_{y^e}G_2 & D_{\pi^e}G_2 - 1 \end{pmatrix}.$$

We start with function G_2. Taking differentials

$$\begin{aligned} D_{y^e}G_2 &= (Q^{-1})'(K_y D_{y^e}G_1 + K_{y^e}) > 0 \\ D_{\pi^e}G_2 &= (Q^{-1})' K_y D_{\pi^e}G_1. \end{aligned}$$

We need to calculate the various derivatives at a steady state:

$$(Q^{-1})' = \frac{1}{2\pi - 1} > 0,$$

$$K_y = \frac{\nu}{\gamma}\left((1+\varepsilon)y^{\frac{1+\varepsilon+\alpha}{\alpha}} + (1-\nu^{-1})\frac{g}{(y-g)^2}\right) > 0,$$

$$K_{y^e} = \frac{\nu}{\gamma}\frac{\beta}{1-\beta}\left((1+\varepsilon)y^{\frac{1+\varepsilon+\alpha}{\alpha}} + \left(1-\nu^{-1}\right)\frac{g}{(y-g)^2}\right) > 0.$$

We also need to compute the following partial derivatives at a steady state

$$\begin{aligned} D_{y^e}G_1 &= (\beta^{-1} - 1)\left(\frac{\bar{\pi}}{1 + f(\bar{\pi}) - \bar{\pi}}\right) = 1, \\ D_{\pi^e}G_1 &= (\beta^{-1} - 1)(\bar{y} - g)\left(\frac{1 + f(\bar{\pi}) - \bar{\pi}f'(\pi^e)}{(1 + f(\bar{\pi}) - \bar{\pi})^2}\right). \end{aligned}$$

Here $1 + f(\bar{\pi}) - \bar{\pi}f'(\bar{\pi}) = (\beta^{-1} - f'(\bar{\pi}))\bar{\pi}$, which is negative at π^* and positive at π_L. Thus,

$$D_{\pi^e}G_1 < 0 \text{ at } \pi^* \text{ and } > 0 \text{ at } \pi_L.$$

For the sign of $D_{\pi^e}G_2$ we have

$$sgn[D_{\pi^e}G_2] = sgn[D_{\pi^e}G_1].$$

Thus, the Jacobian at the normal steady state π^* is

$$DG = \begin{pmatrix} 0 & - \\ + & - \end{pmatrix},$$

implying E-stability of π^*. At the low inflation steady state π_L the Jacobian is

$$DG = \begin{pmatrix} 0 & + \\ + & ? \end{pmatrix}.$$

The $(2,2)$ element is $D_{\pi^e}G_2 - 1$ and for sufficiently small γ $D_{\pi^e}G_2$ becomes large (see the expression for K_y), so the element is positive for small γ and we have E-instability of π_L.

Collecting the results we can state:

Proposition 2 *The model with normal policy has two steady state states π^* and π_L. Under infinite-horizon decision rules with steady-state learning the targeted steady state π^* is locally stable under learning. For γ sufficiently small the low-inflation steady state is locally unstable taking the form of a saddle point.*

For global results we turn to numerical analysis. There is one technical issue that we need to take care of in connection with steady state learning by households. With arbitrary value of inflation expectations, there are regions of the space of expectations in which the expected real interest rate and thus $1 + f(\pi_t^e) - \pi_t^e$ can be negative. This would imply infinite consumption in the preceding formula for the consumption function. To avoid this difficulty we truncate the steady state expectations of the household at some long but finite horizon T and postulate that beyond the horizon agents just assume that real rate of interest has reached its steady state value β^{-1}. With this assumption the consumption function becomes

$$c_t = (1-\beta)\left[y_t - \bar{g} + (y_t^e - \bar{g})\left[\frac{\pi_t^e(1 - (\frac{\pi_t^e}{1+f(\pi_t^e)})^T)}{1 + f(\pi_t^e) - \pi_t^e} + \frac{\beta^T}{\beta^{-1} - 1}\right]\right]$$

and so

$$y_t = \bar{g} + (\beta^{-1} - 1)(y_t^e - \bar{g})\left[\frac{\pi_t^e(1 - (\frac{\pi_t^e}{1+f(\pi_t^e)})^T)}{1 + f(\pi_t^e) - \pi_t^e} + \frac{\beta^T}{\beta^{-1} - 1}\right].$$

In the global analysis we also need to make sure that $\pi \geq 1/2$. This is achieved in the numerics by setting $\pi = 1/2$ if the other temporary equilibrium equations would imply $Q < -\frac{1}{4}$.

Figure 1 illustrates the theoretical results in Proposition 2. We use the parameter values $A = 2.5$, $\pi^* = 1.02$, $\beta = 0.99$, $\alpha = 0.75$, $\beta = 20$, $\nu = 1.5$, $\varepsilon = 1$, $R^* = \pi^*/\beta$, $\bar{g} = 0.1$ and $T = 50$. The figure shows the phase diagram of the system (32) for the evolution of expectations under learning. Given expectations dynamics, it is easy to compute the trajectories of actual inflation and output.

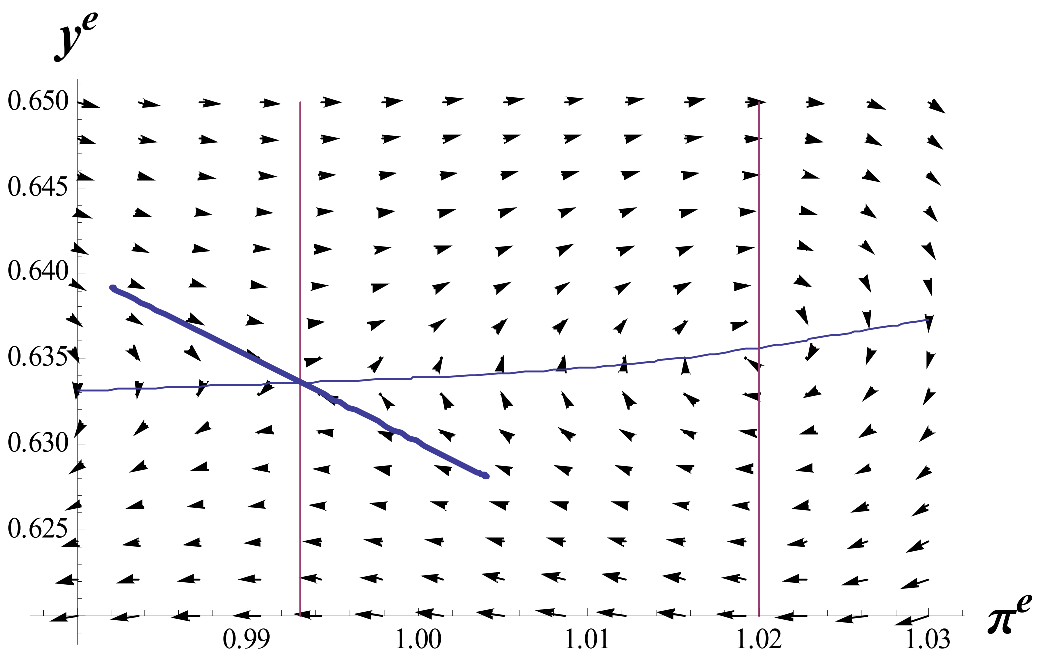

Figure 1: E-stability dynamics under global Taylor rule

Figure 1 shows the global E-stability dynamics that provides an approximation to the real-time dynamics of learning. Examining the aggregate demand equation (29), it is seen that the locus consisting of the two vertical lines gives values for (y^e, π^e) at which $\frac{dy^e}{d\tau} = 0$, while the upward-sloping curve gives values for (y^e, π^e) at which $\frac{d\pi^e}{d\tau} = 0$. The targeted steady state at $\pi^* = 1.02$ is locally stable under E-stability dynamics and convergence toward it is cyclical. The low steady state $\pi_L = 0.993092$, $y_L = 0.633614$ is a saddle point and, most importantly, there is a region of initial expectations implying unstable trajectories with falling inflation expectations and eventually falling output expectations. The same holds true for actual inflation and output. We call these paths deflationary spirals and we call this region the deflationary trap. The downward sloping line through the low steady state gives the local linear approximation of the stable manifold separating the basin of attraction of the targeted steady state from the unstable region.

Figure 1 shows that the problem of deflationary traps for sufficiently pessimistic expectations that was discovered in Evans, Guse, and Honkapohja (2008) for Euler-equation learning, continues to arise under infinite-horizon learning, in which consumption, output and inflation are determined as the first-period decisions of the solution to the infinite horizon optimization problem under subjective expectations based on our learning rule. The intuition for the unstable trajectories is that sufficiently pessimistic expectations π_t^e, y_t^e lead to high expected real interest rates, because of the zero lower bound on net nominal interest rates. These, together with the low expected incomes, leads through the consumption function to low consumption and hence low output, and through the infinite-horizon Phillips curve, to lower inflation. The learning rule can then lead to a downward revision of expectations over time, pushing the economy further along an unstable trajectory. Of course along an unstable path one would expect either private agents or policymakers eventually to alter their actions, but our results nonetheless indicate the potential for major disruptions to the economy resulting from large negative shocks to expectations. We now turn to possible policy changes that can avoid these undesirable outcomes.

4 Alternative Monetary and Fiscal Policies

4.1 Monetary Policy Committing to Low Interest Rates

In earlier work with Eran Guse published as Evans, Guse, and Honkapohja (2008) we considered the implications of aggressive monetary easing triggered by inflation rates below some threshold $\tilde{\pi}$, where $\pi_L < \tilde{\pi} < \pi^*$. That paper studied Euler-equation learning in which agents have short horizons, and and it was found that this type of policy did not provide a fool-proof way to avoid deflationary spirals. In the current framework agents have long-horizons in their decision-making, so that there appears to be more scope for aggressive monetary policy to eliminate these unstable trajectories. Further-more, in models with rational expectations commitment to long periods of low interest rates has been advocated as a way to avoid the consequences of liquidity traps, see e.g. Krugman (1998), Eggertsson and Woodford (2003), and Svensson (2003).

We now modify the interest rate rule to include aggressive monetary easing if gets too low. We formalize this idea by introducing a lower threshold for inflation, so that the interest rate R_t is cut to a low level $\hat{R}$ very close to one. To maintain continuity of the interest rate rule, we introduce two threshold values $\pi_L < \tilde{\pi}_1 < \tilde{\pi}_2 < \pi^*$ with $\tilde{\pi}_1 \approx \tilde{\pi}_2$ and

$$\tilde{f}(\pi^e) = R - 1 \begin{cases} f(\pi^e) \text{ if } \pi^e > \tilde{\pi}_2 \\ \hat{R} + (\pi^e - \tilde{\pi}_1)\frac{f(\tilde{\pi}_2)-\hat{R}}{\tilde{\pi}_2-\tilde{\pi}_1} \text{ if } \tilde{\pi}_1 \leq \pi^e \leq \tilde{\pi}_2 \\ \hat{R} \text{ if } \pi^e < \tilde{\pi}_1 \end{cases} \tag{33}$$

so that $f(\pi^e)$ in the earlier rule (8) is replaced by $\tilde{f}(\pi^e)$.

Figure 2 illustrates the expectation dynamics when aggressive monetary easing. In the numerics we use a continuous interest rate rule, so that between $\tilde{\pi}_1 = 1.009$ and $\tilde{\pi}_2 = 1.01$ interest rate is adjusted linearly down to $R = 1.001 \equiv \hat{R}$. The other parameter values are unchanged. It is evident that the possibility of deflationary spirals remains. However, the policy does help a little bit because it shifts the unstable region south-west, as is evident from comparing Figures 1 and 2. We remark that the constrained low steady state values in Figure 2 are $\pi = 0.990099$, $y = 0.596218$, which are lower than the values of the low-inflation steady state in Figure 1.

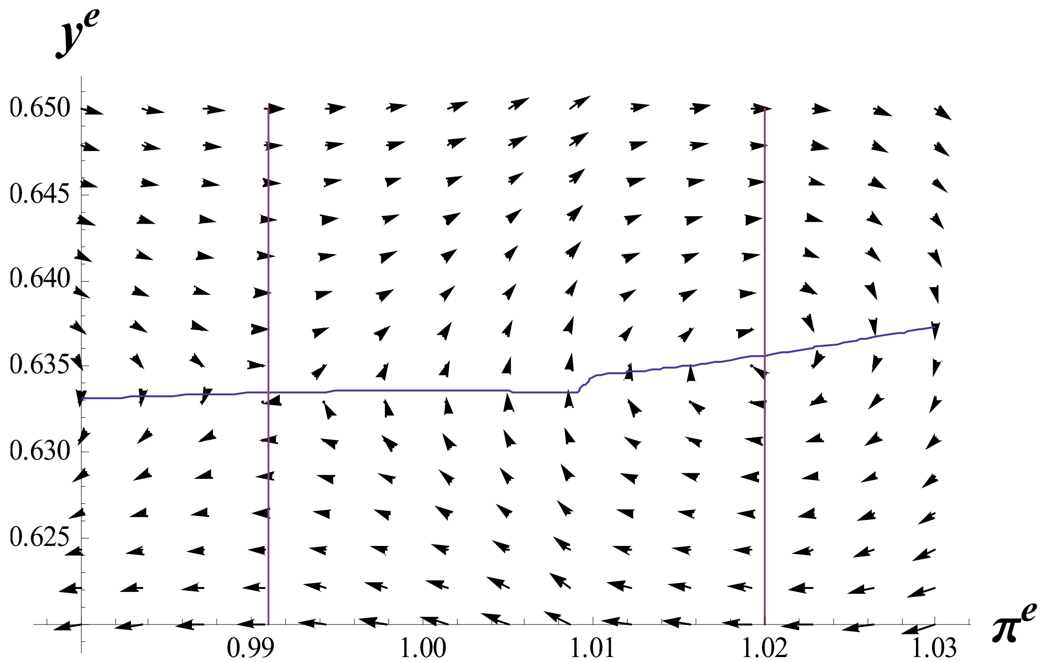

Figure 2: Global expectations dynamics with aggressive monetary easing

In fact, even if policy makers respond to low inflation by committing to the low interest rate policy forever, the possibility of deflation traps remains. This is illustrated in Figure 3.

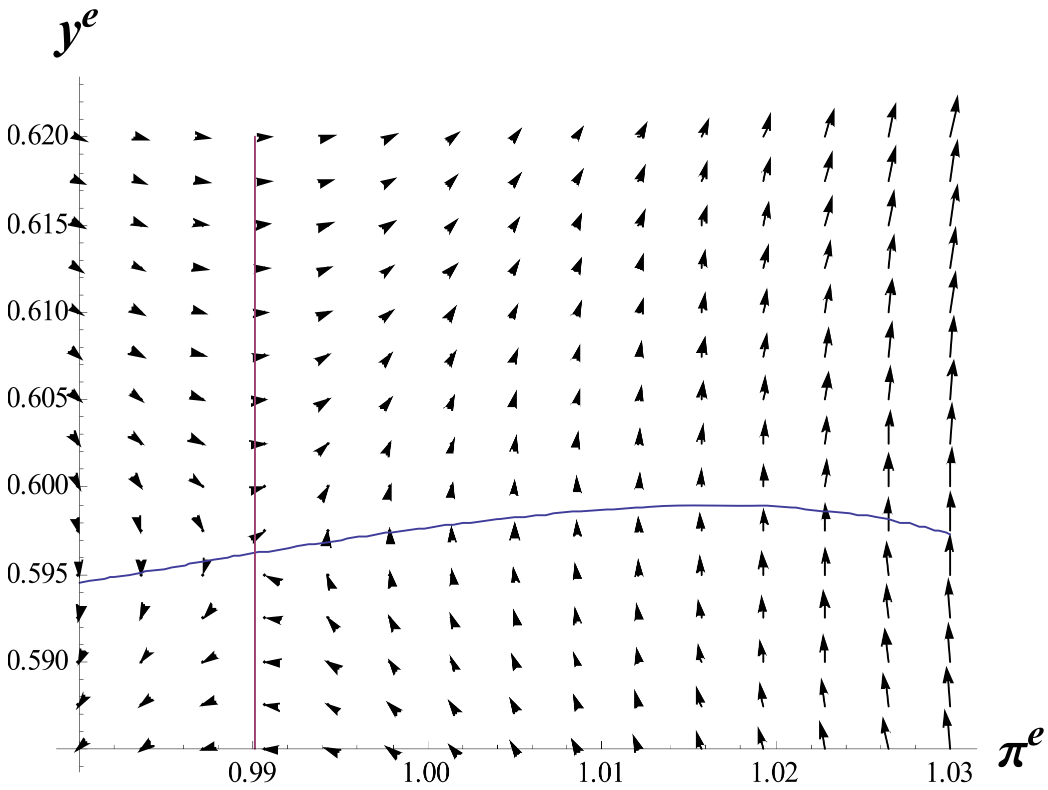

Figure 2: Dynamics with aggressive monetary easing forever

For sufficiently pessimistic expectations the region of deflationary spirals remains. This policy reduces the deflationary region somewhat but at the great cost of converting the previous region of stability into a regime in which inflation would increase without bounds.

4.2 Combined Monetary and Fiscal Easing

We now add aggressive fiscal policy to the preceding monetary easing policy, following the ideas of Evans, Guse, and Honkapohja (2008). The key idea is to temporarily increase government spending to ensure that inflation never falls below a low threshold. With changes in government spending agents now have to forecast both gross and net output, which implies that the expectation dynamics become three-dimensional and phase diagrams cannot be conveniently used to illustrate the dynamics. Instead we will plot some selected time paths of central variables. The formal changes to the model are as follows.

First, we assume that expectations of net output are determined by steady-state learning as we earlier did for output and inflation. Thus, in addition to (26) we have the expectation dynamics for x_t^e as

$$x_t^e = x_{t-1}^e + \phi_t(x_{t-1} - x_{t-1}^e).$$

The temporary equilibrium equations are now given by the following. For gross output we have[10]

$$y_t = g_t + (\beta^{-1} - 1)x_t^e \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1}, \tag{34}$$

$$D_{t,t+j}^e = [(1 + \tilde{f}(\pi_t^e))/\pi_t^e]^j. \tag{35}$$

Net output is given by

$$x_t = y_t - g_t. \tag{36}$$

Evidently, for given expectations net output is independent of g_t, so that in temporary equilibrium the government spending multiplier is one. Inflation is determined by

$$Q(\pi_t) \equiv (\pi_t - 1)\,\pi_t \tag{37}$$

$$Q(\pi_t) \equiv \frac{\nu}{\gamma}\left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right)\frac{y_t}{x_t}\right) + \frac{\nu}{\gamma}\left(\beta(1-\beta)^{-1}\left(\alpha^{-1}(y_t^e)^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right)\frac{y_t^e}{x_t^e}\right)\right). \tag{38}$$

These equations are a generalization of (30)-(31).[11]

The policy of fiscal easing is begun triggered by actual inflation threatening to fall below the threshold $\tilde{\pi}_1$ specified in the modification to the interest rate rule in equation (33) in the preceding section. Specifically we assume that if we would have $\pi_t < \tilde{\pi}_1$ at $g_t = \bar{g}$ then government spending is increased to whatever level is needed to ensure $\pi_t = \tilde{\pi}_1$. This is feasible because of the following Lemma:

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[10] It should be noted that this equation holds only if $(1 + \tilde{f}(\pi_t^e))/\pi_t^e > 1$ and this issue was dealt with by the truncation of the consumption function in the numerical analysis as explained earlier.

[11] As mentioned earlier, these equations hold provided that $Q(\pi_t) > -\frac{1}{4}$ and in this case π_t is taken as the upper root of the quadratic. For $Q(\pi_t) \le -\frac{1}{4}$ we set $\pi_t = \frac{1}{2}$.

Lemma 3 *For given expectations π_t^e, y_t^e, x_t^e,*

$$\frac{d\pi_t}{dg_t} \geq k$$

for some $k > 0$ and g_t sufficiently large.

Proof. As net output is constant, we have $\frac{dy_t}{dg_t} = 1$. Then, it is seen from (37)-(38) that $\frac{\partial Q}{\partial y_t}$ is bounded above zero for y_t sufficiently large and so the same holds for $\frac{\partial \pi_t}{\partial y_t}$. ■

The Lemma implies that under our policy of combined fiscal and monetary easing triggered by the inflation threshold, inflation will never fall below $\tilde{\pi}_1$. This implies the following global uniqueness result:

Proposition 4 *Consider the temporary equilibrium system (34), (35), (36), (37) and (38) with fiscal easing triggered by the threshold $\tilde{\pi}_1$. There is a unique steady state with inflation at π^* and a corresponding value for output. At the steady state $g_t = \bar{g}$.*

Proof. From (34)-(35) in a steady state we obtain the Fisher equation $R = \beta^{-1}\pi$. The interest rate rule provides a second steady state relationship $R = 1+\tilde{f}(\pi)$. These equations have a unique solution at π^*under the specified policy since the policy implies the restriction $\pi \geq \tilde{\pi}_1$. ■

Local stability under learning follows from Proposition 2. Our numerical results indicate that the steady is globally stable under learning. We illustrate this in Figures 4 and 5. Figure 4 shows the time paths of inflation, output and net output expectations from a starting point $\pi^e = 0.995$, $y^e = 0.62$ and $x^e = 0.52$, which are picked from the deflationary region in Figure 2. Figure 5 shows the corresponding dynamics of actual inflation, output and net output. We remark that the time variable plotted here is notional time τ corresponding the the E-stability differential equation. The link to real time t depends on the “gain” ω of the learning rule according to $\tau = \omega t$. Thus if $\omega = 0.10$ per quarter then $\tau = 2$ corresponds to $t = 20$ quarters.

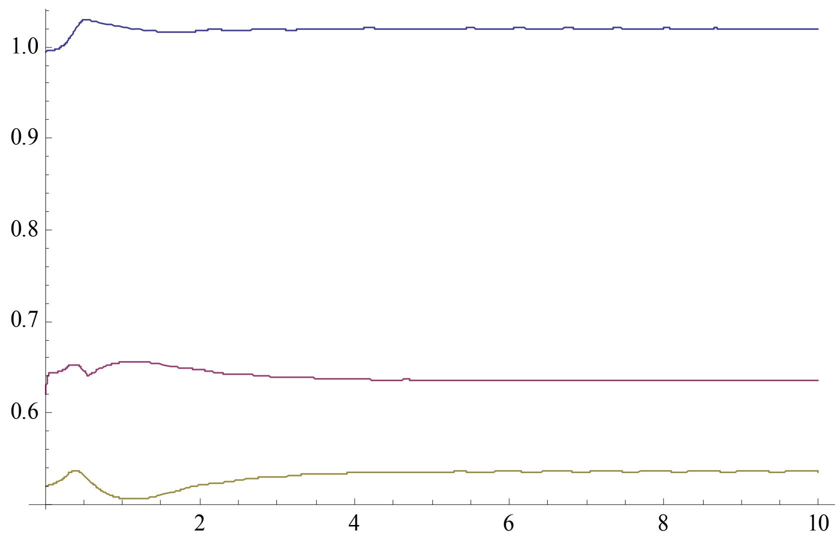

Figure 4: Inflation, output, and net output expectations over time

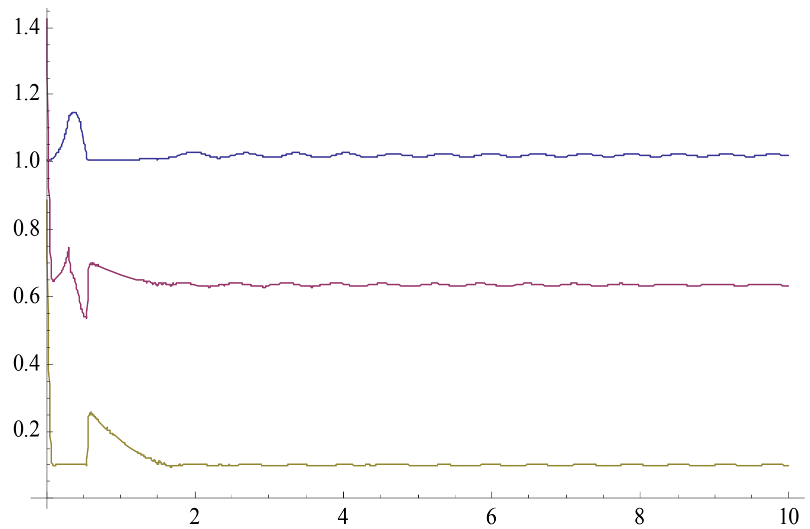

Figure 5: Time paths of actual inflation, ouput, and net output

It evident that there is convergence to the unique steady state and this result appears to be robust numerically. Thus, this policy appears to provide a robust way to avoid a liquidity trap and the associated deflationary dynamics that arise with learning under the basic interest rate policy. The mechanism is that by stabilizing prices through expansionary government spending, low nominal interest rates yield low expected real interest rates, which leads to a recovery of private spending.

While our recommended policy does successfully insulate the economy from the deflation trap, the resulting path is cyclical and exhibits overshooting of the inflation target after the economy is pushed out of the deflationary region. There are big fluctuations in inflation, output and government spending in the initial stages of the dynamics, a feature that was not seen in the short-horizon learning examined in Evans, Guse, and Honkapohja (2008). These numerical results raise the question of whether alternative versions of our combined policy of monetary and fiscal easing can insulate the economy from deflation traps with smaller fluctuations in output and inflation. In Evans, Guse, and Honkapohja (2008) interest rates responded to current rather than expected inflation, and it is possible this would improve performance under infinite-horizon learning. Other possibilities to examine include interest-rate rules that additionally depend on actual or expected output (or net output), fiscal responses that are smoother and that respond countercyclically to high expected output and inflation, and explicit commitments to temporary increases in government spending with a suitable time profile.

5 Conclusions

When monetary policy is conducted using a standard Taylor rule, the intended steady state is locally stable under learning. However the economy is not globally stable under learning, and this remains true even if agents make decisions based on infinite horizon optimization problems. A large exogenous negative shock to expectations can lead to a deflation trap in which expected deflation and low output is reinforced under learning and the economy fails to return to the intended equilibrium. Deflation traps can be avoided by a policy of aggressive monetary and fiscal easing if inflation falls below a suitable threshold, such as zero net inflation. These policies are effective even though households are assumed to make consumption decisions using a perceived life-time budget constraint that incorporates Ricardian equivalence. Although our suggested policy successfully insulates the economy against deflation traps, in some cases there are substantial fluctuations in output and inflation along the transition back to the intended steady state.

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