

# WHEN DOES A CENTRAL BANK'S BALANCE SHEET REQUIRE FISCAL SUPPORT?

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ABSTRACT. The large balance sheet of many central banks has raised concerns that they could suffer significant losses if interest rates rose, and may need capital injection from the fiscal authority. This paper constructs a simple deterministic general equilibrium model, calibrated to US data, to study the the impact of alternative interest rates scenarios on the central bank's balance sheet, as well as the possibility that self-fulfilling equilibria may arise. We show that assumptions about the behavior of seigniorage under high inflation are crucial in determining the need for fiscal backing to achieve price stability. We conclude that for current balance sheet levels fiscal backing would be required only under rather extreme scenarios. Higher balance sheet levels, or a lower currency demand than assumed here, could however lead to price level indeterminacy for a central bank that does not have fiscal support.

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## I. INTRODUCTION

Hall and Reis (2013) and Carpenter, Ihrig, Klee, Quinn, and Boote (2013) have explored the likely path of the Federal Reserve System's balance sheet

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during a possible return to historically normal levels of interest rates. Both conclude that, though a period when the system's net worth at market value is negative might occur, this is unlikely, would be temporary and would not create serious problems.<sup>1</sup> Those conclusions rely on extrapolating into the future not only a notion of historically normal interest rates, but also of historically normal relationships between interest rates, inflation rates, and components of the System's balance sheet. In this paper we look at complete, though simplified, economic models in order to study why a central bank's balance sheet matters at all and the consequences of a lack of fiscal backing for the central bank. These issues are important because they lead us to think about unlikely, but nonetheless possible, sequences of events that could undermine economic stability. As recent events should have taught us, historically *abnormal* events do occur in financial markets, and understanding in advance how they can arise and how to avert or mitigate them is worthwhile.<sup>2</sup>

Constructing a model that allows us to address these issues requires us to specify monetary and fiscal policy behavior and to consider how demand for non-interest-bearing liabilities of the central bank (like currency, or required reserves paying zero interest) responds to interest rates. Results are sensitive to these aspects of the specification, even when we consider a version of the model calibrated to the current situation in the US.

In the first section below we consider a stripped down model to show how the need for fiscal backing arises. In subsequent sections we make the

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<sup>1</sup>Christensen, Lopez, and Rudebusch (2013) study the interest rate risk faced by the Federal Reserve using probabilities for alternative interest rate scenarios obtained from a dynamic term structure model. They reach the similar conclusions as Hall and Reis (2013) and Carpenter, Ihrig, Klee, Quinn, and Boote (2013).

<sup>2</sup>A number of recent papers, including Corsetti and Dedola (2012) and Bassetto and Messer (2013), also study the central bank's and the fiscal authorities' balance sheets separately.

model more realistic and calibrate it to allow simulation of the US Federal Reserve System's response to realistic shifts in the real rate or "inflation scares".

## II. THE SIMPLE MODEL

A central bank that issues fiat money requires fiscal backing if it is to control the price level. To understand why, and how the need for fiscal backing might manifest itself, we first consider a stripped-down model to illustrate the principles at work. We make four main points in this section:

- (1) No policy undertaken by a central bank alone, without fiscal powers, can guarantee a uniquely determined price level. [Cochrane \(2011\)](#) has made this point carefully.
- (2) There are simple, plausible, "backstop" fiscal and monetary policy actions that will make the price level determinate. A commitment to such actions eliminates the need for overt fiscal backing on the economy's equilibrium path.<sup>3</sup>
- (3) When the price level is uniquely determined, it is nonetheless possible that the central bank, if its balance sheet is sufficiently impaired, may need recapitalization in order to maintain its commitment to a policy rule or an inflation target.
- (4) A central bank's ability to earn seigniorage can make it possible for it to recover from a situation of negative net worth at market value without recapitalization from the treasury, while still maintaining its policy rule. Whether it can do so depends on the policy rule, the demand for its non-interest-bearing liabilities, and the size of the initial net worth gap.

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<sup>3</sup>Cochrane has argued that such actions are either impossible, or necessarily so drastic as to be implausible. One contribution of this paper is to provide a counterexample to Cochrane's claim.

A representative agent solves

$$\max_{C,B,M,F} \int_0^{\infty} e^{-\beta t} \log(C_t) dt \quad \text{subject to} \quad (1)$$

$$C_t \cdot (1 + \psi(v_t)) + \frac{\dot{B} + \dot{M}}{P_t} + \tau_t + F_t = \rho F_t + \frac{r_t B_t}{P_t} + Y_t, \quad (2)$$

where  $C$  is consumption,  $B$  is instantaneous nominal bonds paying interest at the rate  $r$ ,  $M$  is non-interest-bearing money,  $\rho$  is a real rate of return on a real asset  $F$ ,  $Y$  is endowment income, and  $\tau$  is the primary surplus (or simply lump-sum taxes, since we have no explicit government spending in this model). Velocity  $v_t$  is given by

$$v_t = \frac{P_t C_t}{M_t}, \quad v_t \geq 0, \quad (3)$$

and the function  $\psi(\cdot)$ ,  $\psi'(\cdot) > 0$  captures transaction costs.

The government budget constraint is

$$\frac{\dot{B} + \dot{M}}{P_t} + \tau_t = \frac{r_t B_t}{P_t}. \quad (4)$$

Monetary policy is an interest-smoothing Taylor rule:

$$\dot{r} = \theta_r \cdot \left( \bar{r} + \theta_\pi \left( \frac{\dot{P}}{P} - \bar{\pi} \right) - r \right). \quad (5)$$

The ‘‘Taylor Principle’’, that  $\theta_\pi$  should exceed one, is the usual prescription for ‘‘active’’ monetary policy.

First order conditions for the private agent are

$$\partial C : \quad \frac{1}{C} = \lambda(1 + \psi + \psi'v) \quad (6)$$

$$\partial F : \quad -\hat{\lambda} = \lambda(\rho - \beta) \quad (7)$$

$$\partial B : \quad -\frac{\hat{\lambda}}{P} + \beta \frac{\lambda}{P} + \frac{\lambda \hat{P}}{P^2} = r \frac{\lambda}{P} \quad (8)$$

$$\partial M : \quad -\frac{\hat{\lambda}}{P} + \beta \frac{\lambda}{P} + \frac{\lambda \hat{P}}{P^2} = \psi'v^2 \quad (9)$$

The  $\hat{z}_t$  notation means the time derivative of the future expected path of  $z$  at  $t$ . It exists even at dates when  $z$  has taken a jump, so long as its future

path is right-differentiable. Below we also use the  $\frac{\hat{d}^+}{dt}$  operator for the same concept.

We are taking the real rate  $\rho$  as exogenous, and in this simple version of the model constant. The economy is therefore being modeled as either having a constant-returns-to scale investment technology or as having access to international borrowing and lending at a fixed rate. Though we could extend the model to consider stochastically evolving  $Y$ ,  $\rho$ , and other external disturbances, here we consider only surprise shifts at the  $t = 0$  starting date, with perfect-foresight deterministic paths thereafter. This makes it easier to follow the logic, though it makes the time-0 adjustments unrealistically abrupt.

Besides the exogenous influences that already appear explicitly in the system above ( $\rho$  and  $Y$ ), we consider an “inflation scare” variable  $x$ . This enters the agents’ first order condition as a perturbation to inflation expectations. It can be reconciled with rational expectations by supposing that agents think there is a possibility of discontinuous jumps in the price level, with these jumps arriving as a Poisson process with a fixed rate. This would happen if at these jump dates monetary policy created discontinuous jumps in  $M$ . Such jumps would create temporary declines in the real value of government debt  $B/P$  which might explain why such jumps are perceived as possible. If the jump process doesn’t change after a jump occurs, there is no change in velocity, the inflation rate, consumption, or the interest rate at the jump dates. Rather than solve a model that includes such jumps, we model one in which the public is wrong about this — there are no jumps, despite the expectation that there could be jumps. After a long enough period with no jumps, the public would probably change its expectations, but there is no logical contradiction in supposing that for a moderate amount of time the fact that there are no jumps does not change expectations. In fact, if we

consider time-varying paths for  $x$ , in which  $x$  returns to zero after some period, there is no way to distinguish whether the “true” model is one with the assumed  $x$  path (and thus a non-zero probability of jumps in  $P$ ) or one with  $x \equiv 0$  if jumps do not actually occur.

The inflation scare variable changes the first order conditions above to give us

$$\partial B : \quad -\hat{\lambda} \frac{1}{P} + \beta \frac{\lambda}{P} + \frac{\lambda}{P} \left( \frac{\hat{P}}{P} + x \right) = r \frac{\lambda}{P} \quad (8')$$

$$\partial M : \quad -\hat{\lambda} \frac{1}{P} + \beta \frac{\lambda}{P} + \frac{\lambda}{P} \left( \frac{\hat{P}}{P} + x \right) = \psi' v^2 \quad (9')$$

Because the price and money jumps have no effect on interest rates or consumption, no other equations in the model need change. These first order conditions reflect the private agents' use of a probability model that includes jumps in evaluating their objective function.

We can solve the model analytically to see the impact of an unanticipated, permanent, time-0 shift in  $\rho$  (the real rate of return),  $x$  (the inflation scare variable), or  $\bar{\rho}$  (the central bank's interest-rate target). We also solve an expanded version below numerically for given exogenous time paths of those variables.

Solving to eliminate the Lagrange multipliers from the first order conditions we obtain

$$\rho = r - \frac{\hat{P}}{P} - x \quad (10)$$

$$r = \psi'(v) v^2 \quad (11)$$

$$-\frac{d^+}{dt} \left( \frac{1}{C \cdot (1 + \psi + \psi'v)} \right) = \frac{\rho - \beta}{C(1 + \psi + \psi'v)}. \quad (12)$$

Using (10) and the policy rule (5), we obtain that along the path after the initial date,

$$\begin{aligned}\dot{r} &= \theta_r \cdot ((\theta_\pi(r - \rho - x) - r + \bar{r} - \theta_\pi \bar{\pi})) \\ &= \theta_r \cdot (\theta_\pi - 1)r - \theta_r \theta_\pi (\rho + x) + \theta_r (\bar{r} - \theta_\pi \bar{\pi}).\end{aligned}\quad (13)$$

With the usual assumption of active monetary policy,  $\theta_\pi > 1$ , so this is an unstable differential equation in the single endogenous variable  $r$ . Solutions are of the form

$$r_t = E_t \left[ \int_0^\infty e^{-(\theta_\pi - 1)\theta_r s} \theta_r \theta_\pi (\rho_{t+s} + x_{t+s}) ds \right] - \frac{\bar{r} - \theta_\pi \bar{\pi}}{\theta_\pi - 1} + \kappa e^{(\theta_\pi - 1)\theta_r t}.\quad (14)$$

In a steady state with  $x$  and  $\rho$  constant (and  $\kappa = 0$ ), this give us

$$r = \frac{\theta_\pi (\rho + x)}{\theta_\pi - 1} - \frac{\bar{r} - \theta_\pi \bar{\pi}}{\theta_\pi - 1}.\quad (15)$$

From (11) we can find  $v$  as a function of  $r$ . Substituting the government budget constraint into the private budget constraint gives us the social resource constraint

$$C \cdot (1 + \psi(v)) + \dot{F} = \rho F + Y.\quad (16)$$

Solving this unstable differential equation forward gives us

$$F_t = E_t \left[ \int_0^\infty \exp \left( - \int_0^s \rho_{t+v} dv \right) (Y_{t+s} - C_{t+s} (1 + \psi(v_{t+s}))) ds \right].\quad (17)$$

Here we do not include an exponentially explosive term because that would be ruled out by transversality in the agent's problem and by a lower bound on  $F$ . With constant  $\rho$ ,  $x$  and  $Y$ ,  $r$  and therefore  $v$  are constant, and (12) therefore lets us conclude that  $C$  grows (or shrinks) steadily at the rate  $\rho - \beta$ . We can therefore use (17) to conclude that along the solution path, since  $\rho$ ,  $Y$  and  $v$  are constant

$$C_t = \frac{\beta \cdot (\rho^{-1} Y + F_t)}{1 + \psi(v)}.\quad (18)$$

This lets us determine initial  $C_0$  from the  $F_0$  at that date. From then on  $C_t$  grows or shrinks at the rate  $\rho - \beta$  and the resulting saving or dissaving determines the path of  $F_t$  from (18).

## III. UNSTABLE PATHS, UNIQUENESS

Our solution for  $r$ , given by (14), tells us that  $r$  could be constant, but nothing in the model to this point tells us that  $\kappa \neq 0$  is impossible. To assess whether these paths are potential equilibria in the model, we need to specify fiscal policy. The standard sort of fiscal policy to accompany the type of monetary policy we have postulated (Taylor rule with  $\theta_\pi > 1$ ) is a “passive” policy that makes primary surpluses plus seigniorage respond positively to the level of real debt. For example, we can assume

$$\frac{\dot{M}}{P} + \tau = -\phi_0 + \phi_1 \frac{B}{P}. \quad (19)$$

Substituting this into the government budget constraint (4) and using (10) gives us

$$\dot{b} = \left( \rho + x + \frac{\hat{P} - \dot{P}}{P} - \phi_1 \right) b + \phi_0. \quad (20)$$

On an equilibrium path,

$$\frac{\hat{P}}{P} = \frac{\dot{P}}{P},$$

that is, actual inflation and model-based expected inflation are equal. Thus if  $\phi_1 > \rho + x$ , this is a stable differential equation, with  $b$  converging to  $\phi_0 / (\phi_1 - \rho - x)$ . In fact, any  $\phi_1 > 0$  is consistent with equilibrium, even though for small values  $b$  grows exponentially. The transversality condition with respect to debt for the private agent who holds the debt is

$$E_0 \left[ e^{-\beta t} \frac{\lambda B}{P_t} \right] = 0. \quad (21)$$

From (7)  $\lambda$  grows at the rate  $\beta - \rho$ , while from (20)  $b$  grows asymptotically at  $\rho + x - \phi_1$ . However the  $E_0$  in the transversality condition is the private agent's expectation operator. Since the agent believes in the possibility of price jumps, the agent thinks that the expected real return on real debt is just  $\rho$ , not  $\rho + x$ . Thus the agent believes that  $b$  grows asymptotically at the rate  $\beta - \rho - \psi_1$ . The agent's transversality condition is therefore satisfied for any  $\psi_1 > 0$ . The agent in such equilibria has ever-growing wealth,



but at the same time ever-growing taxes that offset that wealth, so that the agent is content with the consumption path defined by the economy's real equilibrium.<sup>4</sup>

A passive fiscal policy with  $\psi_1 > 0$ , therefore, guarantees that all conditions for a private agent optimum are met on any of the paths for prices and interest rates we have derived, including those with  $\kappa > 0$ . The inflation rate (not just the price level) diverges to infinity on such a path, along with the interest rate and velocity. So long as  $r$  is an increasing function of  $v$  ( $\psi''(v)v^2 + 2v\psi'(v) > 0$ ), real balances shrink on these paths and, depending on the specification of the  $\psi(v)$  function, may go to zero in finite time.

With  $\kappa < 0$ , the initial interest rate and inflation rate are below the level consistent with stable inflation and both the price level and the interest rate decline on an exponential path. Since negative nominal interest rates are not possible, it is impossible to maintain the Taylor rule when it prescribes, as it eventually must on such a path, negative interest rates. The simplest modification of the policy rule that accounts for this zero bound on the interest rate, has  $\dot{r}$  follows the right-hand side of (5) whenever this is positive or  $r$  itself is positive, and otherwise sets  $\dot{r}$  to zero. With this specification and the passive fiscal rule (19) the economy has a second steady state (assuming  $\psi_1$  large enough to stabilize  $b$ ), at  $r = 0$ ,  $b = \phi_0 / (\phi_1 - \rho - x)$ . In this steady state inflation is constant at  $-\rho - x$ . This steady state is stable.

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<sup>4</sup>Note that, because the realized real rate of return on debt exceeds that on real assets  $F$ , the properly discounted present value of future taxes exceeds the real value of debt on a path with  $x > 0$ , and may even be infinite. "Ricardian" fiscal policy does not guarantee a match between the present value of future taxes and the current real value of debt on this non-rational-expectations path for the economy.

At this point we have approximately matched the model and conclusions of [Benhabib, Schmitt-Grohé, and Uribe \(2001\)](#): This policy configuration produces a pair of equilibria, with only one globally stable. Because the equilibria with  $\kappa > 0$  cannot be ruled out, and because there are many paths for the economy that converge to the stable  $r = 0$  point, the price level is indeterminate.

However, the indeterminacy can be eliminated if we specify plausible modifications of the policy rules for very high and low inflation rates. The passive fiscal rule, when  $r$  and inflation are on an upward-explosive path, requires that the conventional deficit, which includes interest expense, explode upward asymptotically at the same rate as  $r$ . This is required because inflation is tending to reduce the real value of the debt, so large amounts of additional nominal debt must be sold to the public to keep the real value of the debt on its path converging to  $\phi_0/(\phi_1 - \rho - x)$ . This behavior of the policy authorities is implausible. It is natural to suppose that at very high inflation rates the fiscal authorities would try to restrain the conventional deficit by increasing  $\tau$ , and that the monetary authorities would try to refrain from exacerbating the conventional deficit by continuing to push  $r$  upward. In [Sims \(forthcoming\)](#) it is shown in a model with only interest-bearing debt, no currency or transactions costs, that even a tiny positive coefficient on inflation in the fiscal policy rule would make the explosive paths for inflation unsustainable. In such a model it is also straightforward to describe policies that differ from the standard passive active money, passive fiscal rule only far from steady state and that also deliver a unique equilibrium price level.

In a model like that we use here, with transactions costs and non-interest-bearing currency, the details of a modest policy shift at high or low inflation rates that would guarantee a unique equilibrium price level depend on the way transactions costs behave at very high and very low levels of velocity.

This trigger policy may seem complex, but its essence is simple. If inflation gets too high, A modest fiscal and monetary reform is undertaken that “punishes” market participants who have been expecting ever-accelerating inflation by suddenly, but moderately, increasing the value of the currency. If market participants see from the start that this will happen, the explosive equilibrium path can never get started. If market participants are not so far-sighted, the economy might start down such a path, but as soon as people realized where the economy is headed market forces would restore the stable-price equilibrium.

To eliminate the solution paths that converge to the low-inflation steady state, we can invoke a different sort of realistic modification of the simple active-money, passive-fiscal policy rules. The version of passive fiscal policy in [Benhabib, Schmitt-Grohé, and Uribe \(2001\)](#) makes the primary surplus respond positively to the real value of all government liabilities, both interest-bearing and zero-interest. But this makes little sense. There is no need for taxes to increase with the real value of currency. In our model, as the economy approaches the zero lower bound on nominal interest rates (which it does in finite time on these deflationary paths if the standard Taylor rule remains in place), real balances increase without bound. ( $v \rightarrow 0$  as  $r \rightarrow 0$  and  $C$  does not decrease.) So long as these increased real balances are not offset by correspondingly increased taxation and correspondingly increased net *lending* by the government, they make the market value of private wealth increase without bound in finite time. This violates the private sector transversality condition. Less technically, individuals will not be content with consumption satisfying the economy's resource constraint if the market value of their wealth grows arbitrarily large.

We can conclude that there is no internal contradiction in the conventional practice of treating the price level as uniquely determined in models with a Taylor rule. The justification for doing so, though, must appeal to a

backstop fiscal policy commitment — to tighten fiscal policy if inflation becomes too high, and to allow the real value of currency to increase without bound, without raising taxes, if deflation takes hold. Central banks should therefore not be structured to have no institutional link to the treasury, and central bankers should not suggest in their public statements that they can control inflation regardless of fiscal policy.

#### IV. FOUR LEVELS OF CENTRAL BANK BALANCE SHEET PROBLEMS

So far, we have said nothing about the central bank balance sheet, but with the solution path for the economy in hand, assessing the time path of the balance sheet is straightforward. The most severe problem, which we can call level 4, is simply the possible indeterminacy of the price level. To put this in the language of the central bank balance sheet, this is the point that the central bank's assets consist of the market value of its assets and its potential seigniorage, both of which are valueless if currency is valueless. But if it holds nominal debt as assets and issues reserves and currency as liabilities, the central bank has no lever to guarantee the real value of either side of its balance sheet. If the public were to cease to accept currency in payment, it would become valueless, as would both sides of the central bank balance sheet. That this cannot happen, either suddenly or as the end point of a dynamic process, depends on fiscal commitments beyond the central bank's control.

The fiscal backing required for price level determinacy seems quite plausible in the US. In Europe, because fiscal responsibility for the Euro is divided among many countries that seem bent on frequently increasing doubts about their ability to cooperate on fiscal matters, the possibility of a breakdown of the value of the Euro from this source cannot be entirely ruled out.

The next level of possible problem, level 3, arises because the notion of determinacy via a backstop fiscal commitment assumes that the central

bank could maintain its commitment to an active policy rule during an inflationary excursion from the unique stable price path, up to the point that fiscal backing is triggered. If we think of a unified government budget constraint and jointly determined monetary and fiscal policy, this is not an issue. But if the central bank is concerned to maintain its policies without requiring a direct capital injection from the treasury, or possibly even without ever having to set its seigniorage payments to the treasury to zero, then this could be a problem. And of course if markets perceive that the central bank will abandon its policy rule to avoid having to seek treasury support, this undercuts the argument for price determinacy. Showing formally how these issues arise requires solving the model for time varying paths of interest rates and velocity, so it is postponed to later sections of the paper.

If the market value of the assets of the central bank fall to a value below that of their interest-bearing liabilities, it is possible that adherence to the bank's policy rule is impossible without a direct injection of capital. This is only a possibility, however, because the bank has an implicit asset in its future seigniorage. Even with assets below interest-bearing liabilities at market value, the bank may be able to meet all its interest-paying obligations and to restore the asset side of its balance sheet through accumulation of seigniorage. Whether it can do so depends on its policy rule and on the interest-elasticity of demand for currency (or more generally, for its non-interest-bearing liabilities). This issue, of whether the central bank might require a capital injection to maintain adherence to its policy rule in a determinate-price-level equilibrium, is a level 2 balance sheet problem.

Finally, the central bank may be solvent in the sense that with the existing policy rule its assets at market value plus future seigniorage exceed its total liabilities, yet following standard accounting rules and rules for determining how much seigniorage revenue is sent to the treasury each period may lead to episodes of zero seigniorage payments to the treasury.

Extended episodes of this type might be thought to raise issues of political economy, if they led to public criticism of the central bank or to calls for revising its governance.

## V. INFLATION SCARE IN THE SIMPLE MODEL

Our first numerical example uses this simple model to compare a steady state with  $\rho = \beta = \bar{\rho} = .01$  and  $x = 0$  to one in which  $x$  jumps up to  $.02$  at time  $0$ . This is an “inflation scare” scenario. The 2% per year inflation scare shock produces a much larger increase in the nominal interest rate, because the increased inflation expectations shrink demand for money and thereby produce inflation, which prompts the central bank to raise rates further. If the duration of the nominal assets on the central bank’s balance sheet is positive, the permanent rise in rates reduces the time  $0$  market value of the central bank’s assets. The simple model treats the debt as of maturity  $0$ , but this has no consequence except for the initial date capital losses, because for  $t > 0$  the perfect-foresight path requires that long and short debt has the same time path of returns. We show two cases: initial assets of the central bank  $A_0$  are three times the amount of currency outstanding or six times the amount of currency outstanding with the initial deposit liabilities  $V_0$  plus currency matching  $A_0$  in each case.

The nominal capital losses, as a proportion of the new value of the assets, are shown in Table IX. There cannot be any “level 2” problem for the central bank unless the interest increase pushes its initial assets  $A$  below  $V$ . That is, it not only has to have assets less than liabilities  $V + M$ , where  $M$  is currency, it has to have  $A < V$  in order for a level 2 problem to arise. The rise in interest rate reduces the demand for  $M$ , which has to be met either by an decrease in  $A$  through open market sales or an increase in  $V$ . This will dampen the effect on  $V - A$  if the rate rise. The value of  $V - B$  is shown as the “gap” line in Table IX. Whether a level 2 problem actually

arises then depends on the discounted present value of the seigniorage after the initial date, shown as “dpvs” in the table. For this example, even though the gap between  $V$  and  $B$  gets quite large if we assume long durations for the assets, the gap exceeds the discounted present value of seigniorage only for durations of 10 years or more and for the (unrealistically large) balance sheet with  $A_0$  six times outstanding currency.

This example should make it clear that the central bank can suffer very substantial capital losses without needing direct recapitalization. On the other hand, it shows that there are drawbacks to extreme expansion of central bank holdings of long-maturity debt — an expanded balance sheet increases the probability that interest rate changes could require a direct capital injection.

This simple model has omitted two sources of seigniorage, population growth and technical progress, and has considered only a single, stylized, shock to the balance sheet. We now expand the model to include these extra elements and calibrate the parameters and the nature of the shocks more carefully to the situation of the US Federal Reserve. Of course our ability to calibrate is limited by the sensitivity of results to the transactions cost function. We have little relevant historical experience with currency demand at low or very high interest rates. Rates were very low in the 1930's and the early 1950's, but the technology for making non-currency transactions is very different now. It is difficult to predict how much and how fast people would shift toward, say, interest-bearing pre-loaded cash cards as currency replacements if interest rates increased to historically normal levels. We can at best show ranges of plausible results.

## VI. THE MODEL

The model borrows from [Sims \(2005\)](#). The household planner (whose utility includes that of offspring, see [Barro and Sala-i Martin \(2004\)](#)) maximizes:

$$\int_0^{\infty} e^{-(\beta-n)t} \log(C_t) dt \quad (22)$$

where  $C_t$  is per capita consumption,  $\beta$  is the discount rate, and  $n$  is population growth, subject to the budget constraint:

$$C_t(1 + \psi(v_t)) + \dot{F}_t + \frac{\dot{V}_t + \dot{M}_t + q_t \dot{B}^P}{P_t} = \\ Y e^{\gamma t} + (\rho_t - n)F_t + (r_t - n) \frac{V_t}{P_t} + (\chi + \delta - q_t \delta - n) \frac{B^P}{P} - n \frac{M_t}{P_t} - \tau_t. \quad (23)$$

We express all variables in per-capita terms and initial population is normalized to one.  $F_t$  and  $B_t^P$  are foreign assets and long-term government bonds in the hand of the public, respectively,  $V_t$  denotes central bank reserves,  $M_t$  is currency,  $\tau_t$  is lump-sum taxes,  $Y$  is an exogenous income stream growing at rate  $\gamma$ . Foreign assets and central bank reserves pay an exogenous real return  $\rho$  and a nominal return  $r_t$ , respectively. Long term bonds are modeled as in [Woodford \(2001\)](#). They are assumed to depreciate at rate  $\delta$  ( $\delta^{-1}$  captures the bonds average maturity) and pay a nominal coupon  $\chi + \delta$ .<sup>5</sup>

The government is divided into two distinct agencies called “central bank” and “fiscal authority”. The central bank’s budget constraint is

$$\left( q_t \frac{\dot{B}^C}{P_t} - \frac{\dot{V}_t + \dot{M}_t}{P_t} \right) e^{nt} \\ = \left( (\chi + \delta - \delta q_t - n q_t) \frac{B^C}{P} - (r_t - n) \frac{V_t}{P_t} + n \frac{M_t}{P_t} - \tau_t^C \right) e^{nt}. \quad (24)$$

where  $B_t^C$  are long-term government bonds owned by the central bank, and  $\tau_t^C$  are remittances from the central bank to the fiscal authority. The central

<sup>5</sup>We write the coupon as  $\chi + \delta$  so that at steady state if  $\chi$  equals the short term rate the bonds sell at par.



bank is assumed to follow the rule (5) for setting  $r_t$ , the interest on reserves. The central bank is also assumed to follow a rule for remittances, which embodies two principles: i) remittances cannot be negative, ii) whenever positive, remittances are such that the central bank capital (assets minus liabilities) remains constant in nominal terms over time (Hall and Reis (2013) use a similar rule), that is:

$$\left( q_t B_t^C - V_t - M_t \right) e^{nt} = \text{constant}. \quad (25)$$

Differentiating the condition above and plugging the resulting expression into the central bank's budget constraint, one can see that the two principles result in the following rule for remittances:

$$\tau_t^C = \max \left\{ 0, (\chi + (1 - q_t)\delta + \dot{q}_t) \frac{B_t^C}{P_t} - r_t \frac{V_t}{P_t} \right\}. \quad (26)$$

Solving the central bank's budget constraint forward we can obtain its intertemporal budget constraint:

$$q \frac{B_0^C}{P_0} - \frac{V_0}{P_0} + \int_0^\infty \left( \frac{\dot{M}_t}{M_t} + n \right) \frac{M_t}{P_t} e^{-\int_0^t (\rho_s - n) ds} dt = \int_0^\infty \tau_t^C e^{-\int_0^t (\rho_s - n) ds} dt. \quad (27)$$

Equation (27) shows that, regardless of the rule for remittances, their discounted present value  $\int_0^\infty \tau_t^C e^{-\int_0^t (\rho_s - n) ds} dt$  has to equal its left hand side, namely the market value of assets minus reserves plus the discounted present value of seigniorage  $\int_0^\infty \left( \frac{\dot{M}_t}{M_t} + n \right) \frac{M_t}{P_t} e^{-\int_0^t (\rho_s - n) ds} dt$ . We can also compute the constant level of remittances  $\bar{\tau}^C e^{\gamma t}$  (taking productivity growth into account) that satisfies expression (27).

$$\bar{\tau}^C = \left( \int_0^\infty e^{(\gamma+n)t - \int_0^t \rho_s ds} dt \right)^{-1} \left( q \frac{B^C}{P} - \frac{V}{P} + \int_0^\infty \left( \frac{\dot{M}_t}{M_t} + n \right) \frac{M_t}{P_t} e^{-\int_0^t (\rho_s - n) ds} dt \right). \quad (28)$$

We also need to specify the central bank's policy in terms of the asset side of its balance sheet  $B_t^C$ . We assume that these follow some exogenous process  $B_t^C = \bar{B}_t^C$ . Government debt is assumed to be held either by the

central bank or the public:  $B_t = B_t^C + B_t^P$ . The budget constraint of the fiscal authority is

$$\left( G_t - \tau_t + (\chi + \delta - \delta q_t - nq_t) \frac{B}{P} \right) e^{nt} = \left( \tau_t^C + q_t \frac{\dot{B}}{P_t} \right) e^{nt}, \quad (29)$$

where  $G_t$  is government spending. The rule for  $\tau_t$  is given by:

$$\tau_t = \xi_0 e^{\gamma t} + (\xi_1 + n + \gamma) \left( q \frac{B^P}{P} + \frac{V}{P} \right). \quad (30)$$

This rule makes the debt to GDP ratio  $b_t = \left( q \frac{B^P}{P} + \frac{V}{P} \right) e^{-\gamma t}$  converge as long as  $\xi_1 > \beta - n$ . The initial level of foreign assets in the hand of the public, central bank reserves, and currency are  $F_0^P$ ,  $V_0$ , and  $M_0$ , respectively.

As in the simple model the first order condition for the household's problem with respect to  $C$ ,  $F^P$ ,  $B$ ,  $V$ , and  $M$  yield the Euler equation (12), the Fisher equation (10),<sup>6</sup> the money demand equation (11), and the arbitrage condition between reserves and long-term bonds:

$$\frac{\chi + \delta}{q} - \delta + \frac{\dot{q}}{q} = r. \quad (31)$$

The solutions for  $r$  is given by equation (14), and those for inflation  $\frac{\dot{P}}{P}$  and velocity  $v$  follow from equations (10) and (11), respectively. The growth rate of consumption  $\frac{\dot{C}}{C}$ , is given by

$$\frac{\dot{C}}{C} = (\rho - \beta) - \frac{2\psi'(v) + v\psi''(v)}{1 + \psi(v) + v\psi'(v)} \dot{v}, \quad (32)$$

which obtains from differentiating expression (12). Differentiating the definition of velocity (3) we obtain an expression for the growth rate of currency:

$$\frac{\dot{M}}{M} = \frac{\dot{P}}{P} + \frac{\dot{C}}{C} - \frac{\dot{v}}{v}. \quad (33)$$

<sup>6</sup>Note that short term debt was called  $B$  in the simple model, and was issued by the fiscal authority. Here it is called  $V$ , and is issued by the central bank.

The economy's resource constraint is given by

$$C(1 + \psi(v)) + \dot{F} = (Y - G)e^{\gamma t} + (\rho - n)F, \quad (34)$$

where  $F = F^P + F^C$  is the aggregate amount of foreign assets held in the economy (we assume that the central banks foreign reserves  $F^C$  are zero), and where we assumed  $G_t = Ge^{\gamma t}$ . Solving this equation forward we obtain a solution for consumption in the initial period:

$$C_0 \left( \int_0^\infty (1 + \psi(v)) e^{-\int_0^t (\rho_s - \dot{c} - n) ds} dt \right) = F_0 + (Y - g) \int_0^\infty e^{(\gamma+n)t - \int_0^t \rho_s ds} dt, \quad (35)$$

Given velocity  $v$  and the level of consumption, we can compute real money balances  $\frac{\dot{M}}{P}$ , the initial price level  $P_0$ , and seigniorage  $\frac{\dot{M}}{P} + n \frac{M}{P} = \left( \frac{\dot{M}}{M} + n \right) \frac{M}{P}$  (using (33)), and the present discounted value of seigniorage

$$\int_0^\infty \left( \frac{\dot{M}}{M} + n \right) \frac{M}{P} e^{-\int_0^t (\rho_s - n) ds} dt = c_0 \int_0^\infty \left( \frac{\dot{M}}{M} + n \right) v^{-1} e^{-\int_0^t (\rho_s - \dot{c} - n) ds} dt.$$

Finally, solving (31) forward we find the current nominal value of long-term bonds

$$q_0 = (\chi + \delta) \int_0^\infty e^{-\left(\int_0^t r_s ds + \delta t\right)} dt. \quad (36)$$

**VI.1. Steady state.** At a steady state where  $\bar{\rho} = \beta + \gamma$ ,  $\bar{r} = \bar{\rho} + \bar{\pi}$ ,  $\bar{v}$  satisfies  $\bar{v}^2 \psi'(\bar{v}) = r_{ss}$ . Steady state consumption is given by  $\bar{C}_t = \bar{C}_0 e^{\gamma t}$  where  $\bar{C}_0 = \frac{(\beta - n)F_0 + Y - G}{1 + \psi(\bar{v})}$ , and real money balances are given by  $\frac{M}{P}_{ss} = \frac{\bar{C}_0}{\bar{v}} e^{\gamma t}$ . seigniorage is given by  $(\bar{\pi} + \gamma + n) \frac{\bar{C}_0}{\bar{v}} e^{(\gamma+n)t}$  and its present discounted value is given by  $(\bar{\pi} + \gamma + n) \frac{\bar{C}_0}{\bar{v}(\beta - n)}$ .

**VI.2. Central bank's solvency, accounting, and the rule for remittances.**

For some of the papers discussed in the introduction the issue of central bank's solvency is simply not taken into consideration: the worst that can happen is that the fiscal authority may face an uneven path of remittances, with possibly no remittances at all for an extended period. We acknowledge

the possibility that remittances may have to be negative, at least at some point. This is what we mean by solvency.

Like [Bassetto and Messer \(2013\)](#), we approach the issue of central bank's solvency from a present discounted value perspective. If the left hand side of equation (27) is negative, the central bank cannot face its obligations, i.e., pay back reserves, without the support of the fiscal authority. An interesting aspect of equation (27) is that its left hand side does *not* depend on many of aspects of central bank policy that are recurrent in debates about the fiscal consequences of central bank's balance sheet policy. For instance, the future path of  $B_t^C$  does not enter this equation: whether the central bank holds its assets to maturity or not, for instance, is irrelevant from an expected present value perspective. Intuitively, the current price  $q_t$  contains all relevant information about the future income from the asset relative to the opportunity cost  $r_t$ . Whether the central bank decides to sell the assets and realize gains or losses, or keep the assets in its portfolio and finance it via reserves, does not matter. Similarly, whether the central bank incurs negative income in any given period, and accumulates a "deferred asset", is irrelevant from the perspective of the overall present discounted value of resources transferred to the fiscal authority.<sup>7</sup> In fact, we will see later that in some cases scenarios associated with higher remittances in terms of present value are also associated with a deferred asset.

Finally, the issue of "remittances smoothing" is also, from a purely economic point of view, a non issue. In perfect foresight the central bank can always choose a perfectly smooth path of remittances (in fact, this is  $\tau_t^C = \bar{\tau}^C e^{\gamma t}$ ). In a stochastic environment Barro's results on tax smoothing would apply: remittances would move with innovations to the the left

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<sup>7</sup>As we will see later central bank accounting does not let negative income affect capital. The budget constraint (24) implies however that negative income results in either more liabilities or less assets. In order to maintain capital intact, a deferred asset is therefore created on the asset side of the balance sheet.

hand side of equation (27). But there are accounting rules governing central banks' remittances. Hence these may not be smooth and may depend on the central bank's actions, such as holding the assets to maturity or not. We recognize that the timing of remittances can matter for a variety of reasons: tax smoothing, political pressures on the central bank, *et cetera*. For this reason we assume a specific rule for remittances that very loosely matches those adopted by actual central banks and compute simulated paths of remittances under different assumptions. Appendix A.1 discusses this rule.

**VI.3. Functional forms and parameters.** Table 2 shows the model parameters. We normalize  $Y - g$  to be equal to 1, and set  $F_0$  to 0.<sup>8</sup> Since we do not have investment in our model, and  $F_0 = 0$ ,  $Y - G$  in the model corresponds to national income  $Y$  minus government spending  $G$  in the data (data are from Haver analytics, mnemonics are  $Y@USNA$  and  $G@USNA$ , respectively). All real quantities discussed in the remainder of the paper should therefore be understood as multiples of  $Y - G$ , and their data counterparts are going to be expressed as a fraction of national income minus government spending (\$ 11492 bn in 2013Q3). Our  $t = 0$  corresponds to the beginning of 2014. We therefore measure our starting values for the face value of central bank assets  $\frac{B^C}{P}$ , reserves  $\frac{V}{P}$ , and currency  $\frac{M}{P}$  using the January 3, 2014 H.4.1 report (<http://www.federalreserve.gov/releases/h41/>), which measures the Security Open Market Account (SOMA) assets.<sup>9</sup> The model parameters are chosen as follows. The discount rate  $\beta$ , productivity growth  $\gamma$ ,

<sup>8</sup>Note from the steady state calculations that we could choose  $F_0 \neq 0$  and use instead the normalization  $(\beta - n)F_0 + y - g = 1$ , hence setting  $F_0 \neq 0$  simply implies a different normalization.

<sup>9</sup>The January 3, 2014 H.4.1 reports the face value of Treasury (\$ 2208.791 bn), GSE debt securities (\$ 57.221 bn), and Federal Agency and GSE MBS (\$ 1490.160 bn), implying that  $B_0^C$  is \$ 3756.172 bn, the value of reserves  $V$  (deposits of depository institutions, \$ 2374.633 bn) and currency  $M$  (Federal Reserve notes outstanding, net, \$ 1194.969 bn).

and population growth  $n$  are 1 percent, 1 percent, and .75 percent, respectively. These values are consistent with Carpenter et al.'s assumptions of a 2% steady state real rate.

The policy rule has inflation and interest rate smoothing coefficients  $\theta_\pi$  and  $\theta_r$  of 2 and 1, respectively, which are roughly consistent with those of interest feedback rules in estimated DSGE models (e.g., [Del Negro, Schorfheide, Smets, and Wouters \(2007\)](#)); note that  $\theta_r = 1$  corresponds to an interest rate smoothing coefficient of .78 for a policy rule estimated with quarterly data). The inflation target  $\theta_\pi$  is 2 percent (hence  $\theta_0 = \beta + \gamma - (\theta_\pi - 1)\pi_{ss} = -.0025$ ). We use the functional form

$$\psi(v) = \psi_0 \frac{v}{1 + \psi_1 v} \quad (37)$$

for the transaction costs, with  $\psi_0 = 2 \times 10^{-6}$  and  $\psi_1 = -0.055$ . [Figure 1](#) shows the scatter plot of quarterly  $\frac{M}{PC} = v^{-1}$  and the annualized 3-month TBill rate in the data (where  $M$  is currency and  $PC$  is measured by nominal PCE)<sup>10</sup>, where blue crosses are post-1980 data, and crosses are 1947-1980 data (arguably less relevant). The black curve in [figure 1](#) shows the relationship between velocity and interest rates as implied by the model (equation (11)). The parameters  $\psi_0$  and  $\psi_1$  are chosen to i) match currency demand in real terms  $\frac{M}{P}$  in 2013Q4 at current rates ( $r_0 = .0025$ ),<sup>11</sup> ii) so that  $\frac{M}{PC} = v^{-1}$  asymptotes at  $\psi_1 = .055$  when rates go to infinity, which implies that model-implied velocity is roughly in line with the experience of the early 1980s, as shown in [figure 1](#). The implied transaction costs at steady state are negligible - about .03 percent of Y-G. We will later also consider

<sup>10</sup>Data are from *Haver*, with mnemonics C@USNA, FMCN@USECON, and FTBS3@USECON for PCE, currency, and the Tbill rate, respectively.

<sup>11</sup>Matching inverse velocity  $\frac{M}{PC}$  in 2013Q4 as opposed to real money demand  $\frac{M}{P}$  yields very similar results.

alternative parameterizations of currency demand.<sup>12</sup> Finally, we choose  $\chi$  – the average coupon on the central bank's assets – to be 3.5 percent, roughly in line with the numbers reported in figure 6 of [Carpenter, Ihrig, Klee, Quinn, and Boote \(2013\)](#). Chart 17 of the April 2013 FRBNY report on "Domestic Open Market Operations during 2012"<sup>13</sup> shows an average duration of 6 years for SOMA assets (SOMA is the System Open Market Account, which represents the vast majority of the Federal Reserve balance sheet). We therefore set  $1/\delta = 6$ .

## VII. SIMULATIONS

As a baseline simulation we choose a time-varying path of short term nominal interest rates that roughly corresponds to the baseline interest path in [Carpenter, Ihrig, Klee, Quinn, and Boote \(2013\)](#). We generate this path by assuming that the real rate  $\rho_t$  remains at a low level  $\rho_0$  for a period of time  $T_0$  equal to five years, and then reverts to the steady state  $\bar{\rho}$  at the rate  $\varphi_1$ :

$$\rho_t = \begin{cases} \rho_0, & \text{for } t \in [0, T_0] \\ \bar{\rho} + (\rho_0 - \bar{\rho})e^{-\varphi_1(t-T_0)}, & \text{for } t > T_0. \end{cases} \quad (38)$$

Given the path for  $\rho_t$ , equation (14) generates the path for the nominal short term rate (we set  $\kappa = 0$  for the baseline simulation). The baseline paths of  $\rho_t$ ,  $r_t$  and inflation  $\pi_t$  are shown as the solid black lines in the three panels of Figure 2.

<sup>12</sup>We have performed non-linear least squares regression of equation (11) using the interest rate and velocity data shown in figure 1. Some of the estimates of  $\psi_0$  and  $\psi_1$  – particularly those using post-1980 data – are quite close to those reported in table 2. These estimates generally yield a value of  $\psi_1$  close to  $-.06$  in order to fit the high inflation data of the early 1980s, which we as implying too large an asymptote for  $\frac{M}{PC} = v^{-1}$  if rates were to become very large, in light of current transaction technology.

<sup>13</sup><http://www.newyorkfed.org/markets/omo/omo2012.pdf>.

Given the path for  $\rho_t$  and  $r_t$  we can compute  $q$  and the amount of resources, both in terms of marketable assets and present value of future seigniorage, in the hands of the central bank. The first row of table 3 shows the two components of the left hand side of equation (27), namely the market value of assets minus reserves (column 1) and the discounted present value of seigniorage  $\int_0^\infty \left(\frac{\dot{M}}{M} + n\right) \frac{M}{P} e^{\int_0^t (\rho_s - n) ds} dt$  (column 2). The third column shows the sum of the two, which has to equal the discounted present value of remittances  $\int_0^\infty \tau^C e^{\int_0^t (\rho_s - n) ds} dt$ . Column 4 shows  $\bar{\tau}^C$  as defined in equation (28): the constant level of remittances (accounting for the trend in productivity) that would satisfy equation (27), expressed as a fraction of Y-G like all other real variables.<sup>14</sup> Last, in order to provide information about how the numbers in column 1 are constructed, column 5 shows the nominal price of long term bonds  $q$  at time 0.

Under the baseline the real value of the central bank's assets minus liabilities is 14.6 percent of Y-G – which is larger than the difference between the par value of assets minus reserves reported in table 2 given that  $q$  is above one under the baseline. Its value is 1.08, which is above the 1.04 ratio of market over par value of assets reported in [Federal Reserve System \(2014\)](#)<sup>15</sup> The discounted present value of seigniorage is almost an order of magnitude larger, however, at about 99 percent of Y-G, and represents the bulk of the central bank resources (and therefore the present discounted value of remittances), which are 114 percent of Y-G. The constant (in productivity units) level of remittances  $\bar{\tau}^C$  that satisfies the present value relationship is .26 percent of Y-G, about \$ 29 bn per year, quite lower than the amount remitted for 2013 and 2012 according to [Federal Reserve System \(2014\)](#) (\$ 79.6 and \$ 88.4 bn, respectively).

<sup>14</sup>That is, the amount  $\bar{\tau}^C$  such that  $\tau_t^C = \bar{\tau}^C e^{\gamma t}$  satisfies the present value relationship.

<sup>15</sup>Page 23 and 29 shows the par and market (fair) value of Treasury and GSE debt securities, and Federal Agency and GSE MBS, respectively.



The left and right panels of Figures 3 show inverse velocity  $M/PC$  and seigniorage, expressed as a fraction of  $Y-G$ , in the data (1980-2013) and in the model (under the baseline simulation), respectively. A comparison of the two figures shows that the drop in  $M/PC$  as interest rates renormalize under the baseline simulation (from about .085 to .065, left axis) is roughly as large as the rise in  $M/PC$  as interest rates fell from 2008 to 2013. Partly because the model may likely over-predict the fall in currency demand, and more importantly because consumption declines (real interest rates are very low at time 0, inducing unrealistic above trend consumption), seigniorage falls to negative territory for roughly six years. After that, it converges to almost .3 percent of  $Y-G$ , a level that is in the low range of the post-1980 observations. For both reasons the present discounted value of seigniorage reported in table 2 for the baseline simulation is likely to be a fairly conservative estimate.

Finally, the left panel of Figure 4 shows remittances (computed as described in section A.1) under two scenarios for the path of assets  $B^C$ : in the first scenario (solid line) the central bank lets its assets depreciate, while in the second one it actively sells assets at a rate of 20 percent per year. These scenarios highlight the fact that different paths for the balance sheet can imply different paths for remittances, even though their expected present value remains the same (this is the dotted line in figure 4, which shows  $\bar{\tau}^C e^{\gamma t}$ ).

Next, we consider alternative simulations where the economy is subject to different “shocks.” In each of these simulations all uncertainty is revealed at time 0, at which point the private sector will change its consumption and portfolio decisions and prices will adjust. We will use the subscript  $0^-$  to refer to the pre-shocks quantities and prices (that is, the time 0 quantities and prices under the baseline simulation). For each simulation, Table 3 will report the new market value of assets minus reserves in real term

$(q_0 \frac{B_{0-}^C}{P_0} - \frac{V_0}{P_0})$ . By assumption the central bank will not change its assets  $B_{0-}^C$  after the new information is revealed, but the private sector will change its time 0 currency holdings given that interest rates may have changed. This necessarily leads to a change in reserves (given that central bank's assets are unchanged) equal to  $\frac{V_0 - V_{0-}}{P_0} = -\frac{M_0 - M_{0-}}{P_0}$  in real terms. We report this quantity separately in column 5.

Last, for each scenario we also report the level of the balance sheet  $\bar{B}^C$  such that, for any balance sheet size larger than  $\bar{B}^C$ , the present discounted value of remittances (see equation (27)) becomes negative after the shock. We refer to this situation as the central bank becoming “insolvent”, in the sense that at some point it will need resources from the fiscal authority. Specifically, assume the central bank expands its balance sheet by  $\Delta B^C$  at time  $0^-$  (right before the shock takes place) by buying assets at price  $q_{0-}$  and pays with it by expanding reserves by an amount  $\Delta V = q_{0-} \Delta B^C$ . How large can  $\Delta B^C$  be to still satisfy

$$\frac{q_0 (B^C + \Delta B^C) - V - \Delta V}{P_0} + \frac{M_0 - M_{0-}}{P_0} + \int_0^\infty \left( \frac{\dot{M}_t}{M_t} + n \right) \frac{M_t}{P_t} e^{-\int_0^t (\rho_s - n) ds} dt \geq 0 \quad (39)$$

after the “shock”? We report  $\bar{B}/B^C = 1 + \frac{\Delta B^C}{B^C}$ , where  $B^C$  is the 2013Q4 level of the balance sheet reported in table 2. Of course, the reason why with a larger balance sheet the central bank may become insolvent is that  $q$  is lower under the alternative simulations, and hence the central bank may experience losses, in addition to possibly having less seigniorage in present value.

The first alternative scenario we consider is a “higher rates” path similar to one considered by [Carpenter, Ihrig, Klee, Quinn, and Boote \(2013\)](#). Under this new path real rates converge to a 1 percent higher steady state, and so

will short term nominal rates given that the central bank inflation target has not changed. We choose the new starting value for  $\rho$ ,  $\rho_0$ , so that the initial rate remains at 25 basis points. The red solid lines in the three panels of Figure 2 show the “Higher Rates” paths for the nominal and the real short term rates and inflation, respectively. In these simulations we assume that the central bank recognizes the change in the steady state  $\bar{\rho} = \beta + \gamma$ , and adjusts its Taylor rule coefficient  $\bar{r} = \bar{\rho} + \bar{\pi}$  accordingly.

We consider two different reasons why the new steady state  $\rho$  is higher: a higher discount rate  $\beta$  and a higher growth rate of technology  $\gamma$ . While the new value of  $q$  is the same in both cases (the interest rate path is the same), the present value of seigniorage shown in column 2, and therefore the present value of remittances shown in column 3, is quite different. In the high  $\beta$  case the current value of the future income from seigniorage falls by almost one order of magnitude, as future seigniorage is discounted at a higher real rate. In the high  $\gamma$  case the economy is growing faster, and so does money demand and future seigniorage. As it turns out, in both cases (higher  $\beta$  and higher  $\gamma$ ) the level of  $\bar{\tau}^C$  is higher than in the baseline case. This may seem surprising in the higher  $\beta$  case since the present value of seigniorage is lower than under the baseline simulation. However, the central bank is now earning a higher return on its assets, and can therefore afford a higher level of remittances.<sup>16</sup>

We also consider the case where the central bank does not recognize that the steady state real rate has increased, and therefore leaves  $\bar{r}$  unchanged in its reaction function. This is an extreme version of the more realistic case

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<sup>16</sup>Note that our conclusion is different from that reached in the analysis of [Carpenter, Ihrig, Klee, Quinn, and Boote \(2013\)](#), which take seigniorage as given and focus on the effect of the higher nominal interest rates on the value of the central bank's assets  $qB^C$ , which falls following the drop in  $q$ . The effect of the higher real rate of return on future central bank's revenues and, especially in the high  $\gamma$  case, on future seigniorage, trumps in our simulation the negative effect on  $q$ .

where the central bank adjusts its reaction function slowly to changes in the real economy (see Orphanides (2002)). The red dash-and-dotted lines in Figure 2 show the paths of nominal interest rates and inflation, respectively, in this scenario. Inflation is higher relative to the case where the central bank changes  $\bar{r}$  (solid red lines) because the central bank's reaction function calls for rates that are too low. In equilibrium, since inflation is higher, rates end up being also higher. As a consequence,  $q$  falls by more than in the case where the central bank adjusts  $\bar{r}$  (see the rows labeled "Higher rates/same intercept" in Table 3). In addition, because of the higher interest rates the private sector is no longer willing to hold as much currency and turns it into reserves, and hence a higher fraction of the central bank's liabilities becomes interest bearing relative to the baseline scenario. Because of the higher inflation, however, given our assumptions about money demand seigniorage is higher (compare rows 2 and 4 for the higher  $\beta$  case, and 3 and 5 for the higher  $\gamma$  case, of Table 3). As a consequence, the fall in  $q$  does not raise any solvency issues under these scenarios, which the central bank could have withstood even with a balance sheet more than five times as large as the actual one (see column 7). In fact, average yearly remittances  $\bar{\tau}^C$  increase by about .10 percent of Y-G, roughly \$ 10 bn.

The results are very sensitive to the inflation response in the policy reaction function. The blue dash-and-dotted and dotted blue lines in the top left panel of Figure 5 show the interest rate path corresponding to an inflation coefficient  $\theta_\pi$  of 3 and 1.05.<sup>17</sup> As is usually the case in stable rational expectations equilibria, a higher inflation coefficient in the interest rate rule induces a lower equilibrium response of inflation, and therefore a lower equilibrium response of interest rates – and vice versa when the inflation response is lower. In fact, we see that when  $\theta_\pi$  is 1.05, interest rates reach 25

<sup>17</sup>In these simulations we change the time 0 real rate so that under the baseline scenario the nominal rate is still 25 basis points.

percent. Consequently,  $q$  falls to less than half its value under the baseline scenario, and the market value of assets minus reserves  $q_0 \frac{B_{0-}^C}{P_0} - \frac{V_0}{P_0}$  falls to negative levels (see row 11 of Table 3). The implication of this finding is that under a large balance sheet the central bank may want to respond more aggressively to inflation, if it is concerned about fluctuations in the values of its assets.

Even in the  $\theta_\pi = 1.05$  case central bank's solvency is not an issue, however. The present value of seigniorage has a roughly fivefold increase relative to the  $\theta_\pi = 2$  case, so in spite of the large decline in the value of assets, the present value of remittances remains positive. In fact, the present value of remittances would remain positive even if we assumed the central bank balance sheet to be more than three times as large as the current one (column 7). We discuss later what would happen in this scenario under a less favorable outlook for seigniorage (induced by a different parameterization of the money demand function).

Next, we consider simulations where for a given period of time (10 years) the private sector is concerned about a sudden jump in the price level, and therefore demands a premium  $x$  for holding nominal bonds. We call these scenarios "inflation scares" and set  $x$  to 2 percent. The red solid lines in the top right and bottom left panels of Figure 5 show what happens to short term nominal interest rates and inflation, respectively, under this scenario. Because of the higher inflation expectations the central bank, which follows the interest rule, is forced to raise nominal rates over the baseline path. This scenario has a number of effects on the central bank's balance sheet, as shown in row 6 of Table 3. The market value of assets minus reserves  $q_0 \frac{B_{0-}^C}{P_0} - \frac{V_0}{P_0}$  drops to less than half its baseline value, both because  $q$  falls and because the private sector turns currency into reserves. However, the present discounted value of seigniorage computed under our assumptions on money demand is slightly higher than under the baseline case. The

fact that seigniorage holds up, and that its present discounted value is large, implies that the present value of resources in the hands of the central bank is not much affected under the inflation scare simulations. As a consequence, even with a much larger balances sheet the central bank could have withstood the fall in the value of its assets without ever needing any resources from Fiscal Authority, at least in expectations. Also, because seigniorage is still large the decrease in average remittances  $\bar{\tau}^C$  is small relative to the baseline case, from .26 to .24 percent of Y-G.

The blue lines in the top right and bottom left panels of Figures 5 show the responses of interest rates and inflation, respectively, under policy rules different from the baseline for this scenario. As in the “Higher rates/same intercept” case, if policy reacts more aggressively to inflation than under the baseline in equilibrium inflation and short term interest rise less, and vice versa if policy reacts less aggressively to inflation. Rows 9 and 12 of Table 3 document the effect of these alternative policies on the central bank’s balance sheet and show that, again not surprisingly, the effect of the “inflation scare” simulation on  $q$  is stronger the lower the response to inflation  $\theta_\pi$ . In fact, under the lower  $\theta_\pi$  policy, the market value of assets and reserves becomes negative. The central bank’s overall resources (column 3) are still sizable, and average remittances are the same as under the  $\theta_\pi = 2$  policy. Again, this is because the higher inflation experienced under the lower  $\theta_\pi$  policy yields greater seigniorage (column 2).

Finally, we consider explosive paths where  $\kappa$  in equation (14) is different from zero. The solid red line in the bottom right panel of Figure 5 shows one of these paths (with  $\kappa = 10^{-4}$ ) under the baseline policy response. Given the rise in  $r_t$  under this scenario,  $q$  drops by .2 relative to the baseline (row 7 of Table 3). However,  $q_0 \frac{B_0^C}{P_0} - \frac{V_0}{P_0}$  increases to almost four times Y-G. The reason for this counterintuitive result is that the under our assumption

for money demand transaction costs explode as rates reach infinity. Therefore agents will front-load consumption as much as possible while interest rates and transaction costs are still low, and this will drive up money demand. Moreover, under these explosive paths seigniorage will reach very high levels because under our parametrization  $M/Pc$  (inverse velocity) has a lower bound, and the public continues to be willing to hold currency no matter how high rates are. The present discounted value of seigniorage will therefore be very high (we can show that since seigniorage asymptotes to a finite level, the present discounted value is actually a finite number, but is so large that our integration routine does not converge, hence we report "Inf" in Table 3).

The dash-and-dotted and dotted blue lines in the bottom right panel of Figure 5 show the responses under different  $\theta_\pi$  coefficients. In the case of unstable solutions, the inflation response coefficient in the interest rule plays the opposite role relative to the stable solution case (see [Cochrane \(2011\)](#)): the stronger the response, the faster inflation and interest rates explode. The market value of central bank's assets  $q$  surely falls more with a higher  $\theta_\pi$ , but as in the  $\theta_\pi = 2$  case the increase in the demand for money in the initial period and the subsequent large seigniorage imply that central bank's solvency is never in question under these paths – quite the contrary, the central bank transfers very large resources to the fiscal authority.

**VII.1. Calibration with money demand equal to zero for  $r > 100\%$ .** We have seen that under many scenarios where the value of the central bank's assets drops substantially, seigniorage can save the day in terms of central bank's solvency. This conclusion is not robust to the assumptions about money demand, however. Table 4 show the central bank's resources under the baseline, higher rates/same intercept, inflation scare, and explosive paths simulations for a money demand function that drops to zero when interest rates are above 100 percent (the other parameter of the transaction

cost function (37) is still chosen so that currency holdings match 2013Q4 holdings for  $r = .0025$ ). As shown in the left panel Figure 6, this money demand function predicts an unrealistically large drop in money demand as interest rates return to steady state, as well as a too low steady state level of seigniorage. Under the baseline simulation, the present value of seigniorage is therefore below a third of its value under the benchmark money demand function. But it does have the arguably realistic feature that for very high rates of inflation the private sector will use means of transactions other than cash. As a consequence under the explosive path there is no time 0 jump in money demand (people are no longer twisting their consumption profile because of transaction costs) and no dramatic increase in seigniorage to cushion the effect of the fall in  $q$ . Seigniorage is actually negative in present value under the explosive paths, as the public dumps its currency holdings (row 12 of Table 4). Under explosive paths with  $\kappa \geq 10^{-4}$  the central bank, therefore, becomes insolvent: it can no longer stick to the Taylor rule and at the same time honor its liabilities without an intervention from the fiscal authority.

### VIII. SELF-FULFILLING SOLVENCY CRISES

In this section we show that the lack of fiscal backing for a central bank may in principle result in indeterminate equilibria for inflation and interest rates. We also show that in practice in our calibrated version of the U.S. economy such multiple equilibria would only arise for levels of the central bank's balance sheet larger than the current one, although this result depends crucially on the properties of the demand for currency.

What if the public believes that, were the central bank to face the issue of solvency, it would have to resort to seigniorage creation given that it lacks fiscal backing? Entertaining the possibility of central bank's insolvency would therefore lead the public to expect higher future inflation and



nominal interest rates. These expectations would result in a lower value of long term nominal assets today, so that the central bank's assets  $qB^C$  could become worth less than its interest bearing liabilities  $V$ . If the current present discounted value of seigniorage is not large enough to cover this gap, the central bank may have to resort to raising more seigniorage, thereby validating the initial belief. The larger is the size of the central bank's balance sheet, and the longer its duration, the larger is the gap in  $qB^C - V$  that would arise because of future expected inflation, and the likelihood of these alternative equilibria.<sup>18</sup>

If there is the possibility of indeterminacy, these multiple equilibria can take many forms. We focus on a particular type of multiple equilibria, where agents expect that at time  $t = \tilde{T}$  the central bank will change its inflation target to  $\tilde{\pi}$  for a period  $\tilde{\Delta}$ , and revert to the old rule with inflation target  $\bar{\pi}$  afterwards (for  $t > \tilde{T} + \tilde{\Delta}$ ). The appropriately modified version of equation (14) provides the solution for the future path of interest rates. Given the path for  $r_t$  we can solve for all other endogenous variables exactly as in the model above. We can in particular obtain, under this alternative equilibrium, the value of long term assets  $q_0(\tilde{\pi})$ , the initial price level  $P_0(\tilde{\pi})$ , and the present discounted value of seigniorage in real terms at time 0, which we can call  $PDVS_0(\tilde{\pi}) = \int_0^\infty (\frac{\dot{M}_t}{M_t} + n) \frac{M_t}{P_t} e^{-\int_0^t (\rho_s - n) ds} dt$ . All of these objects will be a function of  $\tilde{\pi}$  (and of  $\tilde{T}$  and  $\tilde{\Delta}$  as well). For each  $\tilde{T}$  and  $\tilde{\Delta}$ , we can then find what expected future inflation  $\tilde{\pi}$  needs to be to generate a solvency crisis, that is, we can look for solutions of

$$\frac{q_0(\tilde{\pi})B_0^C - V_0}{P_0(\tilde{\pi})} + PDVS_0(\tilde{\pi}) = 0, \quad (40)$$

if any exist. These solutions are possible self-fulfilling solvency crises, in the sense that the expectation that the central bank will switch to a new rule

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<sup>18</sup>These self-fulfilling equilibria bear close parallel to those described in the currency crisis literature.

with target  $\tilde{\pi}$  will produce a gap in the value of central bank's assets minus liabilities  $\frac{q_0(\tilde{\pi})B_{0-}^C - V_0}{P_0(\tilde{\pi})}$  that will have to be filled with future seigniorage  $PDVS_0(\tilde{\pi})$ . In order to generate this future seigniorage the central bank will have to validate the public expectations and switch temporarily to the rule with higher inflation target.

Using our simple calibrated model we search for these alternative equilibria. In particular, for given  $\tilde{\pi}$ ,  $\tilde{T}$ , and  $\tilde{\Delta}$  we find the minimum level of the balance sheet  $B^C$  for which equation (40) has a solution. The top panel left panel of figure 8 shows this minimum balance sheet level (relative to the current level) as a function of inflation in there alternative regime ( $\tilde{\pi}$ ), and the duration of the alternative regime ( $\tilde{\Delta}$ ). To repeat, these are the balance sheet levels for which the solvency constraint would become binding if the public expects a regime with inflation  $\tilde{\pi}$  and duration  $\tilde{\Delta}$  to materialize in  $\tilde{T} = 3$  years.

The figure shows that under the baseline money demand calibration, these threshold balance sheet limits are much larger than the current one. The left middle and bottom panels of figure 8 explain why this is the case. The middle panel shows what happens to  $\frac{q_0(\tilde{\pi})B_{0-}^C - V_0}{P_0(\tilde{\pi})}$  in the alternative equilibrium under the *current* balance sheet size. The figure shows that for large enough  $\tilde{\pi}$  and duration  $\tilde{\Delta}$  the real value of assets minus interest bearing liabilities does become significantly negative. The bottom panel of figure 8 show that for these values the level of seigniorage  $PDVS_0(\tilde{\pi})$  overshadows this balance sheet loss, however. Hence the results in the top panel: the size of the balance sheet would have to be much larger than the current one for the balance sheet loss to be of the same size of the increase in seigniorage. In other words, under the current level of the balance sheet, the type of alternative equilibria we consider cannot arise because the increase in seigniorage triggered by the temporary higher inflation regime is larger

than the balance sheet loss caused by the fall in  $q$ .<sup>19</sup> The Laffer curve in the left panel of figure 7 shows why the increase in seigniorage is so large under the baseline parameterization of money demand: even for large interest rates money demand asymptotes to positive values, and seigniorage grows linearly with inflation.<sup>20</sup>

Under the alternative money demand the threshold balance sheet limits are much closer to one, as a consequence of the fact that the increase in seigniorage is smaller. The right panel of figure 7 shows that seigniorage peaks at about 40 percent steady state inflation, and becomes zero for inflation larger than one hundred percent. Hence seigniorage increases much less with  $\tilde{\pi}$  than in the baseline case, as shown in the bottom right panel of figure 8 (since the duration of the alternative regime is relatively short we find that seigniorage still increases for values of  $\tilde{\pi}$  up to 500 percent). Still, we find that the threshold balance sheet levels that would generate multiplicity are all larger than one for the  $(\tilde{\pi}, \Delta\pi)$  values shown here. Therefore, even for the alternative money demand multiple equilibria under the current size of the balance sheet could be possible only if the public expected very large inflation and/or very long duration of the alternative regime. Indeed, we find that that for instance  $\tilde{\pi} = 5$ , that is, 500 percent inflation, for 158 periods would be an equilibrium under the current level of the balance sheet. These results are in line with what shown in the previous sections, namely, one has to assume fairly radical scenarios for solvency to become an issue.

## IX. CONCLUSIONS

To Be Written

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<sup>19</sup>We searched for alternative values of  $\tilde{T}$  as well, and the results are not very different.

<sup>20</sup>One caveat to these simulations is that we maintain the hypothesis of passive fiscal policy even under high inflation rates, which may not be realistic.

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## APPENDIX A. APPENDIX

A.1. **Rule for remittances.** The central bank is assumed to follow a rule for remittances, which embodies two principles: i) remittances cannot be negative, ii) whenever positive, remittances are such that the central bank capital measured at historical costs remains constant in nominal terms over time, that is:<sup>21</sup>

$$\tilde{K} = \left( \tilde{q}B^C - V - M \right) e^{nt} = \text{constant}. \quad (41)$$

The historical price  $\tilde{q}$  evolves according to

$$\dot{\tilde{q}} = (q - \tilde{q}) \max \left\{ 0, \frac{\dot{B}^C}{B^C} + \delta + n \right\} \quad (42)$$

where the max operator is there because  $\tilde{q}$  changes only if the central bank is acquiring assets (recall that bonds depreciate at a rate  $\delta$  and that  $B^C$  is defined in per capita terms, so that  $\dot{B}^C = -(\delta + n)B^C$  implies that the central bank is letting its assets mature). Differentiating condition (41) above and using the central bank's budget constraint (24), one obtains a condition for nominal remittances:

$$P\tau^C = (\chi - \delta(\tilde{q} - 1)) B^C + \left( \dot{\tilde{q}} - (q - \tilde{q}) \left( \frac{\dot{B}^C}{B^C} + \delta + n \right) \right) B^C - rV. \quad (43)$$

<sup>21</sup>Hall and Reis (2013) use a similar rule, but measure capital at market prices.

This condition resembles closely the accounting practice of central banks. The first term,  $(\chi - \delta(\tilde{q} - 1)) B^C$ , measures coupon income  $\chi$  net of the amortization of historical costs  $\delta(\tilde{q} - 1)$ , times the par value of bonds  $B^C$ . The second term equals the realized gains/losses,  $-(q - \tilde{q}) \left( \frac{\dot{B}^C}{B^C} + (\delta + n) \right) B^C$ , from assets sales (that is,  $\frac{\dot{B}^C}{B^C} \leq -(\delta + n)$ ), since in this case  $\dot{\tilde{q}} = 0$ . When the central bank is acquiring assets ( $\frac{\dot{B}^C}{B^C} > -(\delta + n)$ ) this second term is zero because  $\dot{\tilde{q}}$  and  $(q - \tilde{q}) \left( \frac{\dot{B}^C}{B^C} + \delta + n \right)$  cancel each other out. The last term,  $rV$ , captures the interest paid on reserves. Whenever net income (the left hand side of equation 43) is negative, the central bank would have to extract negative remittances from the fiscal authority to keep its capital constant. If it cannot do this, its capital declines. Whenever internal accounting rules prevent capital from declining a deferred asset is created. Remittances remain at zero until this deferred asset is extinguished (i.e., capital is back at the original level). Hence our rule for remittances is

$$\tau_t^C = \max \left\{ 0, (\chi - \delta(\tilde{q} - 1)) \frac{B^C}{P} + \left( \dot{\tilde{q}} - (q - \tilde{q}) \left( \frac{\dot{B}^C}{B^C} + (\delta + n) \right) \frac{B^C}{P} \right) - r \frac{V}{P} \right\} \mathcal{I}_{\{\tilde{K} \geq \tilde{K}_0\}}, \quad (44)$$

where  $\mathcal{I}_{\{\tilde{K} \geq \tilde{K}_0\}}$  is an indicator function equal to one only if current capital  $\tilde{K}$  is at least as large as initial capital  $\tilde{K}_0$  (that is, the deferred asset has been extinguished). In practice central bank's capital will not be constant over time, but will likely grow along with nominal income. This implies a net influx of resources for the central bank. At the same time a fraction of net income is devoted to pay dividends on this capital.<sup>22</sup> Moreover the central bank

<sup>22</sup>In the U.S. the central bank's capital is a fixed fraction of the capital of the member banks, and dividends are 6% of capital (see Carpenter, Ihrig, Klee, Quinn, and Boote (2013)). Note also that according to our notation dividends are included in  $\tau^C$ , this quantity

also has operating expenses. We ignore these issues in computing the simulated path of remittances since it would further complicate the description of the remittance rule, and also quantitatively they are not very important in terms of the simulated path for remittances.

In order to compute the path for remittances implied by expression (44) we need to compute paths for  $\frac{B^C}{P}$  and  $\frac{V}{P}$ . For the former, we make assumptions about the path of the central bank's assets. We make two assumptions about the future path of  $B^C$ :

$$\frac{\dot{B}^C}{P} = \begin{cases} -(\delta + n) \frac{B^C}{P} & (1) \\ -(\delta + n + s) \frac{B^C}{P} & (2) \end{cases}, \text{ for } t \leq \tilde{T}, \quad (45)$$

where  $\tilde{T}$  is the time when the size of reserves has reached the early 2008 level (adjusted for inflation, population and productivity growth), after which  $\frac{B^C}{P}$  grows with productivity (i.e.,  $\frac{B^C}{P}e^{-\gamma t}$  is constant over time, yielding  $\frac{\dot{B}^C}{P} = (\gamma + \frac{\dot{P}}{P}) \frac{B^C}{P}$ ). Under assumption (1) the central bank lets its holdings of government debt mature, while under assumption (2) it sells its assets at a rate  $s$  per year (we set  $s = .2$ ). Neither assumption is realistic in the case of the U.S. ( $B^C$  has increased in 2013) but the point is to show that different future paths for sales can yield quite different paths for  $\tau^C$  in the short run, even though the present value of resources remitted to the fiscal authority  $\bar{\tau}^C$  is the same. Given remittances and the path for  $\frac{B^C}{P}$  we use the budget constraint (24) to compute the evolution of reserves in real terms:

$$\left(\frac{\dot{V}}{P}\right) = (r - n - \frac{\dot{P}}{P}) \frac{V}{P} - (\chi + \delta - (n + \delta)q) \frac{B^C}{P} + q \frac{\dot{B}^C}{P} - \left(n + \frac{\dot{M}}{M}\right) \frac{M}{P} + \tau^C. \quad (46)$$

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being the total amount of resources leaving the central bank in any given period. In this sense referring to  $\tau^C$  as "remittances" to the fiscal authority is not entirely appropriate.

A legitimate question (also posed by [Hall and Reis \(2013\)](#)) is whether a rule like (44) keeps the central bank's capital measured at market prices, namely

$$K = (qB^C - V - M) e^{nt}. \quad (47)$$

stationary. Using the budget constraint (24) we write the evolution of detrended capital in real terms  $\left(\frac{K}{P}e^{-(\gamma+n)t}\right)$  as

$$d\left(\frac{K}{P}e^{-(\gamma+n)t}\right) = (\rho - n - \gamma) \left(\frac{K}{P}e^{-(\gamma+n)t}\right) + \left(r\frac{M}{P} - \tau^C\right) e^{-\gamma t} \quad (48)$$

So the rule that stabilizes  $\frac{K}{P}e^{-(\gamma+n)t}$  is

$$\begin{aligned} \tau^C &= r\frac{M}{P} + (\rho - n - \gamma) \left(\frac{K}{P}e^{-(\gamma+n)t}\right) \\ &= r\left(q\frac{B}{P} - \frac{V}{P}\right) - \left(\frac{\dot{P}}{P} + n + \gamma\right) \left(\frac{K}{P}e^{-(\gamma+n)t}\right). \end{aligned} \quad (49)$$

This rule has quite different implications for remittances relative to the rule (44) outside of steady state, but at steady state the two coincide. In fact, at steady state  $\tilde{q} = q = 1$ ,  $K = \tilde{K}$ , and  $\chi = \bar{r}$ , hence the term  $r\left(q\frac{B}{P} - \frac{V}{P}\right)$  coincides with the right hand side of expression (44). The remaining term  $(\pi + n + \gamma) \left(\frac{K}{P}e^{-(\gamma+n)t}\right)$  accounts for the fact that capital increases with inflation, productivity, and population growth (as discussed above), a factor which we ignore in (44). If we did properly account for it, expressions (49) and (44) would be consistent with each other at least at steady state.



	r	v	C	M/P	log(P)	log(M)	x	dpvs
initial	0.015	12.247	1.014	0.083	0.000	0.000	0.000	0.000
new	0.075	27.386	1.012	0.037	0.080	-0.726	0.020	2.667
			duration	2.5	5	10	20	
$A_0 = 3$			A loss	-0.14	-0.28	-0.52	-0.92	
			gap	-0.22	0.35	1.13	1.98	
			duration	2.5	5	10	20	
$A_0 = 6$			A loss	-0.14	-0.28	-0.52	-0.92	
			gap	0.57	1.71	3.25	4.95	

TABLE 1. Change in steady state after 2% inflation scare

TABLE 2. Parameters

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normalization, foreign assets	
$Y - G = 1$	$F0 = 0$
initial assets, reserves, and currency	
$\frac{B^C}{P} = 0.327$	$\frac{V}{P} = .207$
$\frac{M}{P} = .104$	
discount rate, reversion to st.st., population and productivity growth	
$\beta = 0.01$	$\gamma = 0.01$
$\varphi_1 = 0.750$	$n = 0.0075$
monetary policy	
$\theta_\pi = 2$	$\theta_r = 1$
$\bar{\pi} = 0.02$	
money demand	
$\psi_0 = 2.31 \cdot 10^{-6}$	$\psi_1 = -0.055$
bonds: duration and coupon	
$\delta^{-1} = 6$	$\chi = 0.035$

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TABLE 3. Central bank's resources under different simulations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$\frac{qB/P}{-V/P}$	PDV seigniorage	(1)+(2)	$\bar{\tau}^C$ (average remittance)	q	$\Delta M/P$	$\bar{B}/B$
<b>Baseline calibration</b>							
(1) Baseline scenario	0.146	0.998	1.144	0.0026	1.08		
(2) Higher rates ( $\beta$ )	0.133	0.169	0.302	0.0033	1.06	-0.006	14.42
(3) Higher rates ( $\gamma$ )	0.142	1.287	1.429	0.0031	1.06	0.003	64.59
(4) Higher rates ( $\beta$ ) /same intercept	0.097	0.241	0.338	0.0037	1.01	-0.025	5.75
(5) Higher rates ( $\gamma$ ) /same intercept	0.105	1.573	1.677	0.0037	1.01	-0.017	24.56
(6) Inflation scare	0.068	1.007	1.075	0.0024	0.92	-0.024	7.81
(7) Explosive path	3.124	Inf	Inf	Inf	0.88	3.043	Inf
<b>Higher <math>\theta_\pi</math></b>							
(8) Higher rates ( $\beta$ ) /same intercept	0.115	0.216	0.331	0.0035	1.04	-0.019	9.93
(9) Inflation scare	0.085	0.992	1.077	0.0024	0.96	-0.021	10.10
(10) Explosive path	6.124	Inf	Inf	Inf	0.64	6.120	Inf
<b>Lower <math>\theta_\pi</math></b>							
(11) Higher rates ( $\beta$ ) /same intercept	-0.055	1.162	1.107	0.0132	0.52	-0.024	3.38
(12) Inflation scare	-0.012	1.038	1.026	0.0025	0.67	-0.021	3.90
(13) Explosive path	0.241	Inf	Inf	Inf	1.05	0.106	Inf

TABLE 4. Central bank's resources under different simulations. Calibration with money demand = 0 for  $r > 100\%$ 

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$\frac{qB/P}{-V/P}$	PDV seigniorage	(1)+(2)	$\bar{\tau}^C$ (average remittance)	q	$\Delta M/P$	$\bar{B}/B$
<b>Baseline calibration</b>							
(1) Baseline scenario	0.146	0.214	0.360	0.0008	1.08		
(2) Higher rates ( $\beta$ ) /same intercept	0.060	0.033	0.093	0.0010	1.01	-0.062	2.31
(3) Inflation scare	0.012	0.215	0.227	0.0005	0.92	-0.080	2.44
(4) Explosive path	0.079	-0.094	-0.015	-0.0000	0.88	-0.002	0.92
<b>Higher <math>\theta_\pi</math></b>							
(5) Higher rates ( $\beta$ ) /same intercept	0.085	0.017	0.103	0.0011	1.04	-0.048	3.77
(6) Inflation scare	0.029	0.121	0.150	0.0003	0.96	-0.077	2.27
(7) Explosive path	0.001	-0.097	-0.096	-0.0002	0.64	-0.002	0.78
<b>Lower <math>\theta_\pi</math></b>							
(8) Higher rates ( $\beta$ ) /same intercept	-0.111	0.095	-0.016	-0.0002	0.52	-0.080	0.97
(9) Inflation scare	-0.072	0.299	0.227	0.0005	0.67	-0.081	1.64
(10) Explosive path	0.133	0.041	0.175	0.0004	1.05	-0.002	6.46

FIGURE 1. A scatter plot of short term interest rates and M/PC

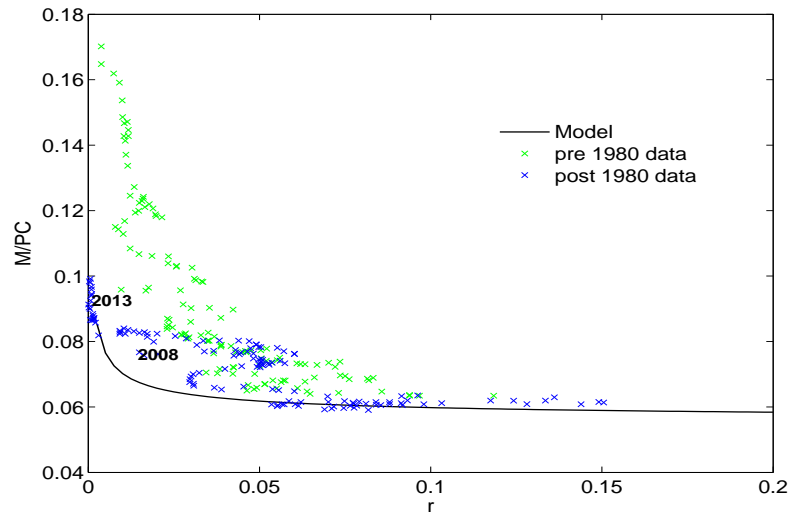


FIGURE 2. Short term interest rates and inflation: baseline vs higher rates

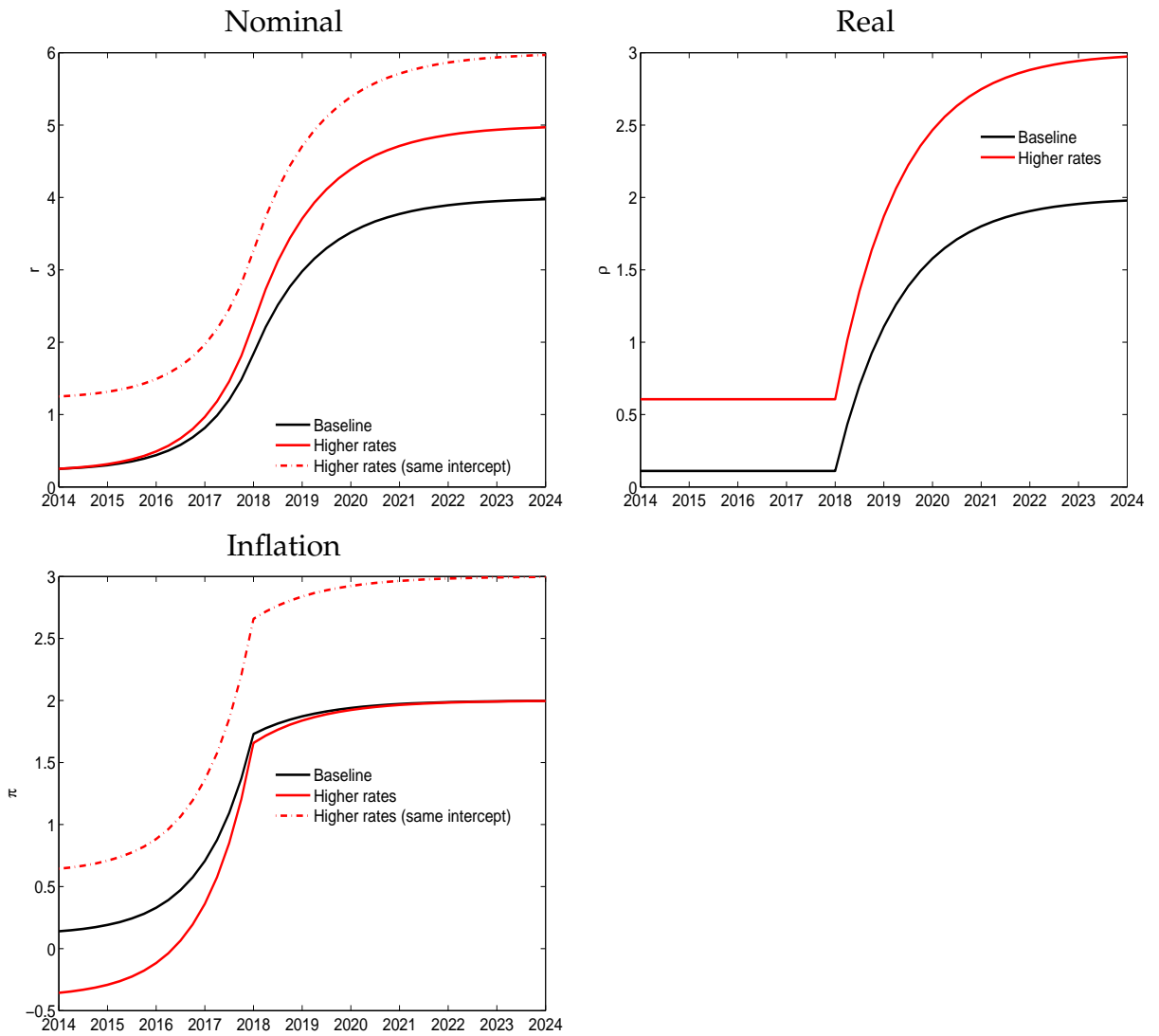
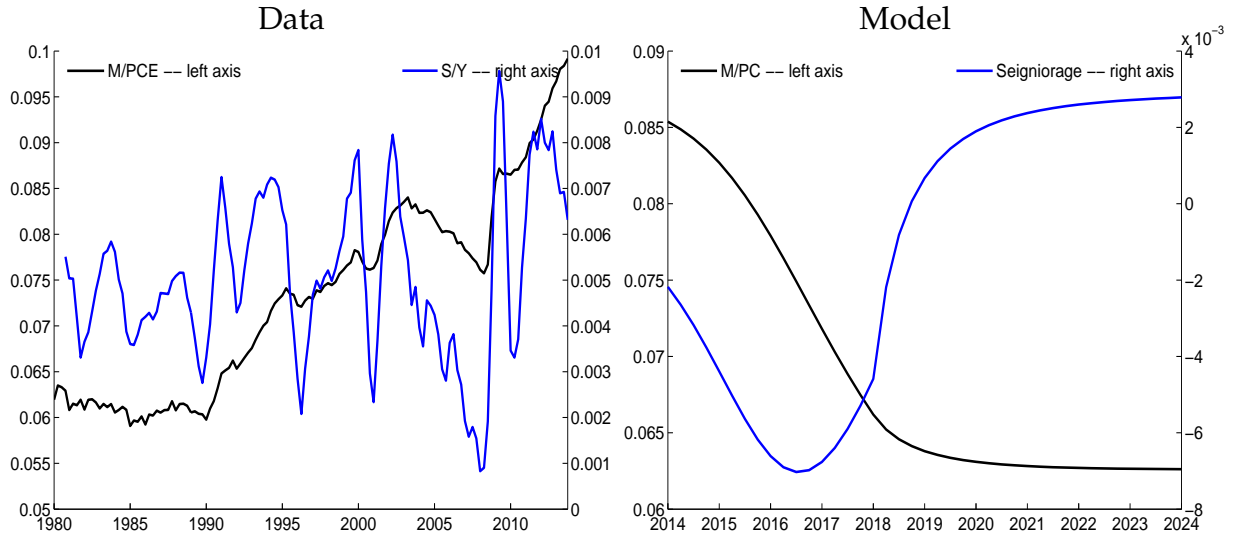
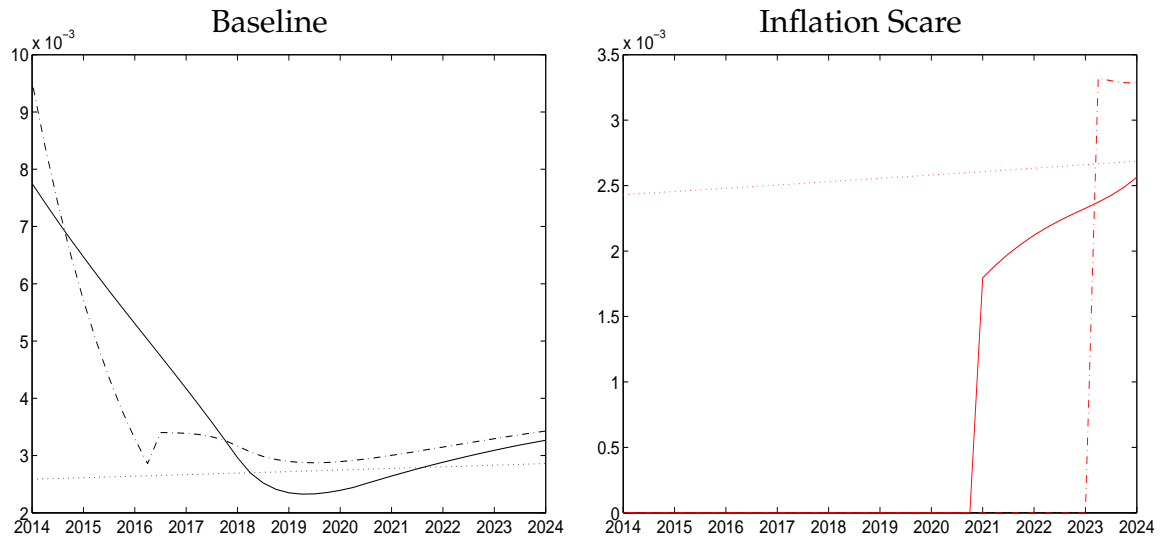


FIGURE 3. seigniorage and M/PC



Notes: Paths for seigniorage and real money balances in right hand panel are obtained under baseline simulation.

FIGURE 4. Paths for remittances



Notes: Each panel shows remittances under two assumptions for the path of assets  $B^C$ : under the first assumption (solid line) the central bank lets its assets depreciate, while in the second one it actively sells assets at a rate of 20 percent per year. The paths in the left and right panels are obtained under the baseline and "inflation scare" scenario, respectively.

FIGURE 5. The effect of different inflation responses in interest rate rule under different scenarios

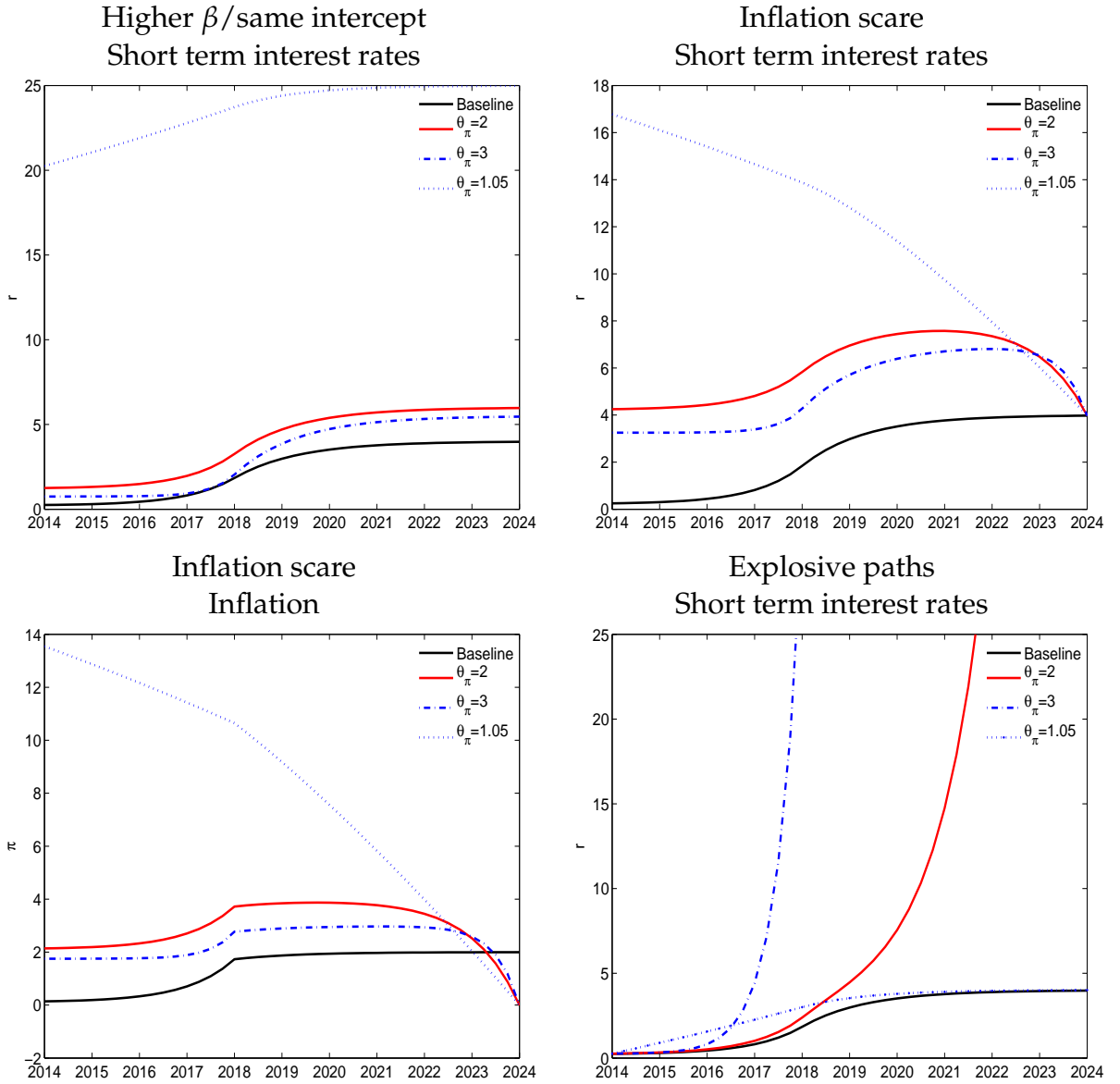
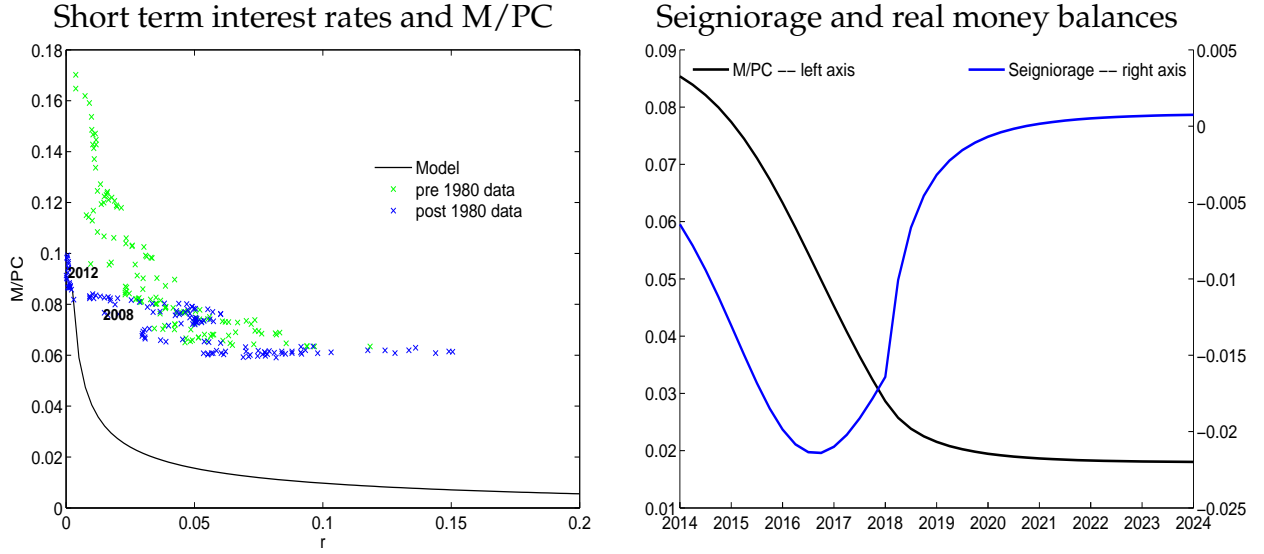




FIGURE 6. Case where money demand = 0 for  $r > 100\%$



Notes: Paths for seigniorage and real money balances in right hand panel are obtained under baseline simulation.

FIGURE 7. Laffer curves

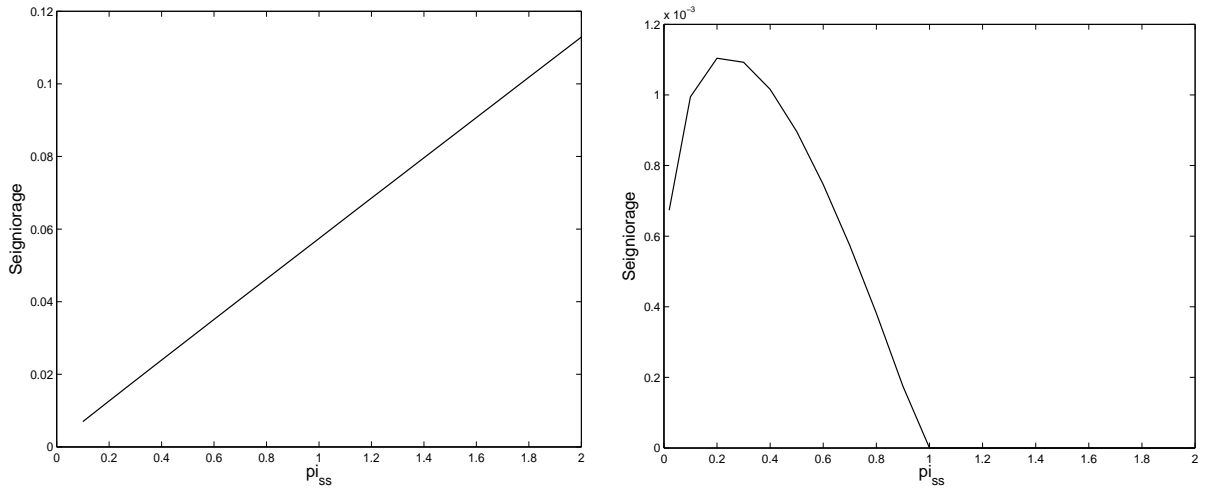
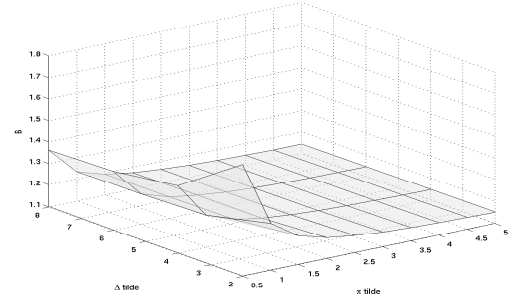
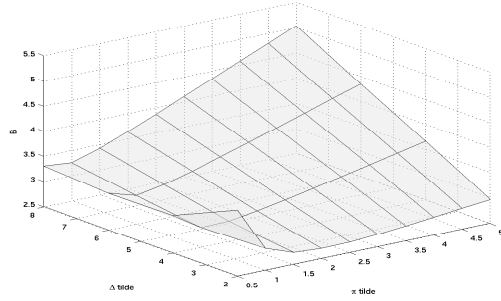


FIGURE 8. Self-fulfilling solvency crises

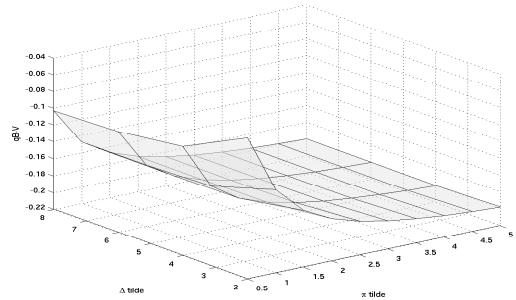
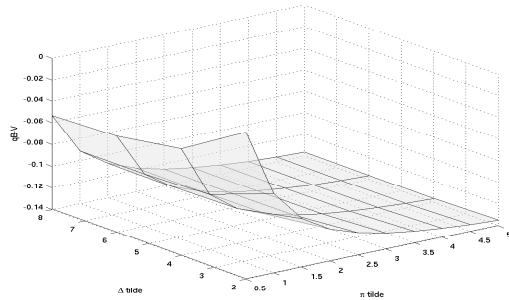
Baseline

Calibration with  
money demand = 0 for  $r > 100\%$

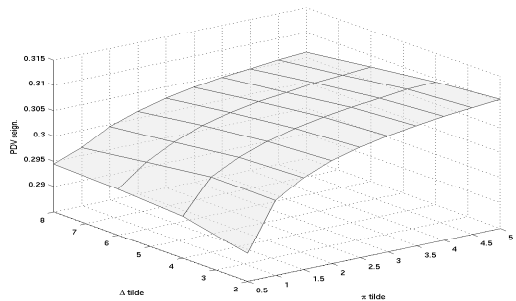
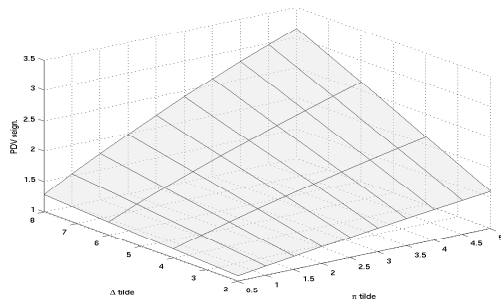
Threshold Balance Sheet Limit  
% of current  $B$



$qB - V$



seigniorage



Notes: The figure shows 1) Top panel: the level of the balance sheet (relative to the current level) for which multiple equilibria are possible; 2) Middle panel: the level of  $qB - V$  as a fraction of income for the current balance sheet size under alternative scenarios; 3) Bottom panel: the level of seigniorage as a fraction of income under alternative scenarios; as a function of a) inflation in there alternative regime ( $\bar{\pi}$ ), b) the duration of the alternative regime ( $\bar{\Delta}$ ). In all simulation the alternative regime is expected to start after 3 years. The left and right figures are for the baseline and alternative transaction technology, respectively. .

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