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Abstract

This paper explains the basic mechanisms of Norges Bank's core model for monetary policy analysis and forecasting (NEMO). NEMO has recently been extended with an oil sector to incorporate important channels of shocks to the Norwegian economy. We show how the effects of a change in the oil price depends on whether the price change is due to demand or supply factors in the international economy. Other extensions of the model include a more detailed modeling of the foreign sector. The paper also uses NEMO to highlight important driving forces of the Norwegian economy after the fall in the oil price. We demonstrate that the model has a reasonable empirical fit compared to VAR models.

Keywords: *Monetary policy; DSGE*

JEL classification: *E12; E52; G01*

*This paper should not be reported as representing the views of Norges Bank. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank. We thank Tord Krogh and Yasin Mimir for useful comments.

Contents

1 Introduction

Since 2006 Norges Bank has maintained and revised an open-economy New Keynesian model for monetary policy analysis and forecasting (NEMO). The overriding goal of the model has been to help policymakers conduct monetary policy. The model is used in several ways: a) to identify the main shocks that are driving the economy, b) to assess the appropriate reaction to shocks to achieve the objectives of monetary policy, and c) to help the central bank to communicate forecasts and policy analysis in a consistent and coherent way. To be useful, the model thus needs to incorporate the main shocks that the economy is exposed to and the most important transmission channels of monetary policy.

The first version of NEMO was documented in Brubakk *et al.* (2006), see also Brubakk and Sveen (2009). The first main extension to NEMO was made in 2013 when it was deemed necessary to introduce an explicit treatment of the credit market and a role for housing services and house prices (Brubakk and Gelain (2014)). The global financial crisis showed that the financial sector can be an important source of shocks and that the financial sector can reinforce or weaken the transmission mechanism of monetary policy. To further facilitate discussions of financial stability concerns in monetary policy within the context of NEMO and improve the empirical fit of the model, a modification to the credit and housing market was made in 2015. Inspired by the work of Gelain *et al.* (2015), the standard assumptions about one-period debt contracts and rational expectations regarding house prices were replaced by long-term debt contracts and simple moving average forecast rules. As a result, the model's ability to produce long cycles in house prices and credit as observed in the data improved greatly.

As an oil-producing country, the Norwegian economy has since 2014 been exposed to a considerable negative shock to oil prices that have worked through channels that were not well captured by previous versions of the model. This has motivated the introduction of an oil sector in the model. Domestic, mainland supply chains deliver goods to off-shore oil producing firms and to rig producers both domestically and abroad. For example, almost 25 percent of all traditional exports from Norway[1] consist of deliveries to the oil sector abroad. According to a report from Statistics Norway (Hungnes *et al.* (2016)), oil activities' share of total employment was about 9 percent in 2013 based on input-output calculations and this employment was mainly in the supply chain.

The introduction of an oil sector in NEMO is mainly based on Bergholt *et al.* (2017). Bjørnland and Thorsrud (2016) has been used to calibrate relevant model parameters. NEMO has been extended to accommodate a set of stylized facts. First, the effects of a change in the oil price depend on whether the price change is due to supply or demand factors in the international economy. The effects of a demand-driven change in the oil price, where we observe a concomitant rise in the oil price and economic activity among Norway's trading partners, are stronger than the effects of a supply-driven change. In the latter case, an increase in the oil price has a contractionary effect on economic activity among trading partners. Second, firms in the oil supply sector are exposed to different shocks than other mainland firms. While oil supply firms are negatively hit by a decline in oil prices due to lower demand from the oil industry both domestically and abroad, other mainland firms often expand their business due to a weaker exchange rate. Thus, while exports from the oil supply sector are reduced, other traditional exports are stimulated.

The paper is organized as follows. Section 2 describes the role of NEMO in Norges Bank's modeling system. Section 3 presents the main structure of the current version of

[1]Exports excluding petroleum.

NEMO. Section 4 presents data and model fit by comparing out-of-sample forecasts from NEMO with those of a weighted average of VAR models. Furthermore, we show some impulse respons functions to highlight the transmission channels of important shocks before we use the model to obtain a more structural interpretation of recent economic developments through a shock decomposition. A summary of the paper is provided in Section 5.

2 Norges Bank's modeling system

In this section we briefly present the modeling system at Norges Bank.[2] NEMO will be described in more detail in the next section.

For Norges Bank, as a flexible inflation-targeting central bank, forecasting is an important ingredient in the monetary policy process. Norges Bank employs a wide range of models depending on the question at hand. Models with good empirical fit and out-of-sample performance are of primary interest for nowcasting and forecasting at a short horizon. For medium- to long-term analysis, models with theoretical consistency and models that are calibrated/estimated to include the likely interaction between monetary policy and the economy play a more prominent role.

The forecasting and policy analysis system is organized around NEMO (see Figure 1). First, nowcasts and short-run forecasts are produced by SAM ("System for averaging models"; see Aastveit *et al.* (2011)) and a broad suite of empirical (reduced-form) models (VARs, one-equation regression models, factors models etc.) that are estimated to provide forecasts for important macro variables. Norges Bank's own regional network provides important inputs into this part of the forecasting process. Projections of economic variables like public consumption and investment, economic developments abroad and the oil price over the whole forecasting horizon, are also determined at this stage. Some of these series, like foreign interest rates and the oil price, are market-based. Projections of fiscal policy are largely technical in nature and based on annual budget of the central government. Moreover, models for recession probabilities (see Aastveit *et al.* (2016)) are used in the comprehensive process of forming a view on economic developments.

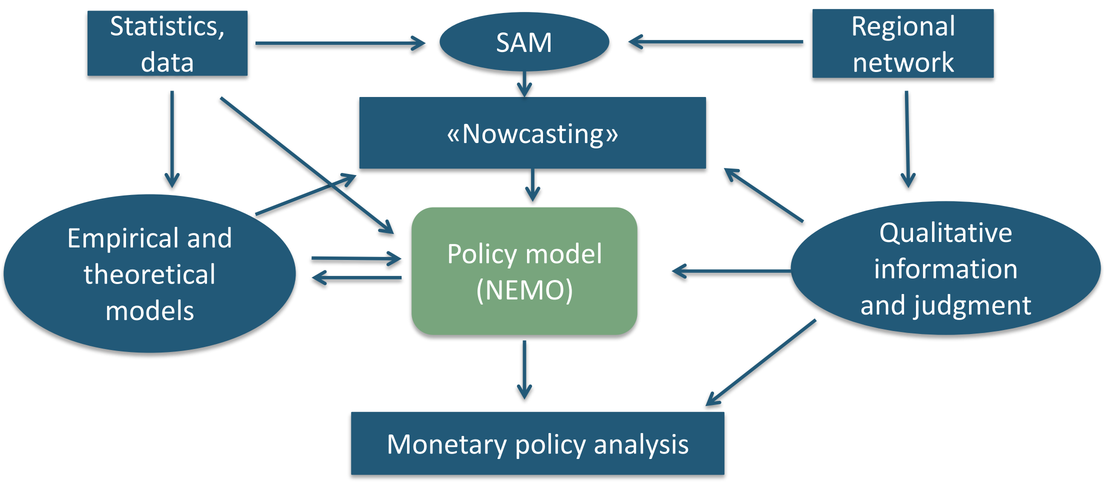

Figure 1: *The forecasting and policy analysis system at Norges Bank.*

[2]See Gerdrup and Nicolaisen (2011) for a more thorough description.

In NEMO, nowcasts and short-run forecasts as well as the trajectory of variables that are taken as "exogenous" to the model, are used as conditioning information. Technically, NEMO is first used to interpret historical developments (including the short-run forecasts) in terms of shocks that have hit the economy and brought it away from a steady-state path (see Appendix D for a technical explanation). The role of monetary policy is then to bring the target variables back on track given the path of exogenous variables. Since 2005 Norges Bank has published the interest rate path that aims to strike a balance between the different objectives of monetary policy. The uncertainty in the forecasts of the main policy variables[3] are published as fan charts. These are in turn based on NEMO's density forecasts.

Judgment is important at all stages of the forecasting process as the forecasting process is largely iterative. As mentioned by Blanchard (2016), all (DSGE) models are flawed, but they can fulfill an important role in offering a core structure around which to build and organize discussions. At Norges Bank, sector experts monitor a large amount of data, including more qualitative information, in conjunction with formal models to form a deeper understanding of the current economic environment. Model experts develop over time a good understanding on the strengths and weaknesses of a policy model like NEMO, and can thus give advice on when one should depart from the model's main predictions or how to modify the model to bring it more in line with the current understanding of economic mechanisms. Smaller, theory-based DSGE models complement NEMO when the purpose is to highlight specific policy questions outside the realm of NEMO. Furthermore, Norges Bank is currently working on developing models that depart from the rational expectations framework of DSGE models as well as models based on micro data (like agent-based models and heterogeneous agent new keynesian models[4]).

3 The model

This section provides a non-technical exposition of the key relationships and workings of NEMO. The presentation below is based on the model used for policy analysis as of September 2017. A technical documentation of all derivations, first-order conditions, the full steady-state solution and the parametrization of the model can be found in Kravik and Paulsen (2017).[5]

NEMO consists of households, domestic (traditional) firms, an oil sector, a government sector and the monetary authority. In addition, there are separate production sectors for housing and non-housing capital goods as well as a banking sector. All agents have rational, or model-consistent, expectations with respect to all prices and quantities, with households' house price expectations being an important exception.

Figure 2 provides a schematic illustration of the model and displays how the different sectors and agents are linked to each other. The numeraire good of the model, *the final good*, is shown next to the top in the figure. This is produced by combining inputs from the domestic firms (Q), labeled intermediate goods producers in the figure, and imports (M). The final goods are converted into household consumption (C), investment (I_C), housing investment (I_H), government expenditures (G) and used as inputs in the oil sector

[3]The main policy variables are: key policy rate, output gap, consumer price inflation, and consumer prices adjusted for tax changes and excluding temporary changes in energy prices.

[4]See Violante *et al.* (2015) for an example of the latter.

[5]Available at http://www.norges-bank.no/en/Monetary-policy/Models-for-monetary-policy-analysis-and-forecasting/NEMO.

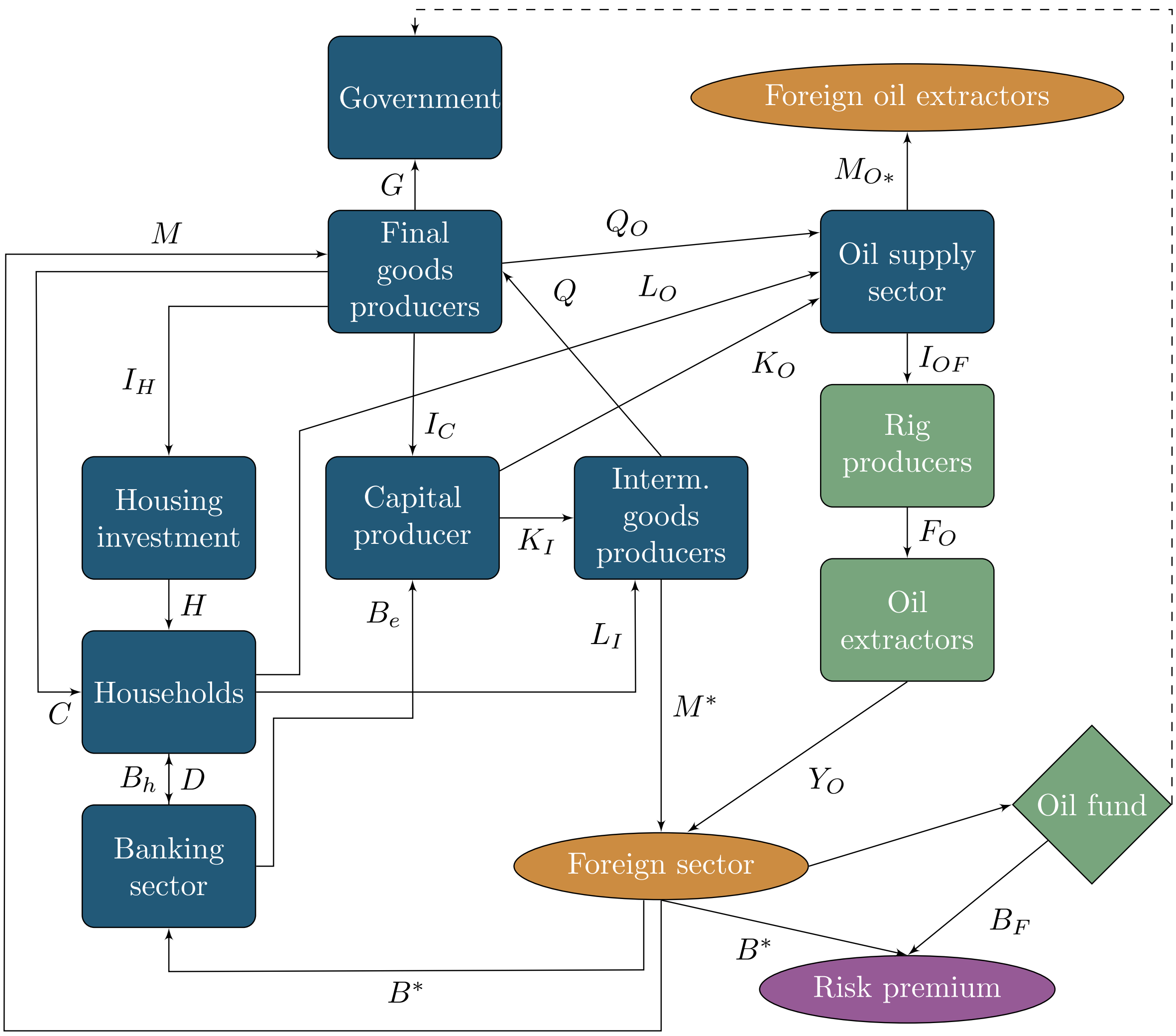

Figure 2: *A schematic illustration of NEMO.*

(Q_O). The intermediate goods producers employ labor (supplied by households, L_I), rent capital (from the capital producers, K_I), and sell their goods to the final goods producers (Q) and as export (M^*). The oil sector uses labor (L_O), capital (K_O) and final goods (Q_O) to produce oil supply goods which are exported (M_{O*}) or sold to the domestic rig producers (I_{OF}). The rig producers invest in oil rigs (F_O) in order to extract oil (Y_O) that in turn is exported in full. The revenues are invested in the Government Pension Fund Global (GPFG). Every period a certain share of the GPFG is transferred lump-sum to the government.

A simplified banking sector lends to households (B_h) and capital producers (B_e), and is funded through equity, deposits (D), and foreign borrowing (B^*). An uncovered interest parity relationship (UIP) together with the country's net foreign assets position (private borrowing, B^*, minus claims on foreigners, B_F) tie down the risk premium to make the real exchange rate return to a steady-state level over time.[6]

Turning to the details of the model, it is convenient to start with households. A continuum of similar households maximize their expected lifetime utility subject to their budget

[6]This is one of the standard ways of closing small open economy models, see Schmitt-Grohe and Uribe (2003).

constraints and have preferences over consumption, deposits, housing and leisure. Total household income comprises labor income and interest income on deposits.[7] Consumers obtain direct utility from deposits to reflect the empirical observation that households are both gross savers and gross lenders at the same time, and is introduced as a short-cut instead of modeling different groups of households (savers and spenders). New loans are constrained by a collateral constraint that depends on the (expected) value of the housing stock and mortgages (i.e. loan-to-value ratio). As house prices increase, this constraint – assumed to always be binding – induces households to borrow more, thus creating a demand for mortgages, which is spent on consumption goods, housing and deposits. When taking up new loans, households enter into long-term loan contracts (à la Kydland *et al.* (2016)) which gives rise to installment schedules found in data, including the realistic feature that the amortization rate is low during the early years of a mortgage. In NEMO, the parameters regarding loan installments are calibrated to match the amortization schedule of a 30-year conventional mortgage.

The labor market is characterized by monopolistic competition. Households supply labor and set wages subject to demand from the intermediate goods sector and the oil supply sector. Real wages are set as a mark-up over the marginal rate of substitution of consumption for leisure. The size of the mark-up depends on workers' market power. As there is assumed to be full labor mobility between the two sectors, there is only one wage level in the economy. Changing (nominal) wage levels is assumed to be costly for workers (see Rotemberg (1982)), giving rise to sluggish wage adjustments.

Agents in NEMO have model consistent expectations. Laborers decide on wages and labor supply not only based on today's prices and demand curves, but also based on all future expected wage curves. The same is true for all agents of the model, regarding all prices. A noteworthy exception is house price expectations. House price expectations are modeled as a weighted sum of model consistent expectations and a moving average process (i.e. partly backward-looking expectations). This generates house price cycles more in line with empirical observations. The housing stock is produced from a separate sector using the final good as input.

On the supply side, a continuum of firms in the intermediate goods sector use capital and labor to produce a differentiated intermediate good which is sold under monopolistic competition to the final goods producers at home and abroad (as exports). Firms choose labor and capital services to minimize factor outlays, taking wages and rental rates of capital as given. Capital is produced by a separate capital producer that uses final goods to invest in new capital, which is then rented out to the intermediate goods producers and the oil sector. Due to perfect capital mobility, the capital rental rate is equated across sectors. The capital producer relies on external funding from the banking sector, and enters into contracts with a duration of about 5-6 years.

Intermediate goods firms have market power and set prices as a mark-up over marginal costs. They charge different prices at home and abroad (local currency pricing). Changing (nominal) prices is however assumed to be costly, which lead firms to change their prices less in response to shocks than they otherwise would have done, i.e. prices are sticky. These assumption is what induces the non-neutrality of monetary policy. Prices are also influenced by expectations. Since changing prices is costly, firms must look into the future when deciding on today's prices. Hence, inflation expectations influence today's inflation.

Banks' deposit and lending rates are also set in a monopolistically competitive fashion,

[7] Households also own the firms and receive all dividends, but this has no effects on marginal decisions in the model.

subject to adjustment costs. The deposit rate is set as a mark-down on the money market interest rate and, conversely, the lending rate is set as a mark-up on the money market rate. The adjustments costs entail sticky nominal interest rates. The banking sector thus becomes a separate source of shocks to the economy and will affect the transmission mechanism of monetary policy.

When making decisions regarding lending and deposits, the banks must adhere to regulatory capital requirements and also take into account that the return on lending is uncertain. Failing to reach the capital requirements will trigger a cost for the banks proportional to their total assets. The combination of capital requirements and uncertain return on lending in the model cause the banks to aim for a cushion above the required capital requirement, and create a potential feedback loop between the real and the financial side of the economy. For example, an economic downturn could possibly hit bank profits and reduce the bank capital ratio, with banks cutting back on lending as a consequence. This would in turn give a further negative impetus to the real economy.

3.1 Recent extensions: Foreign trade and the oil sector

Norway has benefited significantly from its considerable petroleum activity. In spite of its relatively low employment share of merely 2 percent of total employment,[8] oil and gas production have traditionally made up about 45 to 50 percent of total export value and between 20 to 30 percent of total GDP since 2000 (Statistics Norway, 2016). To take into account the significance of the oil sector for the Norwegian economy, an explicit oil sector has been added to NEMO. In addition, the linkages between the foreign sector, the oil price and the domestic economy have been improved.

As the final goods sector combines intermediate goods with imports, imports will constitute a share of domestic consumption, capital investments, housing investments, oil supply inputs and government consumption. Prices for Norway's import goods are set by foreign exporters which enjoy market power, but face price adjustment costs.

The foreign sector is modeled as a block-exogenous set of equations, linking foreign inflation, foreign output, foreign interest rates and the oil price. Moreover, the foreign sector is divided into two groups: Trading partners and non-trading partners. The former is a group of Norway's largest export and import partners; the latter is the foreign sector minus trading partners. The model for the foreign sector is based on a standard New Keynesian model, with a dynamic IS curve representing the relationship between output and the real interest rate, and a Phillips curve linking inflation to output. The oil price has been added to the system of equations to negatively affect trading partners' output and positively affect their rate of inflation. A rise in global production will increase international oil prices.

The oil sector in the model builds on Bergholt *et al.* (2017). The sector consists of supply firms as well as a domestic and a foreign extraction firm. The supply firms combine labor, capital and final goods to produce oil supply goods that are: *a*) used for oil investment by the domestic extraction firm, and *b*) exported to a foreign oil extraction firm. The domestic oil supply firms sell their goods under monopolistic competition, subject to adjustment costs, and charge different prices at home and abroad (local currency pricing).

The representative domestic oil extraction firm undertakes two activities: a) it invests in rigs, using solely oil supply goods as inputs, b) it extracts and exports oil, using rigs and oil in the ground as inputs. If the extraction firm wants to increase production, it can

[8] Up to 9 percent when including all petroleum-*related* employment, as mentioned in Section 1.

increase rig investments. However, changing the investment level is costly and production capacity takes time to build (one period). Alternatively, the extraction firm can expand oil extraction in the current period by increasing existing rig utilization. However, this comes at a cost, as the extraction firms needs to buy more goods from the supply firms to increase the usage of oil rigs. Evidently, there is a trade-off between raising the utilization rate to increase oil production instantaneously, or to invest in rigs that will increase production in future periods.

In Norway, the Government Pension Fund Act stipulates the transfer of the net cash flow from the petroleum industry to the GPFG. A fiscal rule specifies that the transfers from the GPFG to the central government budget shall, over time, follow the expected real return on the GPFG. In NEMO, this relationship is simplified, as the GPFG and fiscal policy are independently treated. The surplus in the oil sector is being taxed at 100% and transferred to the GPFG, but government expenditures are exogenously determined.[9]

The foreign sector and the explicit treatment of the petroleum sector provide a more realistic representation of the division between mainland Norway and the petroleum sector. Hence, NEMO makes it possible to identify how different external shocks affect the home economy in contrasting ways. A rise in international oil prices serves as a good example. In the model, an increase in oil prices makes extraction firms increase oil extraction and their investment in rigs. Naturally, this leads to an increased demand for goods from the supply firms, which boosts the production and raises the demand for labor, capital and final goods. This in turn, other things equal, causes a hike in wages and price inflation as well as higher GDP. How the rest of the economy is affected by the oil price change depends on the nature of the shock that triggered the price hike in the first place. If the oil price increase is due to a negative oil supply shock in an oil-producing country, it has a negative impact on global production (as foreign marginal costs are increased), which in turn weakens exports from the domestic non-oil sector, dampening the positive effect on GDP. On the other hand, higher oil prices caused by increased global demand is likely to amplify the positive effects on the domestic economy.

3.2 Monetary policy

The central bank controls the key policy rate, assumed equal to the money market interest rate (up to a money market risk premium). It sets the interest rate to minimize a loss function ("optimal" monetary policy). The loss function consists of the discounted (weighted) sum of future expected quadratic deviations from steady-state values of inflation, output, the level of the key policy rate and changes in the key policy rate.

Agents in NEMO do not only respond to current nominal interest rates, but also to the entire (expected) future path of *real* interest rates. Nominal interest rate changes affect real interest rates because prices and wages are not fully flexible in the short run. Hence, it is the sluggish price setting assumption that causes monetary policy to have real effects in the model.

The transmission mechanism of the policy rate works through several channels, broadly categorized by the exchange rate, aggregate demand and inflation expectations. First, as the UIP relation must hold in every period, an increase in the interest rate necessitates an immediate strengthening of the domestic currency (or, rather, expectations of a future depreciation), which contributes to a lower imported inflation, lower exports and higher

[9]Although the relationship between fiscal policy and the GPFG is not explicitly modeled in NEMO, the fiscal rule may have a role when the model is used for policy analysis.

imports.

Second, a higher interest rate increases funding costs for banks, driving them to gradually raise lending and deposit rates to maintain mark-ups (-downs). The higher deposit rate induces households to increase saving and reduce consumption. A higher lending rate also raises the required rate of return on capital, in effect making traditional firms, the oil supply sector and the housing producer reduce investment. This entails a lower demand for labor, and lower wage growth. Marginal costs decrease, which over time will be transmitted to domestic prices.

Third, expectations influence how monetary policy works in the model. For instance, if agents see that the monetary authority has a strong preference for interest rate smoothing, a low policy rate in the current period immediately generates expectations of a low interest rate in future periods. As agents' decisions are based on the entire future path of interest rates, a low interest rate has a stronger effect when interest smoothing is high.

4 An analysis of the Norwegian economy

4.1 Data

The dataset in NEMO is quarterly and runs from 1994 Q1. The macroeconomic time series cover Norwegian and international variables: Real domestic variables include GDP, consumption, credit, exports/imports, government expenditures, investments and hours worked. Price variables include wages, consumer prices, house prices, lending rates to households, lending rates to firms, money market interest rates and the key policy rate. Lastly, international variables include the exchange rate, the oil price and foreign GDP, money market interest rates and inflation. Data sources are Statistics Norway, Norges Bank's own calculations, and international sources, particularly the IMF and Thomson Reuters.

The current version of NEMO requires observable variables to be in deviation from steady state (gaps). Trends (and noise) are therefore removed from the raw-data. In total, there are 26 observable variables in NEMO. The gap calculations are reported in Appendix A. Table 2 provides a short description of each variable, including the type of transformation, equilibrium value and its data source.[10]

The Hodrick-Prescott filter (HP filter) is used to estimate the trend for demand components, hours worked, house prices, credit, oil price and foreign GDP. Inflation is detrended with the inflation target of 2.5 percent. The trend in imported inflation is lower than for overall inflation, because of the continuous terms of trade gains from low import price inflation over the sample. These gains have been slightly smaller in the latter years and are expected to be so going forward. An upward trend shift in imported inflation is therefore assumed after 2012.

The trend growth rate of real wages is close to its historical average, with a downward shift in 2012 and 2013. These shifts are introduced due to lower growth rates of trend productivity in recent years (see the Special Feature on low productivity growth in Norges Bank's *Monetary Policy Report* (MPR) 2/16[11]) and a higher trend in import

[10] In Appendix A, 27 variables are graphed. However, as the risk premium is equal to the money market interest rates minus the key policy rate, only two of these need to be fed to the model, i.e. making it 26 observable variables.

[11] All Norges Bank's *Monetary Policy Reports* are available at http://www.norges-bank.no/en/Published/Publications/Monetary-Policy-Report-with-financial-stability-assessment.

price inflation.

The trend shifts in the money market interest rate are consistent with Norges Bank's published estimates of the neutral interest rate (see MPR 3/16, MPR 1/14, MPR 1/12 and MPR 1/10). Norges Bank's estimates are usually expressed as intervals. NEMO requires point estimates in order to compute an interest rate gap. The point estimate in NEMO has been in the lower end of the estimated interval for the neutral interest rate in recent years.

The neutral foreign money market rate is assumed to be 0.5 percentage point lower than in Norway, reflecting the differences in inflation target between Norway and our main trading partners. The neutral level in other interest rates are calculated based on estimates of various risk premiums.

In order to remove the noise component in the detrended series, a three-quarter central moving average procedure is used on some of the variables. Judgment may have been applied to get the gaps, e.g. due to endpoint problems in some of the filters.

4.2 Out-of-sample forecasting

To provide a rough indication of the forecasting abilities of NEMO, we compare unconditional forecasts from NEMO to forecasts from a pool of vector autoregressive (VAR) models, using the root-mean-square error (RMSE) to evaluate the forecasts.[12] In this simple forecast evaluation we concentrate on forecasts for core inflation, GDP mainland Norway and the key policy rate.

Ideally both estimation and calibration of the unobserved variables and model parameters should be based on real-time information in this exercise. Unfortunately, real-time estimation and calibration of the parameters in NEMO would be infeasible. We therefore use the same parameter set for all the recursive forecasts from NEMO. Estimates of trends and unobserved variables are, however, based on real-time data.[13] For the VAR-models, real-time estimates of the parameters are used. One could argue that this is an advantage for NEMO, given that the calibration of the latest version of NEMO is based on information over the entire sample period. On the other hand, NEMO is not calibrated/estimated based on maximizing the forecasting performance of a small set of variables; rather, it is a large system that is meant to give consistent forecasts for a large set of variables and to be useful for policy analysis. Both sets of models only provide forecasts of gap variables, so to be able to map the forecasts to the inflation rate, the GDP growth rate and the interest rate level, we also need to project the trends recursively. For inflation we have assumed a constant trend at 2.5 percent. For GDP growth, the starting point of the recursively projected trend is taken from the growth rate of the HP-filtered trend. It is then assumed that the trend gradually move towards the average growth rate from 1994 up until the date of the forecast according to an AR process with an autocorrelation coefficient of 0.9. Lastly, for the key policy rate we use the detrending displayed in figure 11 except that we first apply the shifts with a lag of one year.

NEMO is normally conditioned on nowcasts and short-term forecasts from a large set of additional data sources (such as SAM (see Section 2), sector experts, market information, Norges Bank's regional network etc.) This additional information is likely to improve the model's interpretation of the state of the economy (the set of shocks hitting

[12]The VAR models selection is presented in Appendix B.

[13]The only exception is that we do not use real time vintages for GDP and the demand components of GDP. This is the case for both the VAR-models and NEMO.

the economy), and thereby improve the medium term forecasts. Without this additional information, the shock identification becomes more uncertain and the unconditional forecasts from the model might deviate substantially from what would have been the case in a monetary policy report forecasting round. This was especially apparent during the financial crisis where additional market information was crucial in understanding economic developments. Hence, our results cannot be interpreted as a test of Norges Bank's full forecasting system.

Recursive forecasts from NEMO and the pool of VAR models are presented in Appendix C. From table 1 we see that NEMO has a slightly lower RMSEs for GDP and the key policy rate, while the VAR models have lower RMSEs for inflation, particularly for the longer forecast horizon. Overall, the out-of-sample forecasting performance of NEMO and the VARs seems to be comparable.

Table 1: *RMSEs for core inflation, GDP mainland Norway and the key policy rate forecast. 2005 Q2 - 2016 Q4.*

	Inflation		**GDP**		**Key policy rate**	
	NEMO	VARs	NEMO	VARs	NEMO	VARs
1	0.24	0.25	0.84	0.89	0.30	0.26
2	0.40	0.45	1.33	1.35	0.61	0.62
3	0.61	0.61	1.82	1.92	0.85	0.93
4	0.89	0.84	2.45	2.60	1.02	1.18
5	1.04	0.83	2.64	2.66	1.15	1.31
6	1.21	0.86	2.71	2.74	1.27	1.44
7	1.31	0.88	2.72	2.78	1.40	1.63
8	1.38	0.90	2.69	2.64	1.51	1.73

4.3 Impulse response functions

In this section we highlight some mechanisms in NEMO using impulse response functions. We show the effects of a shock to monetary policy, household preferences, risk premium in the foreign exchange market and the oil price (demand and supply separately).

Figure 3 shows the effects of a monetary policy shock. Since we model monetary policy as an optimal policy problem, the monetary policy shock is a temporary shock to the inflation target in the loss function. A negative shock to the inflation target leads to a positive inflation gap that the central bank aims to stabilize. The higher interest rate leads to a stronger NOK exchange rate and lower domestic demand. The decline in housing investment and household consumption are accentuated by a fall in house prices. House prices take a long time to adjust back to steady-state levels due to (partly) backward-looking expectations. The result is a quite persistent decline in housing investment and consumption. It takes around four to six years before these two demand components are back to their steady-state levels. Firm investment and exports also decline, but adjust faster as firms are more forward-looking and internalize that the interest rate will be reduced after around six quarters. The NOK exchange rate depreciates for the same reason, which has an expansionary effect on exports (not shown here). Overall, lower domestic demand contributes to lower wage growth and marginal costs in the intermediate sector. Inflation is reduced both as a result of the stronger exchange rate and lower demand.

In Figure 4 we assume that the exchange rate depreciates due to a positive risk premium shock in the foreign exchange market. Agents in the foreign exchange market prefer in this example to hold less NOK than before. The NOK exchange rate depreciates on impact, leading to higher imported inflation. The key policy rate is increased to lessen the depreciation of the NOK exchange rate and the rise in imported inflation and to reduce overall demand. Household consumption, firms investment and housing investment all decrease. The same is true for real house prices. However, exports rise as a result of the stronger NOK exchange rate. Lower domestic demand contributes to less hours worked and a lower real wage growth, and thus to lower domestic inflation. Due to the trade-off between output and inflation, monetary policy authorities do not set the interest rate so high as to fully stabilize inflation. Thus, the inflation rate is higher as a result of the risk premium shock.

The effect of a positive shock to household preferences for consumption is shown in Figure 5. Consumption increases as households value consumption today higher than before. This leads to a higher domestic demand and higher domestic inflation. The central bank increases the key policy rate to stabilize output and inflation. This leads to a change in the composition of demand. House prices, housing investment and business investment all fall. The NOK exchange rate appreciates on impact, but later depreciates due to expectations of a lower key policy rate and a worsening of the net foreign asset position, which in turn increases the endogenous exchange rate risk premium.

Figures 6 and 7 show the effects of a positive shock to global demand (the blue lines in the figures) and a positive shock to oil supply (yellow lines) that in both cases lead to an increase in the oil price of 10 percent. First, in the latter case, the positive oil supply shock leads to higher inflation and lower demand among Norway's trading partners. This leads to a lower demand for traditional export goods from Norway. The higher oil price boosts production by oil extractors abroad, which increases demand for exported goods from the oil supply chain in Norway. Domestic oil investment is also stimulated. Overall, a positive oil price shock has an expansionary effect on activity in Norway. Real wages and household consumption increase. The central bank increases the key policy rate somewhat. The NOK exchange rate appreciates both as a result of better terms of trade and as a result of a higher interest rate in Norway. Inflation is first reduced due to a stronger NOK exchange rate, but increases after around a year due to higher domestic demand.

The effects of a global demand shock are much stronger on the Norwegian economy. In this case, the oil price increases due to higher global activity, which also results in a higher activity among Norway's trading partners. This leads to a higher demand for traditional export goods from Norway in addition to a higher demand for goods from the oil supply chain. The key policy rate is increased much more than in the case of a positive oil price shock. A higher interest rate abroad also contributes to a higher key policy rate in Norway. This crowds out to some extent household consumption the first two years after the shock. House prices stay low for an extended period. The appreciation of the NOK exchange rate contributes to lower inflation in the first couple of quarters, but inflation eventually becomes higher as the effect of lower imported inflation is phased out and higher domestic inflation becomes dominant.

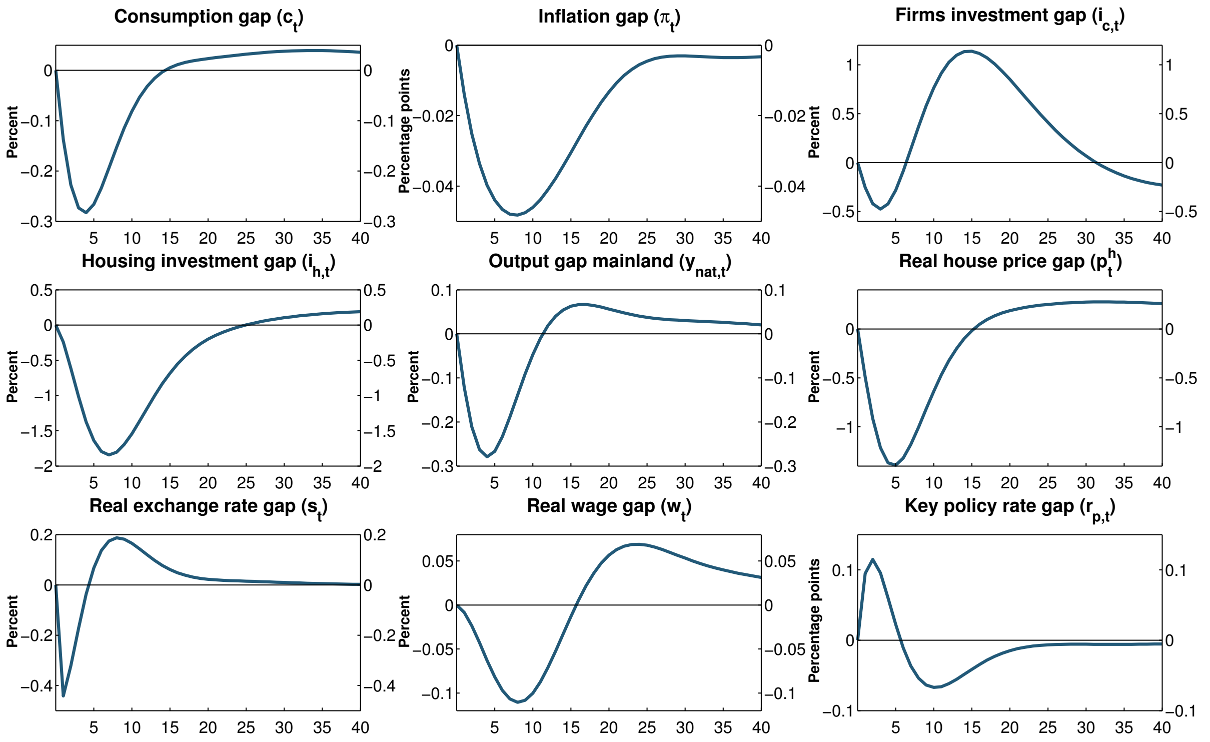

Figure 3: *Impulse response functions of a monetary policy shock equal to one standard deviation. Measured as deviations from steady-state. The key policy rate gap is not annualized.*

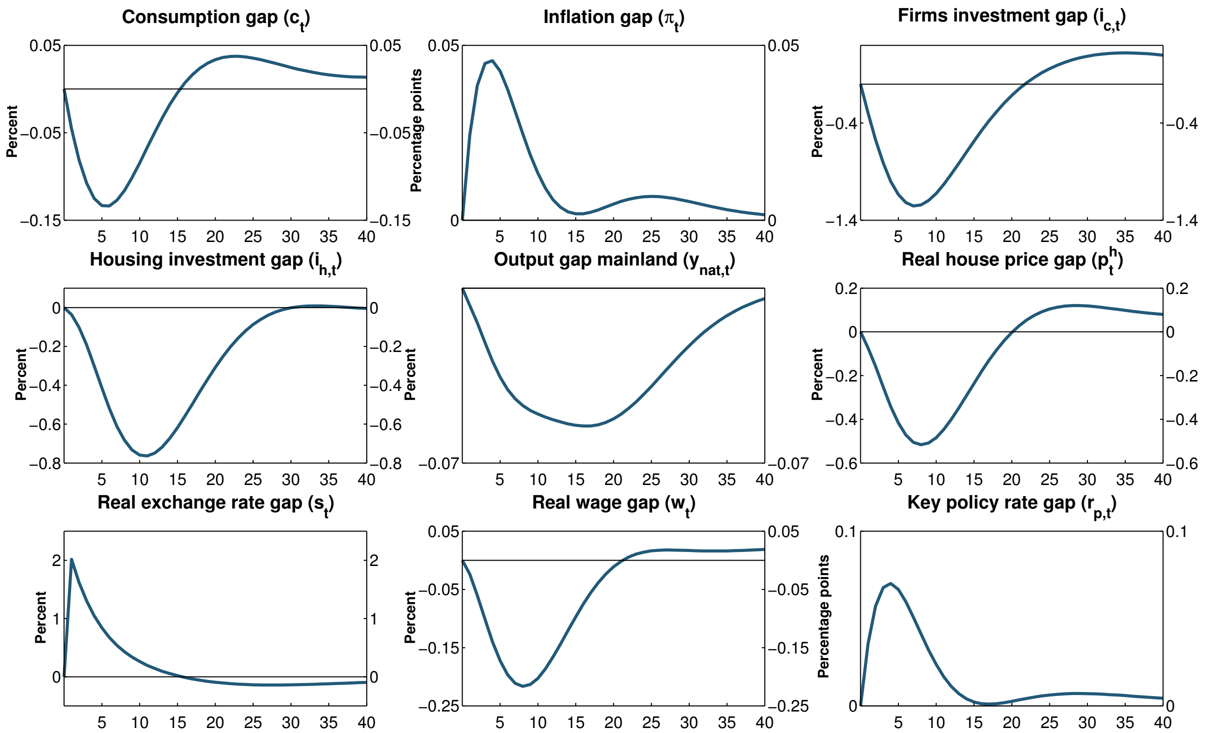

Figure 4: *Impulse response functions of a risk premium shock equal to one standard deviation. Measured as deviations from steady-state. The key policy rate gap is not annualized.*

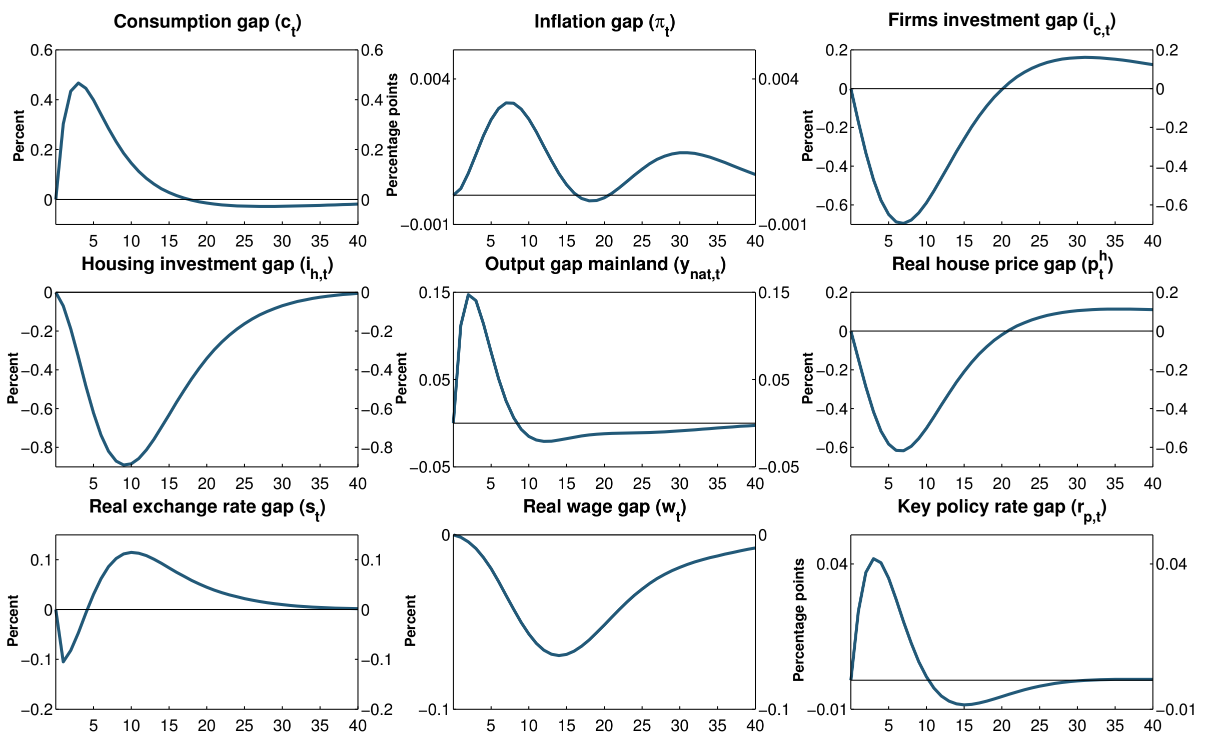

Figure 5: *Impulse response functions of a shock to household preferences equal to one standard deviation. Measured as deviations from steady-state. The key policy rate gap is not annualized.*

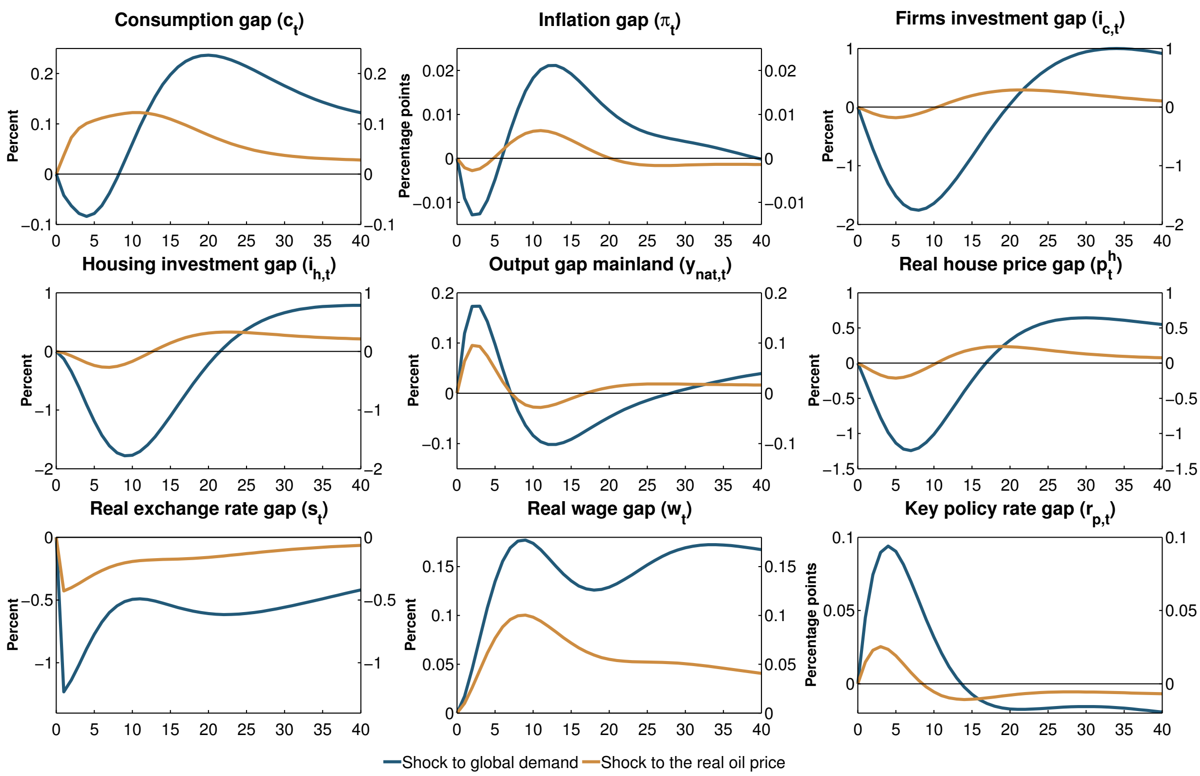

Figure 6: *Impulse response functions of a shock to global demand and the oil price such that the oil price increases by 10 percent in both scenarios. Measured as deviations from steady-state. The key policy rate gap is not annualized.*

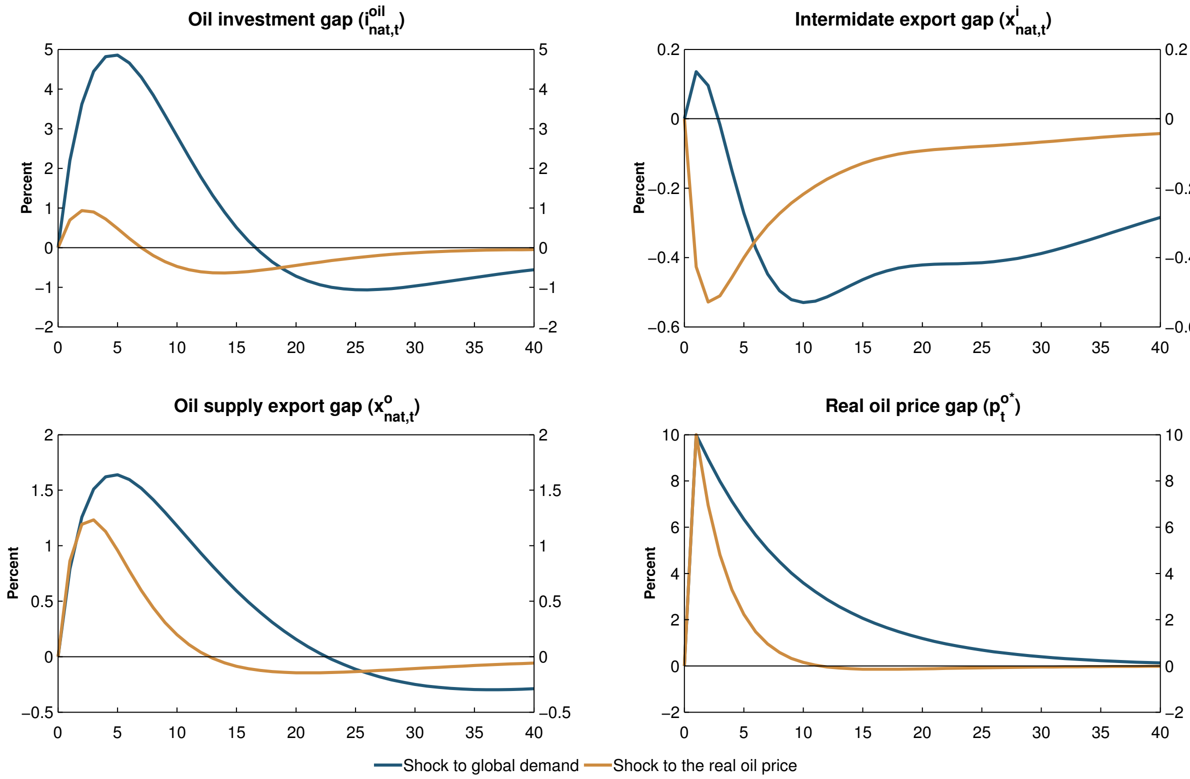

Figure 7: *Impulse response functions of a shock to global demand and the oil price such that the oil price increases by 10 percent in both scenarios. Measured as deviations from steady-state.*

4.4 Shock decomposition

In this section we show how historical developments in inflation measured by the CPI-ATE, the output gap and the key policy rate can be explained in terms of shocks that have hit the Norwegian economy according to NEMO. We limit the discussion to developments since the sharp fall in oil prices in 2014 to highlight the new features of the model.

The key policy rate has been reduced by one percentage point since 2014, from 1.50% to 0.50% (see Figure 8). Over this period, a series of both negative and positive shocks have hit the economy. Overall demand from households and enterprises were already low in 2014 (see Figure 9). This contributed to a lower interest rate. In the course of 2016, demand picked up due to higher government expenditures and housing investment, contributing in isolation to a higher interest rate.

The decline in the oil price from 2014 has mainly been interpreted as a set of positive global oil supply shocks. Oil investments have also been lower than the model had otherwise predicted (also given the fall in the oil price), which has been interpreted as a set of negative oil investment shocks. Both these shocks have reduced the demand for labor, capital and goods from the mainland economy, which in turn has reduced inflation and the output gap. To partly counteract this negative impact on the Norwegian economy, the key policy rate has been lower than what it otherwise would have been. In sum, oil-related shocks contributed to a somewhat lower inflation (see Figure 10), to a lower key policy rate (see Figure 8) and a lower output gap (see Figure 9).

The Norwegian economy has since 2014 been hit by a series of supply domestic shocks as well, in particular negative wage mark-ups (as interpreted by the model), that have reduced overall wage growth. Lower wage growth has lead to disinflation and contributed to a lower interest rate. In isolation, the lower interest rate has contributed to a higher output gap.

The NOK exchange rate has depreciated much more than that explained by the UIP condition in the model. This is interpreted as a risk premium shock in the model (labeled *Exchange rate* in the figures). The weaker NOK exchange rate has led to higher imported inflation and represented an important stimulus for demand, in particular non-oil exports. In isolation, this has contributed to a higher interest rate. Other foreign shocks, mainly foreign demand shocks, have on the other hand contributed to a lower interest rate.

Financial frictions have also affected economic developments (labeled *Banking sector* in the figures), although to a small extent compared to other shocks. The *Banking sector* bar includes shocks to the money market risk premium (the difference between the money market rate and the key policy rate), bank lending mark-up to households and non-financial enterprises, respectively, and loan-to-value "requirements" (LTV) for households. The LTV shock is not well identified in the model since we do not observe new lending in the data, only the stock of credit, but general movements in the stock of credit and house prices can provide some information on the ability of households to accumulate debt as house prices increase. In the past couple of years, financial shocks have contributed to somewhat lower output, inflation and key policy rates. This can be due to a stricter regulation on requirements for new residential mortgage loans from July 1, 2015, and higher money market premium from the second half of 2015.

Monetary policy cannot be represented exactly by a simple loss function in a model. This gives rise to monetary policy shocks as a further explanation of why the economy deviates from a steady-state path. In the past couple of years, monetary policy has been tighter than that derived from the model. One explanation for this is that financial stability concerns against the backdrop of high house price inflation have gained a more prominent role in monetary policy considerations. Another explanation is that Norges Bank has put weight on moving with caution as the uncertainty surrounding the effects of monetary policy increases when the interest rate approaches a lower bound.

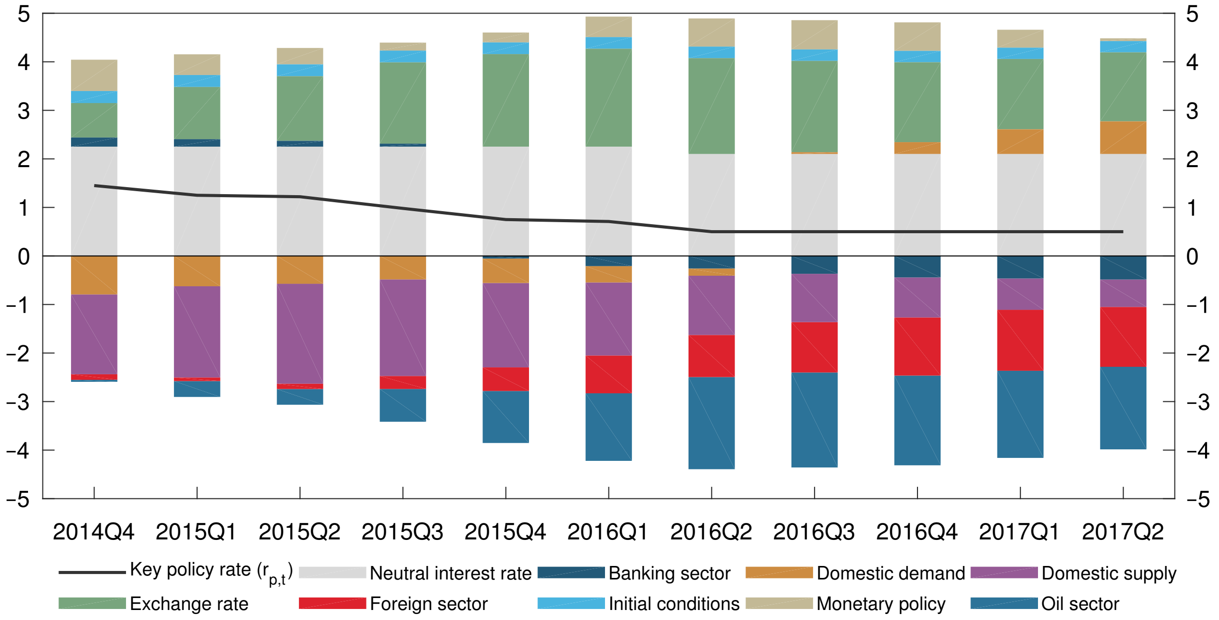

Figure 8: *Historical shock decomposition of the key policy rate. The key policy rate is adjusted for exogenous changes in the nominal neutral rate of interest. Quarterly average. Annualized.*[14]

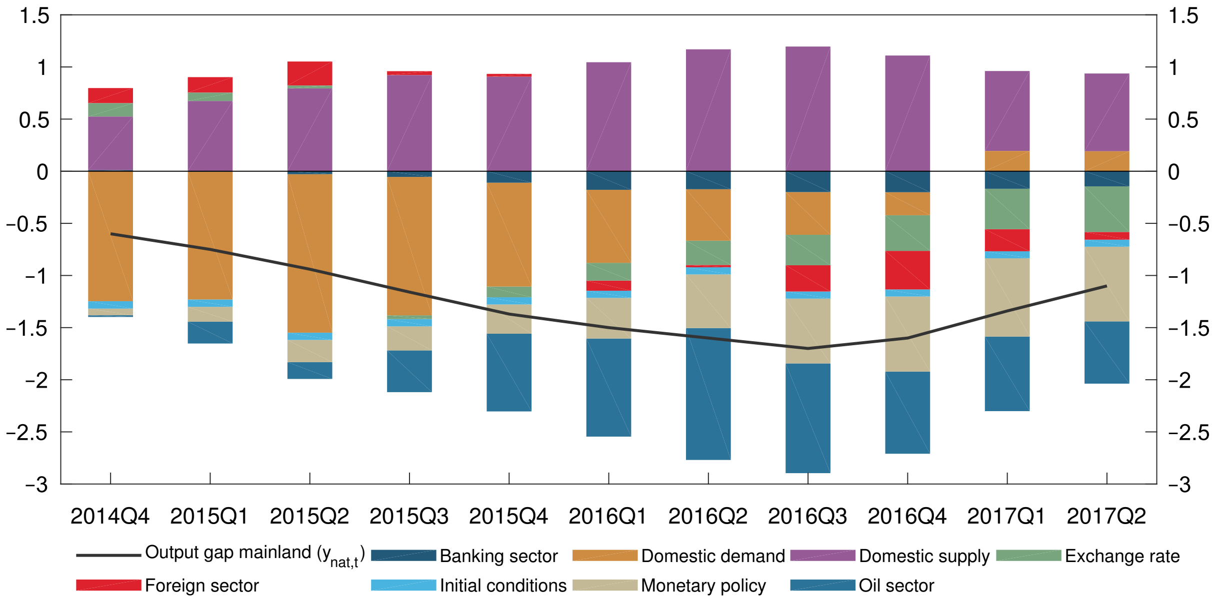

Figure 9: *Historical shock decomposition of the output gap. The output gap is defined as mainland GDP as the percentage deviation from a trend estimated with a Hodrick-Prescott filter with* $\lambda = 3000$. *To remove noise from the detrended data, a three-quarter central moving average procedure is then used.*[14]

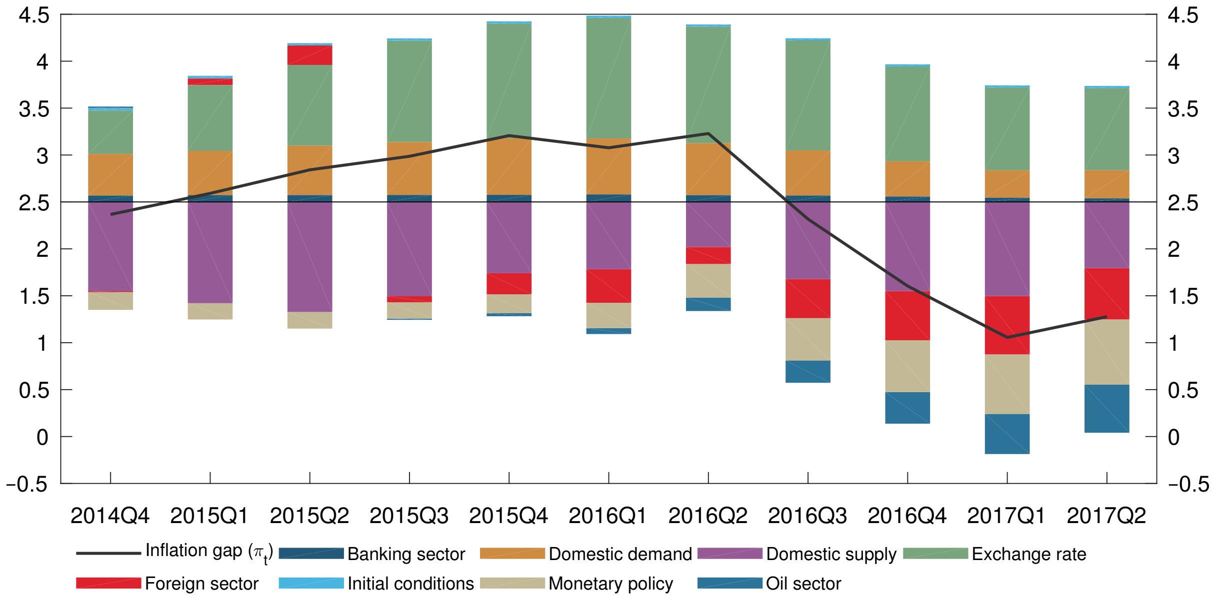

Figure 10: *Historical shock decomposition of the inflation gap. The inflation gap is defined as consumer prices adjusted for taxes and excluding energy products as a deviation from the inflation target of 2.5 per cent. Annualized quarterly inflation.*[14]

5 Summary

Forecasting and monetary policy analysis at Norges Bank are organized around the macroeconomic model NEMO, which represents the Bank's core model. In this paper we provide a non-technical documentation of NEMO and the recent extensions made to this model, in particular the introduction of an oil sector to incorporate important channels of shocks to the Norwegian economy. The introduction of an oil sector also enables the Bank to separate exports from mainland Norway into oil supply exports and other traditional exports.

The paper explains the basic mechanisms of NEMO, and we show that the model has a reasonable empirical fit compared to VAR models. Next, we highlight the transmission channels of important shocks by looking at a series of impulse-response functions. These and other impulse-response functions are the building blocks of a shock decomposition of the key policy rate, the output gap and inflation from 2014. We focus on this period to highlight important driving forces of the Norwegian economy after the fall in the oil price. A technical documentation of all derivations, first-order conditions, the full steady-state solution and the parametrization of the model can be found in Kravik and Paulsen (2017).

[14] The bars contain the following shocks: Banking sector: Money market risk premium, LTV households, LTV entrepreneurs, lending rate to households mark up and lending rate to entrepreneurs mark up. Domestic demand: Government expenditures, consumption preferences, housing preferences, import and GDP measurment error. Domestic supply: Temporary productivity, investment costs, housing investment, price markup and wage markup. Exchange rate: Risk premium. Foreign sector: Foreign marginal costs, global demand, money market interest rate abroad, inflation abroad, output abroad and foreign home preferences. Monetary policy: Monetary policy preferences. Oil sector: Real oil price, oil investment and oil production abroad.

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Appendices

A Data transformations

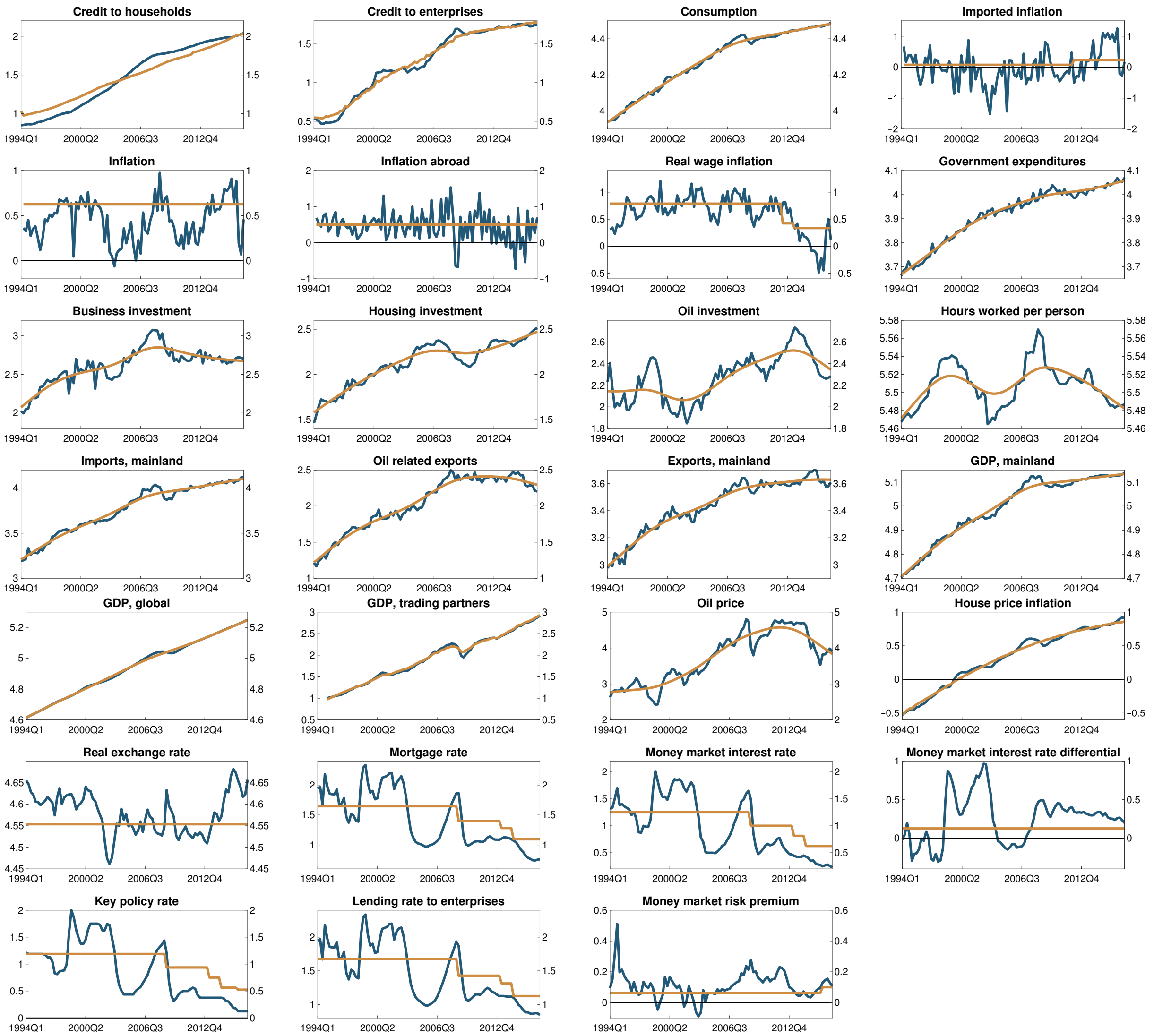

Figure 11: *Observable variables in NEMO. Levels (blue) and trends (yellow). A description of the detrending method for each observable can be found in Table 2. Noise filtering and judgment (e.g. due to endpoint problems) are not displayed in this figure, so the differences between the levels and trends do not necessary correspond to the gaps displayed in figure 12.*

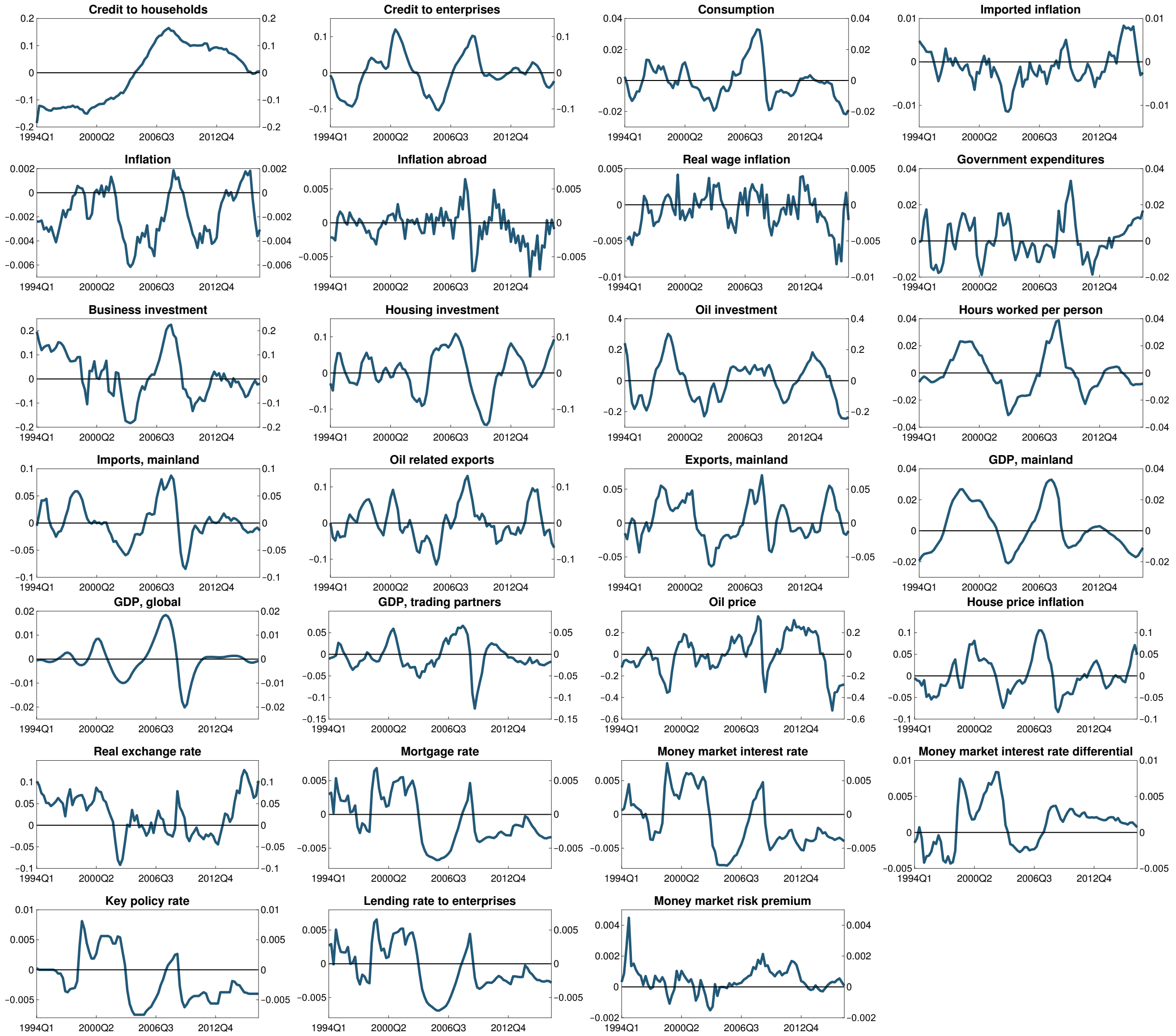

Figure 12: *Observable variables in NEMO. Gaps. Noise filtering and judgment (e.g. due to endpoint problems) have been applied on the series. This means that the differences between the levels and trends in figure 11 do not necessary correspond to the gaps displayed in this figure.*

Observable	Description	Transformation
Consumption (c_t)	Private consumption per capita. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Credit to enterprises ($b_{e,t}$)	Credit indicator for non-financial enterprises per capita for mainland Norway (C3). Source: Statistics Norway.	Divided by population aged 15-74 and CPI adjusted for taxes and excluding energy prices (CPI-ATE), taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Credit to households ($b_{h,t}$)	Credit indicator for households per capita (C2). Source: Statistics Norway.	Divided by population aged 15-74 and CPI-ATE, taken log and HP-filtered with lambda 400000. Calculated 3-quarter moving average.
Exports, mainland ($x_{nat,t}$)	Total exports per capita for mainland Norway. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average. Constructed at Norges Bank.
GDP, mainland ($y_{nat,t}$)	Estimated output gap per capita for mainland Norway. Source: Norges Bank.	Official estimates.
GDP, global ($y^{tot}_{nat,t}$)	World GDP. Sources: IMF and Norges Bank.	Weighted by market shares. Taken log and HP-filtered with lambda 1600.
GDP, trading partners ($y^*_{nat,t}$)	GDP in 25 of Norway's most important trading partners. Source: Thomson Reuters and Norges Bank.	Weighted by export shares.
Government expenditures (g_t)	Public sector consumption and gross investment per capita. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Hours worked (l_t)	Total hours worked per capita. Source: Statistics Norway.	Divided by population aged 15-74, taken log and calculated 3-quarter moving average.
Housing investment ($i_{h,t}$)	Housing investment per capita for mainland Norway. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
House price inflation (p^h_t)	Nominal house prices deflated by CPI-ATE. Sources: Eiendom Norge, Eiendomsverdi. Finn.no, Norges Bank, and Statistics Norway.	Divided by CPI-ATE, taken log and HP-filtered with lambda 30000. Calculated 3-quarter moving average.
Imports, mainland ($m_{nat,t}$)	Import per capita for mainland Norway. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Imported inflation (π^m_t)	Import consumer goods in CPI-ATE. Source: Statistics Norway.	Taken log-approximated growth rate (3-quarter moving average of seasonally adjusted series) and subtracted 0.3 before 2012Q1 and 0.9 after. Seasonally adjusted by X12-ARIMA.
Inflation (π_t)	CPI-ATE. Sources: Statistics Norway and Norges Bank.	Taken log-approximated growth rate (3-quarter moving average of seasonally adjusted series) and subtracted 2.5. 3-month moving average. Seasonally adjusted by X12-ARIMA.

Observable	Description	Transformation
Inflation abroad (π_t^*)	Import weighted CPI for 25 of Norway's trading partners. Sources: IMF, Thomson Reuters and Norges Bank.	Taken log-approximated growth rate and subtracted 2.
Business investment ($i_{c,t}$)	Firms gross investment per capita for mainland Norway. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Lending rate to enterprises (r_t^e)	Lending rate to enterprises. Source: Statistics Norway.	Subtracted 6.71 before 2008Q3, 5.71 before 2013Q1, 5.25 before 2014Q2 and 4.5 afterwards.
Key policy rate ($r_{p,t}$)	Nominal sight deposit rate. Source: Norges Bank.	Subtracted 4.75 before 2008Q3, 3.75 before 2013Q1, 3 before 2014Q2 and 2.25 before 2016Q1 and 2.1 afterwards.
Money market interest rate (r_t)	3-month nominal interest rate, Norwegian Interbank Offered Rate (Nibor). Source: Norges Bank.	Subtracted 5 before 2008Q3, 4 before 2013Q1, 3.25 before 2014Q2 and 2.5 afterwards.
Money market interest rate abroad (r_t^*)	3-month nominal money market interest rates for trading partners. Sources: Thomson Reuters and Norges Bank.	Subtracted 4.5 before 2008Q3, 3.5 before 2013Q1, 2.75 before 2014Q2 and 2 afterwards.
Money market risk premium ($z_{prem,t}$)	Interest rate difference. Source: Norges Bank.	Subtracted 0.25 before 2016Q1 and 0.4 afterwards.
Mortgage rate (r_t^f)	Loans to households. Source: Statistics Norway.	Subtracted 6.59 before 2008Q3, 5.59 before 2013Q1, 5.13 before 2014Q2 and 4.38 afterwards.
Oil investment ($i_{nat,t}^{oil}$)	Total investments in oil activity. Source: Statistics Norway.	Divided by population aged 15-74, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Oil price (p_t^{o*})	Brent blend USD per barrell. Sources: Norges Bank and Statistics Norway.	Taken log and and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Oil related export ($x_{nat,t}^o$)	Total exports from oil services. Source: Norges Bank and Statistics Norway.	Divided by population between 15-74 years, taken log and HP-filtered with lambda 3000. Calculated 3-quarter moving average.
Real exchange rate (s_t)	Import weighted exchange rate from 44 countries (I-44). Source: Thomson Reuters and Norges Bank.	Taken log and subtracted the average calculated from 1994Q1 to 2000Q4 before 2002Q1 and the average from 2010Q1 afterwards. Divided by CPI-ATE.
Wage inflation ($\pi_{w,t}$)	Nominal wage growth. Source: Norwegian Technical Calculation Committee for Wage Settlements and Statistics Norway.	Taken log and subtracted 5.65 before 2012Q1 and 4 afterwards.

Table 2: *Data descriptions and transformations. Additional judgment may have been applied to construct the final gaps that is used in NEMO, e.g. due to endpoint problems of the filters applied. The detrending and noise filtering have been applied up until 2017 Q2.*

B VAR selection

For each variable of interest, we create a pool of vector autoregression (VAR) models to select from. We use data on the variables; inflation gap, key policy rate gap, money market rates abroad gap, oil price gap, output gap, output gap abroad and real exchange rate gap. The pool of VAR models for each variable of interest is created by adding 0-6 variables from these variables excluded itself (without replications). The reason to pick maximum six additional variables is to keep the number of possible VARs at solvable level. We also replicate all these models for 1-4 lags. That makes a total of 256 models for each variable of interest. We then evaluate the out-of-sample recursive forecast performance of each model using root mean squared errors (RMSE). The 20 ($= M$) best performing models are selected for each variable at each horizon and for each recursive period, i.e. the best performing models will vary with the forecasting horizon and the period of the forecast. We are using the following score to evaluate the models:

$$score_{h,v,T,m} = \frac{1}{\sqrt{\frac{\sum_{s=t}^{T}(F_{h,v,s,m}-Y_{s+h,v})^2}{T-t+1}}}, \tag{1}$$

where we have defined t to be the start period of the evaluation, T the end period of the evaluation, the point forecast at time s at horizon h for model m for variable v by $F_{h,v,s,m}$ and the actual data at time s for variable v as $Y_{s,v}$. Note that $m \in [1, M]$ and $v \in [1, V]$, where V is the number of variables of interest. We then combine the forecasts using a weighted average,

$$CF_{h,v,s} = \sum_{m=1}^{M} w_{h,v,s,m} F_{h,v,s,m}. \tag{2}$$

$w_{h,v,s,m}$ is the weight on each model in the combined forecast, and is calculated as

$$w_{h,v,s,m} = \frac{score_{h,v,s,m}}{\sum_{m=1}^{M} score_{h,v,s,m}}. \tag{3}$$

Note that the weights also change with time s and forecasting horizon h.

C Recursive forecasts

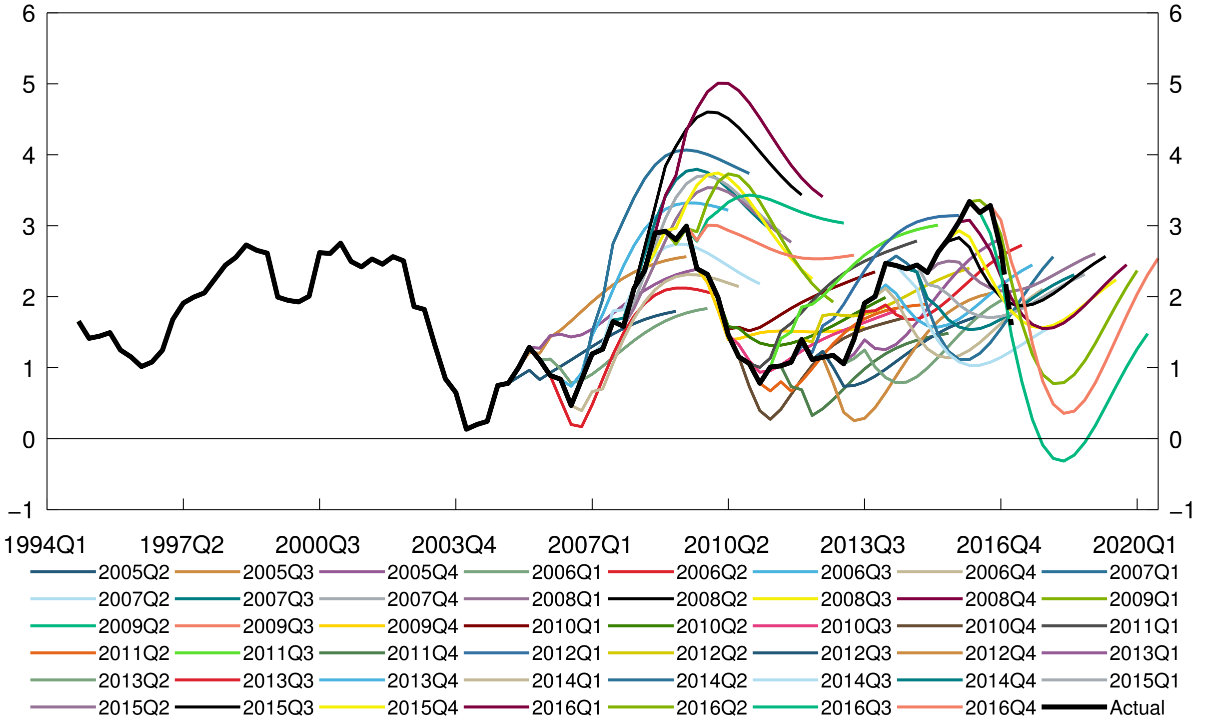

Figure 13: *Unconditional (quasi) out-of-sample forecast of inflation. Four-quarter change. NEMO*

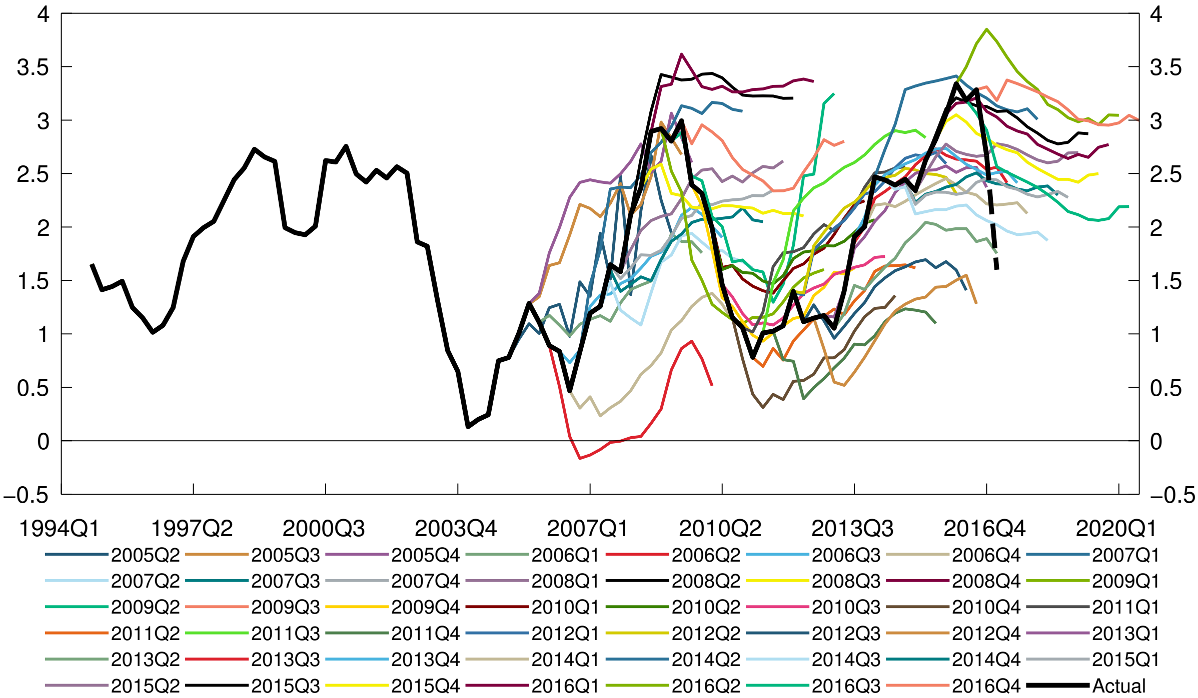

Figure 14: *Unconditional out-of-sample forecast of inflation. Four-quarter change. VAR models*

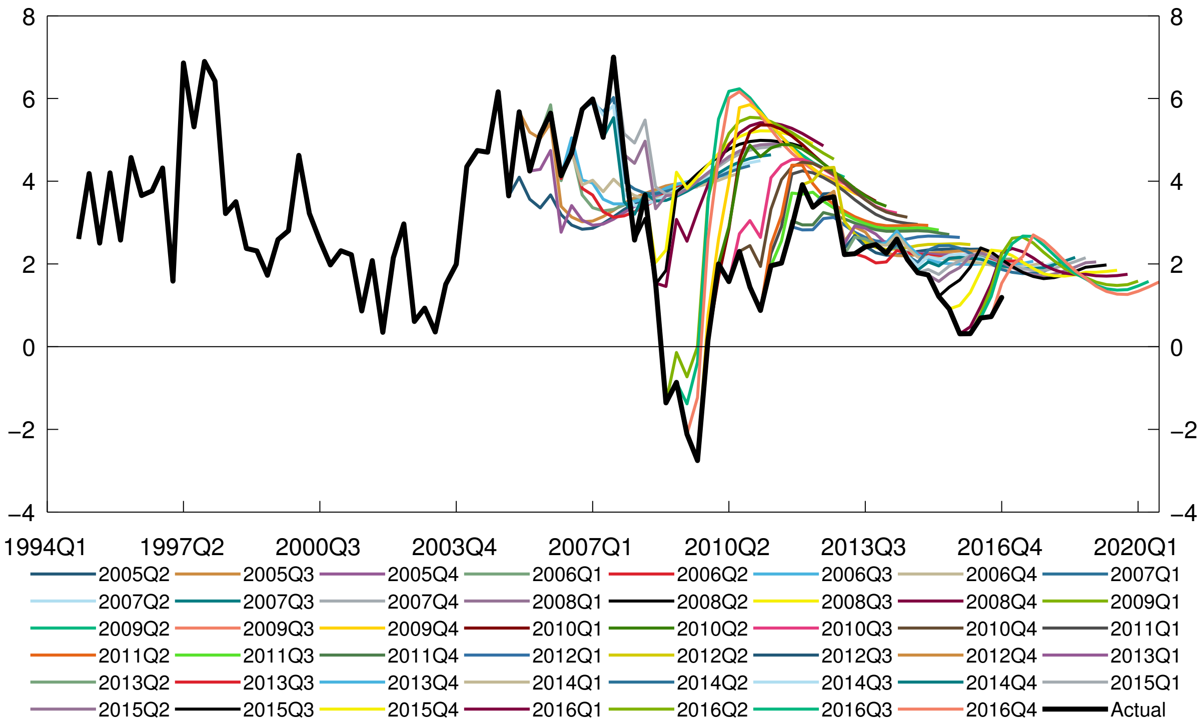

Figure 15: *Unconditional (quasi) out-of-sample forecast of GDP. Four-quarter growth. NEMO*

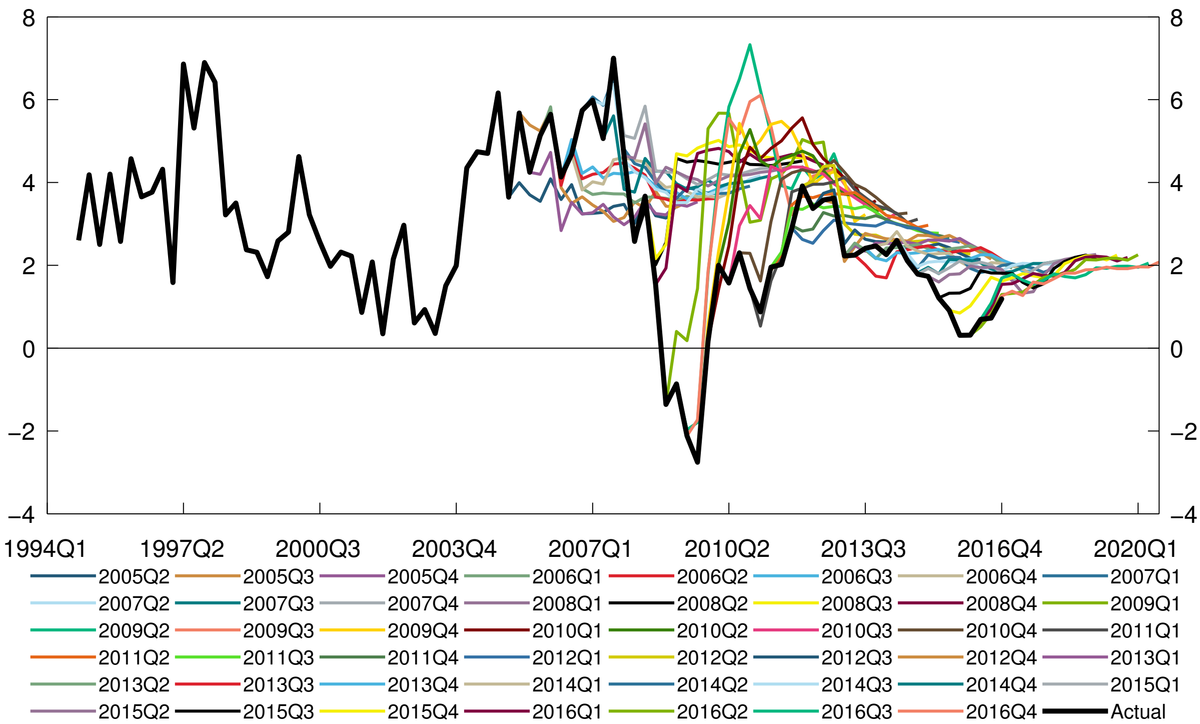

Figure 16: *Unconditional out-of-sample forecast of GDP. Four-quarter growth. VAR models*

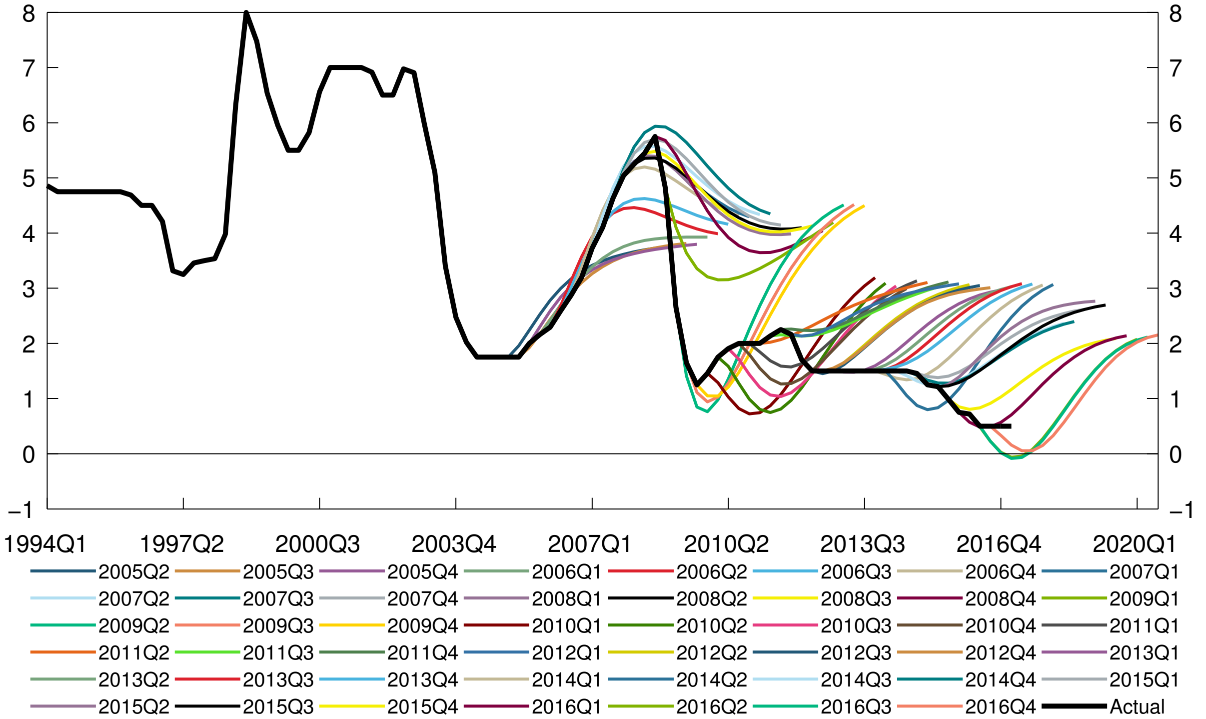

Figure 17: *Unconditional (quasi) out-of-sample forecast of the key policy rate. Level. NEMO*

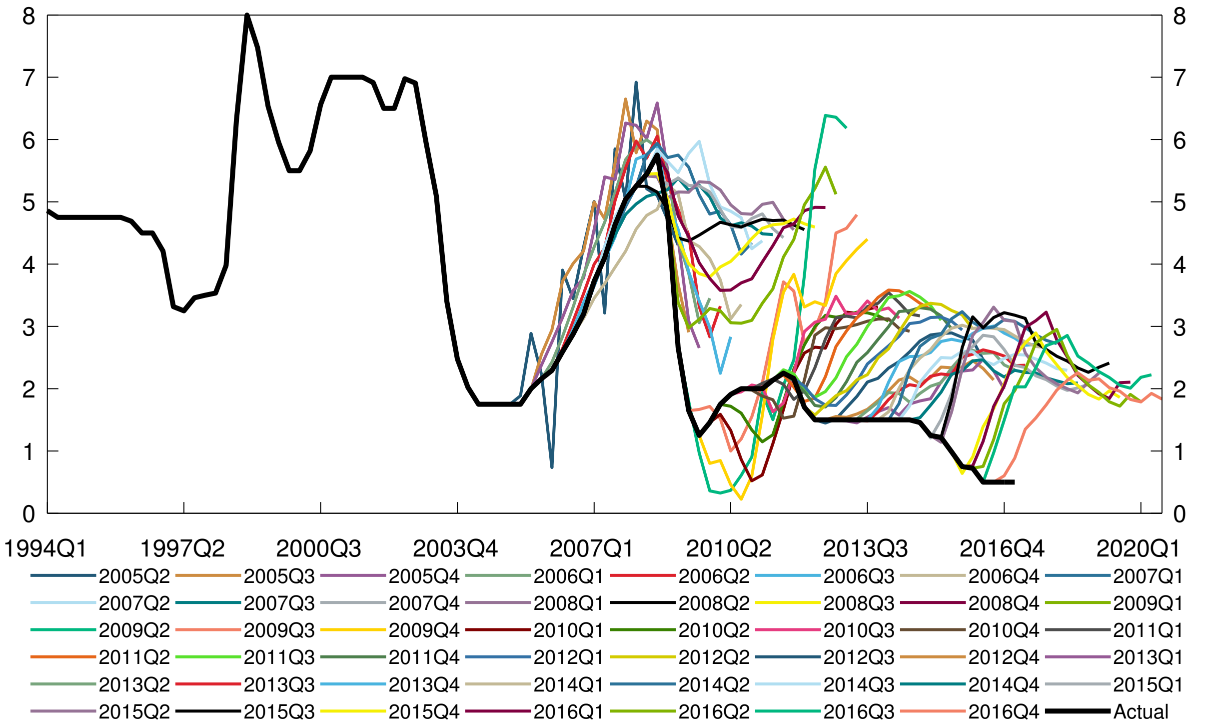

Figure 18: *Unconditional out-of-sample forecast of the key policy rate. Level. VAR models*

D Impulse response functions and interest rate account

To solve NEMO, we log linearize the model and solve it under the assumption of model-consistent expectations. The solution of the model may then be written as a vector autoregressive process with one lag,

$$y_t = Ay_{t-1} + B\varepsilon_t, \tag{4}$$

where y_t is a vector with the endogenous variables of the model with size $M \times 1$[15], ε_t is a vector with the structural innovations with size $N \times 1$, A is a matrix with size $M \times M$ and B is a matrix of size $M \times N$. The matrix A describes how the model will evolve without being subject to new disturbances, while the matrix B steers the contemporaneous effects on the endogenous variables due to changes in the structural innovations of the model.

Since equation 4 is a covariance stationary process we can equally represent the process for y_t using Wold's decomposition,

$$y_t = \beta_0\varepsilon_t + \beta_1\varepsilon_{t-1} + ... + \beta_{\inf}\varepsilon_{t-\inf}. \tag{5}$$

As we have limited length of our time-series we can write the process as a sum of past innovations and initial conditions as follows:

$$y_t = \beta_0\varepsilon_t + \beta_1\varepsilon_{t-1} + ... + \beta_{t-1}\varepsilon_1 + \gamma_t y_0, \tag{6}$$

where β_s for $s = 0, ..., t-1$ has size $M \times N$, while γ_t has size $M \times M$.

Now we will describe what we mean about concepts such as: impulse response function and shock decomposition. In an impulse response exercise, we want to see how the system evolves after being subject to a disturbance to one of the innovations. Let the innovation of interest be the first innovation in the vector ε. It is common in the literature to study a one standard deviation disturbance to the innovation of interest. I.e. we start at period 0 with $\varepsilon_0 = [\sigma_1, 0, .., 0]'$, where σ_1 is the standard deviation of the first innovation of the process. Then, using equation 4, we see that the contemporaneous effect on y is given by

$$y_0 = B\varepsilon_0 = \beta_0\varepsilon_0, \tag{7}$$

assuming $y_{-1} = 0$. Iterating one more period using equation 4, we see that

$$y_1 = Ay_0 + B\varepsilon_1. \tag{8}$$

In an impulse response exercise we assume that $\varepsilon_s = 0$ for all $s > 0$. Using this and substitute for y_0 from equation 7 we get

$$y_1 = AB\varepsilon_0 = \beta_1\varepsilon_0. \tag{9}$$

By iteration in the same way forward we get

$$y_s = A^s B\varepsilon_0 = \beta_s\varepsilon_0, \tag{10}$$

for $s \geq 0$. I.e. we have shown the response on y_s at all future periods s due to a disturbance to one of the innovations in period 0, which is just what an impulse response function is

[15] Contemporaneous and lagged values of the endogenous variables are stacked in the vectors y_t and y_{t-1} respectively.

in the linear case. During these steps we have also solved for β_s for all $s = 0$,...,inf. γ_t on the other hand can be found by iterating equation 4 t times and assuming that $\varepsilon_s = 0$ for all $s = 1, ..., t$, i.e. $\gamma_t = A^t$.

In the shock decomposition exercise, we are, on the other hand, interested in describing the evolution of the endogenous variables in terms of identified innovations. We use a Kalman smoother approach to estimate the most likely innovations that the model has been subject to given the parameters and data on the observables of the model. We then get an estimate of ε_s for $s = 1, ..., t$, which we, in combination with equation 6, can use to do the shock decomposition. I.e. for period $s = 1$ we get

$$y_1 = \beta_0\varepsilon_1 + \gamma_1 y_0, \tag{11}$$

for period $s = 2$,

$$y_2 = \beta_0\varepsilon_2 + \beta_1\varepsilon_1 + \gamma_2 y_0, \tag{12}$$

and for the general s we can substitute in for $t = s$ in equation 6. For more information on the concepts presented in this appendix, see Hamilton (1994).

In the periodic *Monetary Policy Reports* (MPR), Norges Bank presents an interest rate account which shows how the news and new assessments have affected the changes in the interest rate forecast. The accounting between to reports is constructed using the following decomposition:

$$y_t = \beta_{+s}\varepsilon_{t+s} + ... + \beta_{+2}\varepsilon_{t+2} + \beta_{+1}\varepsilon_{t+1} + \beta_0\varepsilon_t + \beta_1\varepsilon_{t-1} + ... + \beta_{t-1}\varepsilon_1 + \gamma_t y_0. \tag{13}$$

Here we have also added expected shocks with horizon s to decomposition 6. We do this to take into account our use of conditional information about the future when producing forecasts with NEMO for the MPR, see Maih (2010). Let T be the last historical observation of the current MPR, while $T - 1$ is the last historical observation in the previous MPR. Let H be the forecasting horizon, then we can decompose the forecasts from the current MPR from period $T + 1$ to $T + H$ as:

$$\begin{aligned} y^c_{T+h} =& \beta_{+s}\varepsilon^c_{T+h+s} + ... + \beta_{+2}\varepsilon^c_{T+h+2} + \beta_{+1}\varepsilon^c_{T+h+1} + \beta_0\varepsilon^c_{T+h} + \\ & \beta_1\varepsilon^c_{T+h-1} + ... + \beta_{T+h-1}\varepsilon^c_1 + \gamma_{T+h} y^c_0, \end{aligned} \tag{14}$$

for $h = 1, ..., H$. The superscript c indicate identified shocks, estimated initial conditions and forecasts at the current MPR. For the previous MPR we get equivalently for the y_{T+h-1} forecast

$$\begin{aligned} y^p_{T+h-1} =& \beta_{+s}\varepsilon^p_{T+h-1+s} + ... + \beta_{+2}\varepsilon^p_{T+h+1} + \beta_{+1}\varepsilon^p_{T+h} + \beta_0\varepsilon^p_{T+h-1} + \\ & \beta_1\varepsilon^p_{T+h-2} + ... + \beta_{T+h-2}\varepsilon^p_1 + \gamma_{T+h-1} y^p_0, \end{aligned} \tag{15}$$

where the superscript p indicate identified shocks, estimated initial conditions and forecasts at the previous MPR. By combining equation 4 and 15 we can produce a decomposition of the forecast at period y_{T+h} at the previous MPR as

$$\begin{aligned} y^p_{T+h} =& A\beta_{+s}\varepsilon^p_{T+h+s-1} + ... + A\beta_{+2}\varepsilon^p_{T+h+1} + A\beta_{+1}\varepsilon^p_{T+h} + \beta_1\varepsilon^p_{T+h-1} + \\ & \beta_2\varepsilon^p_{T+h-2} + ... + \beta_{T+h-1}\varepsilon^p_1 + \gamma_{T+h} y^p_0. \end{aligned} \tag{16}$$

Note, due to the Kalman smoother, the historically identified innovations and initial conditions may also change due to revision in the data. To produce the decomposition between the different forecast from the to periods we take the difference between equation 14 and 16:

$$\begin{aligned} y^c_{T+h} - y^p_{T+h} =& \beta_{+s}\varepsilon^c_{T+h+s} + \beta_{+s-1}\varepsilon^c_{T+h+s-1} - A\beta_{+s}\varepsilon^p_{T+h+s-1} + ... + \\ & \beta_{+1}\varepsilon^c_{T+h+1} - A\beta_{+2}\varepsilon^p_{T+h+1} + \beta_0\varepsilon^c_{T+h} - A\beta_{+1}\varepsilon^p_{T+h} + \\ & \beta_1\left(\varepsilon^c_{T+h-1} - \varepsilon^p_{T+h-1}\right) + ... + \beta_{T+h-1}\left(\varepsilon^c_1 - \varepsilon^p_1\right) + \\ & \gamma_{T+h}\left(y^c_0 - y^p_0\right). \end{aligned} \tag{17}$$

Given that the initial conditions are so far back in history, we have that $\gamma_{T+h}\left(y^c_0 - y^l_0\right) \approx 0$, and they are therefore excluded from the interest rate account presented in the MPR. Forecast may also change due to changes in the parameters. This is also incorporated into the interest account, but it is not shown in equation 17. Finally, as the model has many innovations, we group some of them together by summing to make a clearer picture. This grouping of shocks may change from report to report.