Optimal Inflation Target in an Economy with Menu Costs and Zero Lower Bound*

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Abstract

This paper studies the optimal inflation target in a menu cost model with an occasionally binding zero lower bound on interest rates. I find that the optimal inflation target is 3%, larger than the rates currently targeted by the Fed and the ECB, and also larger than in other time- and state-dependent pricing models. In my model resource misallocation does not increase greatly with inflation, unlike in previous sticky price models. The critical additions for this result are firms' idiosyncratic shocks together with state dependent pricing. Higher inflation does indeed increase the gap between old and new prices, but it also increases firms' responsiveness to idiosyncratic shocks. These two effects are balanced using idiosyncratic shocks consistent with micro-price statistics. By increasing the inflation target, policymakers can reduce the probability of hitting the zero lower bound and the magnitude of the deflationary spirals, avoiding costly recessionary episodes.

JEL: E3, E5, E6.

Keywords: menu costs, (S,s) policies, monetary policy, inflation target.

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1 Introduction

Since the end of the 1980s, many countries around the world have adopted inflation targeting models, where the central bank explicitly declares a medium-term target inflation rate. New Zealand first adopted the model, which has since been taken up by 28 developed and developing countries. Even the United State had a historic shift on January of 2012, when the U.S. Federal Reserve set a 2% target inflation rate.[1] However, though international consensus increasingly favors the existence of such targets, policymakers struggle to find the right size for them. This paper asks the following question: What is the inflation target a Central Bank should have?

Before the last great recession, there was a consensus among economists for a low or even negative optimal inflation target—see Schmitt-Grohé and Uribe (2010). The last recession placed this consensus in doubt. Central banks hit the zero lower bound on nominal interest rate so quickly that policy makers have wondered whether a higher inflation target would leave more room to stimulate the economy during recession.[2] According to these economists, a higher inflation target would increase average nominal interest rates, thus giving central bankers more room to react to adverse macroeconomic shocks.

While the benefits of a higher inflation target are well-understood, much less is known about the costs of permanently higher rates of inflation. In existing sticky price models, the major cost of inflation is that it induces inefficient dispersion in relative prices—markups—and therefore productivity losses in the economy due to dispersion in firms' marginal product. Intuitively, a higher inflation target increases the gap between recently-adjusted prices and those of producers that have not adjusted in a while, opening a gap between their marginal productivity.

This paper quantifies the benefits and costs of a higher inflation target in a model consistent with micro-pricing behavior and macroeconomic dynamics. I build a random menu cost model with idiosyncratic cost shocks that reproduces the micro-price data at low and high levels of inflation. I incorporate a Taylor rule for monetary policy, occasionally subject to a zero lower bound, as well as a rich source of aggregate dynamics arising from several aggregate shocks. Thus the model not only matches micro-facts, the model is also able to reproduce US macroeconomic time series during the great moderation, as does the Calvo model.

My main result is that the optimal inflation target in this environment is about 3%, much greater than in other leading time- and state-dependent pricing models studied in the literature —Calvo (1983), Taylor

[1]See "FOMC statement of longer-run goals and policy strategy" report about their long-run target inflation.

[2]See Ball (2013), Blanchard, DellAriccia and Mauro (2010), and Williams (2009) for a revival of this old proposal by Summers (1991).

(1980), and Dotsey, King and Wolman (1999). I show that with the same preference and technology a model with Calvo pricing has an optimal inflation target of 1%. Thus, my results sharply contrast with those of Coibion and Gorodnichenko (2012), who find a robust result of an optimal inflation target between 1% by quantifying the optimal inflation target with an occasionally binding zero lower bound in several time- and state-dependent pricing models.

The optimal inflation target is higher in my model than in previous studies due to the small cost of inflation implied by the interaction between large idiosyncratic shocks and menu cost. Since firms are exposed to real fluctuations of idiosyncratic marginal cost, misallocation and productivity losses depend on the dispersion of markup. If a firm is hit by negative idiosyncratic cost shocks, then a positive inflation decreases the firm's markups and offsets the idiosyncratic cost shock. In this case, higher inflation offsets idiosyncratic shocks, decreasing the dispersion of markups. In the other case, whenever a firm is hit by positive idiosyncratic cost shocks, higher inflation generates a price change whenever the idiosyncratic shocks are sufficiently large. Since these are the firms that largely contribute to misallocation, this increase in the frequency of price change *in the firms where it is most needed* decreases misallocation due to idiosyncratic shocks. I show that menu cost alone, without idiosyncratic shocks, generates more misallocation than the Calvo model at low levels of inflation—less than 2%—since it loses the two mechanisms described above. Since, trivially, the Calvo model with idiosyncratic shocks generates more misallocation than without idiosyncratic shocks, it is the interaction between idiosyncratic shocks and menu cost that generates much lower cost of inflation at all levels of inflation.

We continue with an overview of the ideas in this paper.

Framework for Optimal Inflation Target and Positive Implication of the Model: The model I use to compute the optimal inflation target departs from the standard medium scale New Keynesian model, where I replace Calvo pricing with state dependent pricing in which there is a fixed cost of adjustment and firms level cost shocks. The last two modifications allow the model to better fit micro-pricing behavior and decreases the cost of inflation. Business cycle dynamics of output and inflation are generated by a rich set of aggregate shocks given by productivity, government expenditure, monetary and risk premium shocks. I model monetary policy with a Taylor rule subject to an occasionally binding zero lower bound. The constraint on the nominal interest rate restricts the Central Bank in order to stabilize inflation and out-gap; higher inflation target increases Central Bank's ability to stabilize the economy since the economy is further away from the zero lower bound. Additionally, my model has strategic complementarities in the form of intermediate input—marginal cost is a function of labor and the aggregate price of the economy. This feature together with Greenwood-Hercowitz-Huffman (GHH) allows the model to generate a flat Phillips curve as in the data, a critical component for the magnitude of the deflationary spiral and the cost of the zero lower bound. These strategic complementarities–like others–increase the cost of inflation; thus, my analysis takes into account the relation between a flat Phillips curve due to strategic complementarities and the cost of inflation.

The pricing model I use to analyze the optimal inflation target is a CalvoPlus model, where the menu cost is a i.i.d. binominal random variable, and a fixed normal distribution for firm's shocks. This model matches, among others, these facts in the price statistics: small price changes, large size of average price change and fat tails in the price change distribution. I estimate the model to match micro-price facts and the data in price identifies how much of price changes are due to Calvo or menu cost. Additionally, this model has the capacity to generate a positive relation between higher inflation environments and frequency of price change and extensive margin component of inflation volatility.

In addition to carefully choosing my pricing model, I assume preferences that can capture the cost of business cycle without affecting the inter-temporal elasticity of substitution. I use Epstein-Zin preferences to generate reasonable business cycle fluctuation together with the cost of business cycle observed in asset prices. Following Alvarez and Jermann (2004), I calibrate the risk sensitivity parameter to match the excess of return of an asset that pays aggregate consumption with business cycle fluctuations and an asset that pays trend consumption. The cost of business cycle in my model is in the lower bound of empirical estimates.

Without zero lower bound and at zero inflation target, my model generates similar business cycle statistics to a Calvo model with the same preferences and technology. To formalize this argument, I show that an econometrician with only aggregate data though VAR or business cycle statistics could not distinguish which model generates it in a finite sample with the same length of the great moderation. Moreover, both models have a good fit with the US macroeconomic time series during the great moderation—1984Q1-2006Q4—for a medium scale DSGE model. The result of similar business cycle dynamics without zero lower bound depends on three assumption: CalvoPlus pricing model, mixed normal distribution of firm's shocks and general equilibrium effects. Since in my model monetary policy responds to inflation, the effect of a steeper Phillips curve in equilibrium inflation dynamics is partially offset by movement in the nominal interest rate. To explain this argument, assume a structural shock that increases the output gap. Since nominal interest rate depends on inflation, in the CalvoPlus model nominal interest will increase by more; consequently, the equilibrium output gap responds by less and therefore so does inflation.

The equivalence result between the Calvo and the CalvoPlus model breaks with zero lower bound. To understand this, notice that in models with price rigidities, an initial drop in the output gap decreases inflation; and if the zero lower bound is binding, this initial drop in inflation increases real interest rates, even further depressing the output gap. The important property in the CalvoPlus model is that the deflationary spiral decreases with the inflation target. At low levels of inflation, periods of ZLB binding are associated with deflations, thus triggering large downward price adjustments and a higher deflationary spiral. But for a sufficiently high inflation target, during periods where nominal interest rate is constrained there is no downward price adjustment in periods of binding ZLB since there is no deflation. Comparatively, at 1% inflation target the deflationary spiral is larger in the CalvoPlus model than in the Calvo model, but at 3% inflation target they are the same.

I estimate the model to match micro-prices behavior at 2% inflation. Micro-price statistics discipline the calibration of the CalvoPlus model between a menu cost and random pricing together with the magnitude of idiosyncratic shocks, key parameters of the cost of inflation. Since my model abstracts from product heterogeneity, I need to control for the large heterogeneity at product level that consumer price index (CPI) databases have. In order to control for heterogeneity at product level, I use monthly price quotes recollected for the CPI micro dataset of the United Kingdom's Office for National Statistics (ONS). The UK's ONS calculates aggregate inflation departing from product level inflation—even more, they compute inflation at stratum level—and using CPI weights and price indexes at item level they aggregate to compute aggregate CPI. Given this methodology, UK's ONS sample a large set of stores and location for a given item. This large sampling at item level allows me to filter heterogeneity at the lowest level of disaggregation. For example, without controlling for heterogeneity, the kurtosis of price change is 9; in my model this will imply almost Calvo calibration of the CalvoPlus model. After controlling for heterogeneity at product level, the kurtosis falls to 3.7, with 30% of price changes due to free price adjustment. Additionally, I clean the data with respect to sales, product substitution, measurement error, out-of-season and outliers—as previous papers have done. Thus in my model, consistent with micro-price statistics facts at low inflation target, I find that the traditionally high cost of inflation in the sticky price models is comparatively small in a model with state dependent price and idiosyncratic shocks.

Normative Implication of the Model: The interaction between idiosyncratic shocks and optimal repricing decision is critical to decrease cost of inflation and therefore to justify an optimal higher inflation target. To understand the interaction between optimal repricing decision with menu cost and idiosyncratic shocks, I analyze the cost of inflation in a model with menu cost—and random menu cost—only. I show that this model, without idiosyncratic shocks but with the same pricing technology as my model, generates a higher cost of inflation than the Calvo model at levels of inflation target less than 2% (and much higher than in my original model). To understand this result, notice that in the menu cost or random menu cost model without idiosyncratic shocks, inefficient price dispersion is zero at zero inflation target. At any level of inflation, since the Ss bands are always positive, the distribution of relative price is close to a uniform distribution within the Ss bands. Therefore the distribution of relative price jumps from a state in which all prices are at the optimal level at a zero inflation target to a uniform distribution at any positive inflation level. Instead, in the Calvo model, price dispersion increases continuously since prices do not translate from one Ss band to another. Therefore, the menu cost model alone penalizes more inflation at low inflation level.

Consider next the benefits of having a higher inflation target. A higher inflation target decreases the business cycle volatility of the output gap for two reasons. First, it decreases the probability of hitting the zero lower bound. Second, in the CalvoPlus higher inflation target reduces the deflationary spiral during liquidity traps as I explain before.

Methodological Contribution: There are two challenges to numerically solve a menu cost model in a New Keynesian framework. First, even without the ZLB constraint, the firm problem has kinks, eliminating perturbation methods as a means to solve these economies. Thus, I rely on global projection methods. Due to the curse of dimensionality, I use the Smoliak sparse-grid method as in Judd, Maliar, Maliar and Valero (2014) and Krueger and Kubler (2004). Second, since standard application of the Krusell-Smith algorithm fails in these economies, I develop a modified version of the algorithm.[3] Typically, the Krusell-Smith algorithm projects price and quantities on a small set of moments of the distribution of the idiosyncratic state. In this model, it consists of projecting inflation to some moments of the distribution of relative prices. But an exogenous inflation function and a Taylor rule on nominal interest rates imply that these depend only on the state of the economy—thus they don't react to inflation and output endogenously, generating indeterminacy (see **?**). To avoid this problem, I modify the Krusell-Smith algorithm to incorporate the intensive margin of the Phillips curve in the inflation function, making inflation partially react to output—thus nominal interest rates react to inflation and output endogenously, generating determinacy.

Section 2 describe related literature. Section 3 describes the model. Section 4 presents a simplified

[3]See Krusell and Smith (2006), Midrigan (2011) and Khan and Thomas (2008) for applications of the Krusell-Smith algorithm.

version of the model and shows the modified Krusell-Smith algorithm together with the projection method I used. Section 5 calibrates the model. Section 6 analyzes the positive dimensions of the model with respect to micro-pricing behavior and business cycle properties and section 7 analyses the optimal inflation target. Section 8 concludes.

2 Related Literature

My paper contributes to the optimal inflation target level literature—see Schmitt-Grohé and Uribe (2010) for a thorough and rigorous description of the cost and benefits of a higher inflation target. The case without idiosyncratic shocks has been extensively studied by Coibion, Gorodnichenko and Wieland (2012). They study the optimal inflation target in leading time- and state-dependent pricing models in the literature—Calvo (1983), Taylor (1980), and Dotsey *et al.* (1999)—that are able to match macroeconomics dynamics but not micro-pricing behavior, since they don't have idiosyncratic shocks. My model with state-dependent pricing and idiosyncratic shocks matches micro-pricing behavior; additionally, the interaction of idiosyncratic shocks and menu cost reduces the cost of inflation. Moreover, this paper explains Coibion *et al.* (2012)'s numerical result of a lower optimal inflation target in the state dependent pricing model of Dotsey *et al.* (1999) than Calvo. As I explain above, only the interaction between menu cost and idiosyncratic shocks reduces the cost of inflation in menu cost models.

This paper relates to Gertler and Leahy (2008) and Midrigan (2011). These papers have shown that the menu cost and Calvo models generate similar business cycle dynamics; I extend these results to a medium-scale general equilibrium model of the US economy. This result is not obvious, since in the case of Gertler and Leahy (2008), they assume that firms cannot change their price if they don't receive an idiosyncratic shock—an assumption that I don't make. Midrigan (2011) uses only small money shocks; my model uses larger, empirically relevant shocks for the US economy. Additionally, since I have a Taylor rule, a steeper Phillips curve in the CalvoPlus model compared to the Calvo model is partially offset in inflation fluctuations by movement in the nominal interest rate. This property breaks down at the zero lower bound, since nominal interest rate cannot react to inflation.

This paper also participates in the larger discussion of macroeconomic consequences of higher trend inflation. Many studies examine the consequences of higher trend inflation in the Calvo model—see Ascari and Sbordone (2013) for a review of the literature[4]. Much less is known of the macroeconomic effect of

[4] Also see Ascari and Ropele (2007), Ascari and Ropele (2009) and more recently Ascari, Phaneuf and Sims (2015)

higher inflation target with menu cost in a rich macroeconomic environment. My model is a first attempt to study business cycle in a medium scale DSGE model. To my knowledge, I'm the first economist to study the consequences of ZLB in a general equilibrium model with state dependent pricing.

Finally, my paper is related to the literature of solution methods in models with heterogeneous agents—see Algan, Allais and Den Haan (2010) for a literature review. In typical applications of Krusell-Smith, aggregate equilibrium conditions are static; thus projecting price and quantities on a small set of moments of the distribution of the idiosyncratic state does not generate any complication in the solution algorithm. In my model, aggregate equilibrium conditions are dynamic; therefore, projecting inflation on a small set of moments of the distribution of the idiosyncratic state generates multiplicity of equilibrium when solving the aggregate equilibrium solution. Given this characteristic of the problem I modified Krusell-Smith to be able to compute the equilibrium in this economy. Additionally, given that the zero lower bound generates non-convexities, methods like the one used in Reiter (2009) and Winberry (2014) are not fit to compute the equilibrium in this economy.

3 Model

Time is discrete. There is a continuum measure one of intermediate firms indexed by $i \in [0, 1]$, a final competitive firm, a representative household, a central bank and a government.

The intermediate firm produces final output Y_t using intermediate firms' production y_{ti} subject to random idiosyncratic shocks A_{ti}

$$Y_t = \left(\int_0^1 \left(\frac{y_{ti}}{A_{ti}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

where the final output uses a Dixit-Stiglitz aggregator with elasticity γ.

Intermediate firms are monopolistically competitive. The intermediate good firm i produces output y_{ti} using labor l_{ti} and material n_{ti}, and the productivity of the firm is given by an idiosyncratic component A_{ti}, an aggregate component η_{tz} and a labor augmented productivity $\Gamma_t = (1 + g)^t$according to

$$y_{ti} = A_{ti} \eta_{tz} n_{ti}^{\alpha} \left(\Gamma_t l_{ti} \right)^{1-\alpha} \tag{2}$$

The firm's production function features intermediate inputs. In the data, almost 70% of total gross value is given by intermediate inputs, which adds real rigidities in the model: the firm's policy function depends on its own marginal cost and aggregate price level. There are two consequences of this assumption: a flatter Phillips curve, which affects inflation dynamics; and a larger elasticity of productivity losses with respect to inefficient price dispersion, which affects the cost of inflation.

Following the literature,[5] I refer to A_{ti} as a quality shock. On one hand, a decrease in A_{ti} increases the marginal product of the final producer, but at the same time, it reduces the marginal product of the intermediate producer. These two effects offset each other, in such a way that the marginal product of labor in firm i with respect to the final output is independent of A_{ti}. The main reason to add this quality shock in the Dixit-Stiglitz aggregator is to decrease the state space of the firm and the aggregate state of the economy (see section 4 for a detailed explanation).

The quality shock A_{ti} follows a compound Poisson process given by

$$\Delta \log(A_{ti}) = \begin{cases} \eta^1_{t+1i} & \text{with prob. } p \\ \eta^2_{t+1i} & \text{with prob. } 1-p \end{cases} \quad ; \quad \eta^k_{ti} \sim_{i.i.d} N(0, \sigma_{ak}) \tag{3}$$

The main motivation for adding idiosyncratic shocks to the model is to match empirical facts with respect to firms' micro-price behavior. For example, in the low-inflation economies, the average absolute value of price change is around 10% (see table 2 for the micro-price statistics to calibrate the model) and around half of them are price decreases. These facts can only be matched in a model that includes idiosyncratic as well as aggregate shocks. Moreover, as I show in section 7, the interaction between idiosyncratic shocks and menu cost is critical to have a low cost of inflation.

Firms face a stochastic physical cost of changing their price. Every time the firm changes her nominal price, she has to pay a *menu cost* given by θ_t units of labor. The menu cost is an *i.i.d.* random variable over time with the following process

$$\theta_t = \begin{cases} \theta & \text{with prob. } 1-hz \\ 0 & \text{with prob. } hz \end{cases} \tag{4}$$

The menu cost is fixed to a constant number with a free price change with i.i.d. probability. The random menu cost allows the model to generate small price changes—as in the data—that cannot be matched with a constant menu cost. Moreover, the CalvoPlus model can span from the menu cost model with low cost of inflation to the Calvo model with high cost of inflation. Micro-price statistics will identify the best model to describe the firm's pricing behavior.

[5] This formulation was first used by Woodford (2009) to keep his model tractable. It was also used by Midrigan (2011), Midrigan and Kehoe (2011), Alvarez and Lippi (2012) and others.

Idiosyncratic quality shocks and random menu cost allows the model to match micro-price behavior at low and high inflation targets. Fat-tail idiosyncratic quality shocks and random menu cost generate the price change distribution in the model similar to the data for low levels of inflation. Menu cost allows the model to match micro-price statistics (frequency of price change, intensive and expensive margin of inflation) for high levels of inflation.

Households' preferences are given by

$$
\begin{aligned}
U_t &= u(C_t, L_t) + \beta \mathbb{E}_t [U_{t+1}^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}} \\
u(C_t, L_t) &= \bar{u}\,(1-\sigma_{np})^{-1} \left(C_t - \Gamma_t \kappa\,(1+\chi)^{-1} L_t^{1+\chi} \right)^{1-\sigma_{np}}
\end{aligned} \tag{5}
$$

where σ_{ez} is the risk sensibility parameter and measures the departure from expected utility. Period utility follows Greenwood-Hercowitz-Huffman (GHH) preferences specification, where C_t is aggregate consumption and L_t is labor supply. The term Γ_t affects the disutility of labor and implies a balance growth path in the model. GHH preferences allow the model have low volatility of inflation, since it decreases the elasticity of the output with respect to the marginal cost for a given Frisch elasticity.

This paper computes the optimal inflation target; therefore, it is crucial to capture the benefit of business cycle stabilization. With this objective in mind, I depart from expected utility and I calibrate the risk sensibility parameter to match asset pricing facts with respect to the price of risk. As Alvarez and Jermann (2004) have shown, there is a one-to-one mapping between the cost of business cycle and the excess of return of an assets that pays aggregate consumption; therefore, my model with Epstein-Zin preferences has a better fit with respect to asset prices and the cost of business cycle than a model with expected utility.

The consumer faces the following budget constraint given by

$$
P_t C_t + B_t = W_t L_t + \int \Phi_{ti} di + \eta_{t-1q} R_{t-1} B_{t-1} + T_t \tag{6}
$$

where W_t and P_t are the nominal prices of labor and consumption, Φ_{ti} are nominal profits for the intermediate producer and T_t are lump sum transfers from the government. B_{t-1} is the stock of one-period nominal bonds with a rate of return $R_{t-1}\eta_{t-1q}$, where the second element generates a wedge between the nominal interest rate controlled by the central bank and the return of the assets held by households. For this reason it is defined as a risk premium shock. This shock can be micro-funded as a net-worth shock in models with the financial accelerator and capital accumulation. I calibrate it to match the probability of hitting the zero lower bound.

The behavior of monetary policy is described by a Taylor rule given by

$$
\begin{aligned}
R_t^* &= \left(\frac{1+\bar{\pi}}{\beta(1+g)^{-\sigma_{np}}}\right)^{1-\phi_r} \left(R_{t-1}^*\right)^{\phi_r} \left[\left(\frac{P_t}{P_{t-1}(1+\bar{\pi})}\right)^{\phi_\pi} X_t^{\phi_y}\right]^{1-\phi_r} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_{dy}} \eta_{rt} \\
R_t &= \max\{1, R_t^*\}
\end{aligned}
\tag{7}
$$

R is the nominal interest rate, $\bar{\pi}$ is the target inflation, η_r is a money shock, and X_t is the output gap, i.e., the ratio between current output and the natural level of output defined in an economy with zero menu cost (an economy without price rigidities). This quantitative Taylor rule describes the behavior of monetary policy with binding and non-binding zero lower bound. I define R^* as the shadow interest rate. Importantly, the shadow interest rate can be below zero even after an economic recovery as optimal policy and empirical evidence in this framework suggests.

Aggregate output is equal to aggregate consumption plus government expenditure

$$
Y_t - \int N_{ti} di = C_t + \eta_{tg} \tag{8}
$$

where I used η_{tg} to denote government expenditure. The government follows a balanced budget each period.

Aggregate shocks follow an AR(1) given by

$$
\log(\eta_{tj}) = (1-\rho_j)\eta_j + \rho_j \log(\eta_{t-1j}) + \sigma_j \epsilon_{tj} \quad \epsilon_{tj} \sim_{i.i.d.} N(0,1) \tag{9}
$$

where $j \in \{r, q, z, g\}$. Next I will describe each agent's problem and the equilibrium definition.

Household: The representative consumer problem is given by

$$
\max_{\{C,L,B\}_t} U_0 \tag{10}
$$

subject to (5) and (6) for all periods. From the problem of the representative consumer we have the stochastic nominal discount factor

$$
Q_{t+1} = \beta \left(\frac{U_{t+1}}{\mathbb{E}_t[U_{t+1}^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \frac{U_c(C_{t+1}, L_{t+1})}{U_c(C_t, L_t)} \frac{P_t}{P_{t+1}} Q_t \tag{11}
$$

with $Q_0 = 1$.

Final Firm Producer: The final producer problem is given by

$$\max_{\{Y_t,\{y_{ti}\}_i\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t \left(P_t Y_t - \int_0^1 p_{ti} y_{ti} di \right) \right] \tag{12}$$

subject to (1). Given constant return to scale and zero profits conditions, we have that the aggregate price level and the firm's demand are given by

$$P_t = \left(\int_0^1 (p_{ti} A_{ti})^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \qquad y_t(A_{ti}, p_{ti}) = A_{ti} \left(\frac{A_{ti} p_{ti}}{P_t} \right)^{-\gamma} Y_t \tag{13}$$

Intermediate Firm Producer: The intermediate firm's problem is given by

$$\max_{p_{ti}} \mathbb{E} \left[\sum_{t=0}^{\infty} Q_t \Phi_{ti} \right] \qquad s.t.$$

$$\Phi_{ti} = y_t (A_{ti}, p_{ti}) \left(p_{ti} - \iota \frac{((1-\tau_L) W_t)^{1-\alpha} P_t^{\alpha}}{A_{ti} \eta_{tz}} \right) - I(p_{t-1i} \neq p_{ti}) W_t \theta_{ti} \tag{14}$$

subject to (3), (**??**) and A_{-1}, p_{-1} given. Note that I've already included the optimal technique in the marginal cost of the firm with $\iota = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$.

There is a subsidy to labor denoted by τ_L that allows my model to match two important objects for the optimal inflation target: the elasticity of labor misallocation across firms with respect to the relative price dispersion and the average level of markups. The elasticity of demand with respect to price is around minus 3 to 5 according to micro-studies. Without the labor subsidy, this elasticity of demand implies an aggregate markup around 50 percent, an inconsistent number according to micro-evidence. I'll calibrate the labor subsidy to match an aggregate markup of 20 percent with an elasticity of demand of minus 5 percent at a 2 percent inflation target.

Equilibrium definition *An equilibrium is a set of stochastic processes for (i) consumption, labor supply, and bonds holding* $\{C, L, B\}_t$ *for the representative consumer; (ii) pricing policy functions for firms* $\{p_{ti}\}_t$ *and inputs demand* $\{N_{ti}, l_{ti}\}$ *for the monopolistic firms; (iii) final output and inputs demand* $\{Y_t, \{y_{ti}\}_i\}_t$ *for the final producer and (iv) nominal interest rate* $\{R\}_t$*:*

1. *Given prices,* $\{C, L, B\}_t$ *solve the consumer's problem in* (10).

2. *Given prices,* $\{Y_t, \{y_{ti}\}_i\}_t$ *solve the final good producer problem in* (12).

3. *Given the prices and demand schedule, the firm's policy p_{ti} solves* (14) *and inputs demand is optimal.*

4. *Nominal interest rate satisfies the Taylor rule* (7).

5. *Markets clear at each date:*

$$\begin{aligned}\int_0^1 \left(l_{ti} + I(p_{ti} \neq p_{t-1i})\theta_{ti}\right) di &= L_t \\ Y_t - \int_0^1 n_{ti} di &= C_t + \eta_{tg}\end{aligned}$$

Notice that menu cost implies real resources in the economy and therefore it has welfare implication with respect the optimal inflation target.

4 Equilibrium Conditions and Solution Method

This section describes the equilibrium conditions and the solution method applied to compute the equilibrium. Computing the equilibrium poses two challenges: 1) the combination of a Taylor rule for monetary policy and an infinite dimension in the aggregate state requires a non-standard application and evaluation of Krusell-Smith; 2) both aggregate and idiosyncratic policy functions have non-convexities..

For exposition, and for exposition only, I simplify the model in several dimensions in this section: preferences are given by expected utility $\sigma_{ez} = 0$ with period utility $u(C, L) = \log\left(C - \frac{L^{1+\chi}}{1+\chi}\right)$; the only input of production is labor $y_i = A_i l_i$ and menu costs are constant; the risk premium shock is the only structural shock; and the Taylor rule is given by $R_t = \max\left\{\frac{1+\bar{\pi}}{\beta}\left(\frac{\Pi_t}{1+\bar{\pi}}\right)^{\phi_\pi}, 1\right\}$. I abstract from growth rate; thus $g = 0$. The equilibrium conditions of the main model are in the appendix G and the equilibrium conditions of the Calvo model are in the online appendix. The description of numerical methods with all the steps to solve each model are in the online appendix.

4.1 Equilibrium Conditions

Firm's Equilibrium Conditions: the relevant state variable for firm i at time t is $\tilde{p}_{ti} = \frac{p_{ti}A_{ti}}{P_t}$, the relative price multiplied by productivity. For simplicity, I define this object as relative price. The relative price is the important idiosyncratic variable for the firm since the firm's demand and the static profits depend on it. Let $v(\tilde{p}_-, S)$ be the present discounted value of a firm with previous relative price $\tilde{p}_-$ and current aggregate state S. Then $v(\tilde{p}_{-}, S)$ satisfies

$$\begin{aligned}
v(\tilde{p}_{-}, S) &= \mathbb{E}_{\Delta a}\left[\max_{\text{change,no change}}\left\{V^c(S), V^{nc}\left(\frac{\tilde{p}_{-}e^{\Delta a}}{\Pi(S)}, S\right)\right\}\right] \\
V^{nc}(\tilde{p}, S) &= mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\} \\
V^c(S) &= -\theta w(S)mu(S) + \max_{\tilde{p}}\left\{mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\right\} \qquad (15)
\end{aligned}$$

where $mu(S), C(S), w(S)$ and $\Pi(S)$ denote the marginal utility, aggregate consumption, real wage and inflation respectively. The timing of the firms' optimization problem is as follows: first, aggregate and idiosyncratic uncertainty are realized, then the firm sets its price subject to the menu cost constraint.

There are two sources of fluctuation in the relative price: inflation and idiosyncratic quality shocks. After the change in the relative price due to these components, the firm has the option either to change the price or keep it the same. If it changes the price, it has to pay the menu cost θ.

The firm's problem depends only on what I define as relative price; any combination of nominal price and productivity that generates the same value of $\tilde{p}$ yields the same profits. This comes from the assumption that productivity shocks also affect the demand of the intermediate input.

The policy of the firm is characterized by two objects: (1) a reset price and (2) a continuation region. Let $P^*(S)$ be the reset price, i.e. the firm's relative price with respect to the aggregate price level. Then

$$P^*(S) = \max_{\tilde{p}}\left\{mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\right\} \qquad (16)$$

$P^*(S)$ does not depend on the idiosyncratic shock; it only depends on the aggregate state of the economy and therefore is the same across resetting firms. The continuation region is given by all relative prices such that the value of changing the price is less than the value of not changing the price. Let $\Psi(S)$ be the continuation region. Then

$$\Psi(S) = \{\tilde{p} : V^{nc}(\tilde{p}, S) \geq V^c(S)\} \qquad (17)$$

Since the firm makes the pricing decision after aggregate and idiosyncratic shocks are realized, the firm's policy is given by

$$\text{change the price and set a relative price equal to } P^*(S) \text{ if and only if } \frac{\tilde{p}_{-}e^{\Delta a}}{\Pi(S)} \notin \Psi(S) \tag{18}$$

As is typical in models with heterogeneity, the firm needs to forecast equilibrium prices and quantities and the aggregate state law of motion. If the firm knows these functions, then it has all the elements to take the optimal decision in (15).

Aggregate conditions: The aggregate conditions are given by the household optimality conditions, feasibility and the monetary policy rule

$$\begin{aligned} mu(S) &= \beta R(S)\eta_q(S)\mathbb{E}_{S'}\left[\frac{mu(S')}{\Pi(S')}|S\right] \\ \kappa L(S)^{\chi} &= w(S) \quad ; \quad mu(S) = \bar{u}(C(S) - (1+\chi)^{-1} L^{1+\chi})^{-1} \\ R(S) &= \max\left\{\frac{1+\bar{\pi}}{\beta}\left(\frac{\Pi(S)}{1+\bar{\pi}}\right)^{\phi_\pi}, 1\right\} \quad ; \quad C(S) = \frac{L(S) - \Omega(S)\theta}{\Delta(S)} \end{aligned} \tag{19}$$

where $\Delta(S)$ is labor productivity due to price dispersion given by

$$\Delta(S) = \int \tilde{p}^{\gamma} f(d\tilde{p}) \tag{20}$$

and $f(\tilde{p})$ is the distribution of $\tilde{p}$, after repricing—these are the relative prices that firms sell. $\Omega(S)$ is the measure of firms changing the price, with per price change cost in labor given by θ.

The system of equations (19) is the standard equilibrium conditions given by the Taylor rule, feasibility, the Euler equation and the intra-temporal Euler equation. Aggregate equilibrium conditions depend on two outcomes of the firm problem: inflation and price dispersion. Price dispersion only depends on the distribution of relative price and not the distribution of relative price and idiosyncratic productivity shocks. This is a direct consequence of the structure of the *quality* idiosyncratic shocks. To see this, notice that the technological assumptions over quality shocks imply that labor demand is given by

$$\int l_i(S)di = C(S)\left[\int \tilde{p}^{\gamma} f(d\tilde{p})\right] + \theta\Omega(S) \tag{21}$$

where $l_i(S)$ are firm's demand functions.

The key cross-equation restriction where the CalvoPlus model deviates from the Calvo model is the cross-equation restriction with respect to inflation. Next proposition shows the equilibrium condition for inflation:

Proposition 1 *Define*

$$\mathcal{C}(S) = \left\{ (\tilde{p}_-, \Delta a) : \frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)} \in \Psi(S) \right\}$$

Inflation dynamic is given by

$$\begin{aligned}
\Pi(S) &= \left(\frac{1 - \Omega(S)}{1 - \Omega(S) P^*(S)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi(S) \\
\Omega(S) &= \int_{(\tilde{p}_-, \Delta a) \notin \mathcal{C}(S)} f_-(d\tilde{p}_-) g(d\Delta a) \\
\varphi(S) &= \left(\int_{(\tilde{p}_-, \Delta a) \in \mathcal{C}(S)} \frac{\left(\tilde{p}_- e^{\Delta a}\right)^{1-\gamma}}{1 - \Omega(S)} f_-(d\tilde{p}_-) g(d\Delta a) \right)^{\frac{1}{1-\gamma}}
\end{aligned} \tag{22}$$

where $g(\Delta a)$ is the distribution of quality shocks innovations and $f_-(\tilde{p}_-)$ is the distribution of relative prices before repricing.

I define $\varphi(S)$ as menu cost inflation. The menu cost inflation characterizes the distribution of relative prices conditional of no price change. For example, in the Calvo model this variable is identical to 1.

Inflation is a function of three elements: reset price, the firm's inaction set, and the distribution of relative price at the beginning of the period. Inflation depends on two forward-looking variables (reset price and the Ss bands), and a backward-looking variable (the distribution of $\tilde{p}$). The forward-looking variables are a function of the firm's problem. The menu cost inflation and the frequency of price change are the outcome of the interaction between the distribution of relative prices—the state variable—and the Ss bands.

Aggregate State: Given that that inflation is the aggregation of the relative prices, the distribution of relative prices is the state in the economy. I denote with S the state of the economy with the law of motion $\Gamma(S'|S)$. Therefore the state of the economy is $S = (f(\tilde{p}_-), \eta^Q)$ with law of motion $\Gamma(S'|S)$.

4.2 Solution Method

To solve this model numerically, I modified the Krusell-Smith algorithm and I developed projection methods to approximate firm's policy function. Next, I describe each development in numerical methods.

Modification of Krusell-Smith Algorithm: Given that the distribution of relative prices is part of the state, I use the Krusell-Smith algorithm to solve this problem. However, the standard method to implement Krusell-Smith does not work for this problem. To explain why, first I will start with the simplest example that demonstrates both why this problem requires Krusell-Smith and why it cannot use that algorithm's standard implementation. Then, I briefly describe the modification of the Krusell-Smith algorithm and its evaluation in this model.

Krusell-Smith algorithm replaces a model's equilibrium conditions with an approximation of them. In this model—as in many others—this could generate indeterminacy at the time of solving the aggregate equilibrium equations. To explain this argument in detail, I will make the same assumptions as in Gertler and Leahy (2008) that idiosyncratic shocks are uniformly distributed and that firms can only change the price is they receive an idiosyncratic shock. Under these two assumptions at zero inflation target, it is possible to obtain the equilibrium equations of inflation and price dispersion.

Proposition 2 *Assume that firms can only change her price if they receive an idiosyncratic shock and*

$$A_{t+1}/A_t = \begin{cases} 1 & \textit{with probability } 1-p \\ U_t & \textit{with probability } p \end{cases} \tag{23}$$

where U_t is an uniform support, with support larger than the continuation region. Then at a first order approximation for the Phillips curve and a second order approximation for price dispersion, inflation and price dispersion follow

$$\hat{\pi} = \frac{(1-p)(1-\beta p)}{p}\hat{w}_t + \beta \mathbb{E}[\hat{\pi}_{t+1}] \quad ; \quad \hat{\Delta}_t = (1-p)\hat{\Delta}_t + \frac{p}{1-p}\hat{\pi}_t^2 \tag{24}$$

where $\hat{\pi} = \log(\Pi_t)$ and $\hat{w}_t = \log\left(\frac{w_t}{w_{ss}}\right)$ and w_{ss} is the steady state real wage.

Proof. The proof is a simple extension of Gertler and Leahy (2008) for the Phillips curve and Woodford and Walsh (2005) for the dynamic law of price dispersions. ∎

Under the approximation of the Phillips curve and the law of price dispersion, an equilibrium is a stochastic process for inflation, consumption, labor supply, marginal utility, real wage and nominal interest rate that satisfies the system of equations (19) together with (24). Assuming there is no ZLB, given that inflation and marginal utility are forward looking variables, there exists an unique equilibrium if this economy satisfies the Taylor rule principle, i.e. the interest rate respond strongly to inflation and real wage. The main reason why nominal interest rate responds to real wage is because there is an equilibrium equation given by the Phillips curve that relates inflation to real wage. *This is the critical property that Krusell-Smith breaks.*

Krusell-Smith algorithm consists of projecting price and quantities to a small set of moments of the distribution of relative prices—one of the states of this economy—and the exogenous state. Let us define S_{SK} the set of finite moments of the distribution in Krusell-Smith, $\Gamma(S'_{SK}|S_{SK},\eta'_q)$ the law of motion of the state, and $\Pi(S_{SK})$, $\Omega(S_{SK})$ and $\Delta(S_{SK})$ the projection of inflation and price dispersion with respect to the state. Formally, the algorithm is given by

1. Given $\Pi(S_{SK}),\Delta(S_{SK})$, $\Omega(S_{SK})$ and $\Gamma(S'_{KS}|S_{KS})$ solve aggregate conditions (19)
2. With the solution of (19), solve the firm's value function (15).
3. Simulate and update $\Pi(S_{SK}),\Delta(S_{SK}),\Omega(S_{SK})$ and $\Gamma(S'_{KS}|S_{KS})$ .Check convergence. If $\Pi(S_{SK}),\Delta(S_{SK})$ and $\Gamma(S'_{KS}|S_{KS})$ don't converge, go to step 1.

To my knowledge, all Krusell-Smith formulations have this approach.[6] Note that Krusell-Smith replaces equilibrium equations of inflation and price dispersion given by (24) with exogenous function of the state for inflation and price dispersion. The next proposition shows how the standard method generates multiplicity of equilibriums at the step of solving aggregate conditions.

Proposition 3 *For any $\Pi(S_{KS}),\Delta(S_{KS}),\Gamma(S'_{KS}|S_{KS},\eta'_q)$ and $\lambda>0$, if*

$$\{mu(S_{KS}),C(S_{KS}),L(S_{KS}),R(S_{KS}),w(S_{KS})\} \tag{25}$$

is a solution for (19), then $\{\lambda mu(S_{KS}),\tilde{C}(S_{KS}),\tilde{L}(S_{KS}),R(S),\tilde{w}(S)\}$ is a solution, where $\tilde{C}(S_{KS})$, $\tilde{L}(S_{KS})$ and $\tilde{w}(S)$ solves

$$\begin{gathered}\kappa\tilde{L}(S_{SK})^{\chi}=\tilde{w}(S_{SK}) \quad ; \quad \tilde{C}(S_{SK})=\frac{\tilde{L}(S_{SK})-\Omega(S_{SK})\theta}{\Delta(S_{SK})}\\ \lambda mu(S_{SK})=\bar{u}\left(\tilde{C}(S_{SK})-\frac{\tilde{L}^{1+\chi}}{1+\chi}\right)^{-1}\end{gathered} \tag{26}$$

Proof. It is easy to see that $\{\lambda mu(S_{KS}),\tilde{C}(S_{KS}),\tilde{L}(S_{KS}),R(S),\tilde{w}(S)\}$ satisfies all the equilibrium conditions. ■

[6]See Krusell and Smith (2006), Midrigan (2011) and Khan and Thomas (2008) for implementations Krusell-Smith implementation in different environments.

The reason why Krusell-Smith fails is similar to the Taylor principle in the New Keynesian model: nominal interest rates should respond strongly to inflation and real marginal cost. Krusell-Smith implementation generates an exogenous function for inflation and therefore an exogenous nominal interest rate. Importantly, nominal interest is not reacting to real marginal cost and consumption whenever solving aggregate conditions. It is a standard result that if nominal interest rates do not react to consumption, then there exist infinite solutions to the aggregate conditions (See **?**).

The main problem is that in the aggregate equilibrium conditions, there is no information on the relationship between inflation and real marginal cost, i.e. the Phillips curve. Replacing the Phillips curve with an exogenous function for inflation changes the equilibrium conditions of the model, implying multiplicity of equilibria whenever solving the aggregate equilibrium conditions.

If there were no ZLB constraint, I would have two options for solving the problem described above. The first is the one used in Reiter (2009), and implemented in menu cost models by Costain and Nakov (2011). Even without ZLB, I cannot use this method since the the aggregate state of the economy is unmanageably large due to the histogram approach for the approximation of the distribution. Additionally, the ZLB discards any perturbation methods due to the non-convexities at aggregate level. For this same reason, I cannot apply the method developed by Winberry (2014).

The solution I propose is to apply Krusell-Smith to the frequency of price change and menu cost inflation, and solve the aggregate conditions *together* with the firm's problem. Even if this method generates some numerical challenges—that I have solved— it provides the central bank with the cross-equation restriction of the intensive margin of the Phillips curve, breaking the multiplicity mentioned before. Through numerical computation, it seems a reliable method since it provides an unique solution whenever solving equilibrium conditions.

Before describing the solution, we need to find the state. In models with nominal rigidities, the important object for the repricing of the firm is the markup, the ratio between $\tilde{p}$ and real marginal cost. Therefore I use real marginal cost in the previous period as the state in Krusell-Smith. This variable is significant to predict menu cost inflation. Moreover, I use price dispersion in the previous period. This variable approximates the second moment of the relative prices distribution and predicts menu cost inflation and itself. Next I describe the algorithm used to solve the model.

1. Guess $\Delta(w_-, \Delta_-, \eta^Q), \Omega(w_-, \Delta_-, \eta^Q), \varphi(w_-, \Delta_-, \eta^Q)$ as a function of the state.

2. Solve for the equilibrium conditions: the joint system of (15), (19), and (22). Get the law of

motion for inflation and real wage $(mu, C, w, \Pi)(w_-, \Delta_-, \eta^Q)$, and the continuation set and reset price $(\Psi(w_-, \Delta_-, \eta^Q), P^*(w_-, \Delta_-, \eta^Q))$.

3. Simulate a measure of firms and compute $\{\Omega, \varphi, S\}_t$.

4. Project Ω, φ on the state. Check convergence. If not, update and go to step 2.

Note that the only law of motion of the state given in step 3 is the law of motion for price dispersion—what I define as Δ. The law of motion for the real marginal cost comes from the solution of the aggregate equilibrium equations. Since the business cycle fluctuations of price dispersion are small, note that law of motion for the endogenous state comes from step 2, solving the aggregate conditions, not from the simulation. Secondly, even without ZLB, I need to solve the model globally given the kinked property of idiosyncratic policies. Third, this methods breaks the separability in the equilibrium solution. With previous implementation of Krusell-Smith, it was possible to solve the *aggregate conditions separate from idiosyncratic conditions*, dividing the system into two sub-systems. Now, the aggregate and idiosyncratic equilibrium conditions must be solved together. The exact steps for the implementation of this method are in the online appendix, section CalvoPlus model.

Validity of the Modified Krusell-Smith Algorithm: I verify the validity of the Krusell-Smith method checking the equilibrium conditions with the simulated inflation, price dispersion and frequency of price change. I did not use the R^2 as most papers do, mainly because, when this variable has a small effect on the ergodic set of the equilibrium equation, its projections do not inform us of the fit of the model. To give a concrete example, price dispersion in the CalvoPlus model has low volatility at business cycle frequency and it has a R^2 of 0.96. Using standard criteria, we would conclude that Krusell-Smith didn't work in this case, but notice that the model satisfies feasibility with the predicted price dispersion at the third decimal in the simulation. Thus, using realized or predicted price dispersion does not affect the model solution for aggregate variable like consumption, interest, etc.

To verify the validity of the Krusell-Smith approximation, I construct the solution of the equilibrium equations in the simulation and with the Krusell-Smith approximation for each variable.Let $X(S_t^s)$ be the variable that solves the equilibrium conditions with the Krusell-Smith approximation and $\hat{X}_t^s$ be the solution of this variable with the simulated price dispersion, inflation and frequency of price change. The statistic I use to evaluate the fit of the approximation for the variable X is the ratio of the error in each variable with respect to the total variable, i.e. $\frac{Var(\log(X_t^s/)X(S_t^s))}{Var(X(S_t^s))}$. Constructing $\hat{X}_t^s$ is straightforward for the majority of variables, since they are the solution of a static system of equations. This is not the case when constructing simulated marginal utility, as this variable depends on expected inflation. To construct this variable, first I estimate inflation in the simulation and then I construct marginal utility in the simulation using the projected inflation. Appendix G describes the construction of the errors in each variable to verify the Krusell-Smith algorithm and figure 15 plots the predicted and simulated aggregate time series at zero inflation without zero lower bound.

As we can see in figure 15, the time series of each variable are on top of each other. Table 7 shows the errors $\frac{Var(\log(X_t^s/)X(S_t^s))}{Var(X(S_t^s))}$ for the levels of inflation target from 0 to 6. As we can see, even if the fit of Krusell-Smith decreases with inflation, even at 6% inflation target the fit is good enough. For example, the variance of the error of consumption with respect to the total consumption volatility decreases from 0.02% at zero inflation to 0.05% at six percent inflation. Moreover, marginal utility and period utility in the simulation are almost the same as with predicted equations. Table 8 shows the predicted error with ZLB. As we can see when the deflationary spiral is large enough, the fit decreases. This lost in prediction understate the effect of the zero lower bound and therefore its reinforces the normative results.

Projection Method: Due to the curse of dimensionality, I use Smoliak sparse-grid method as in Judd *et al.* (2014) and Krueger and Kubler (2004) with anisotropic construction. I found that sparse grid methods generates a good approximation for the aggregate conditions, but not for the idiosyncratic equilibrium conditions. Figure XXX shows firms' value function using Smoliak sparse-grid method for the idiosyncratic variable. As we can see in the figure, there is a bad fit with respect to the value function in different points of the state. The main intuition why Smoliak sparse-grid method fails to generate a good approximation is because this method cannot deal with large kinks and also does not generate enough grid points in the point near the Ss bands. To solve this problem, I extended Judd *et al.* (2014) and Krueger and Kubler (2004) Smoliak sparse-grid methods with splines projection methods. Given a grid for the exogenous idiosyncratic state and a sparse-grid for the aggregate state, I generate firms state grid using tensor product only between these two dimensions. I also generate the polynomial in this projection method using tensor product. Figure XXX plots firms' value function and in the online appendix I describe each step for this projection method.

5 Calibration

I calibrate the model to perform the quantitative evaluation of the optimal inflation target. I solve the model for two different calibrations: (1) the CalvoPlus model with finite menu cost and idiosyncratic quality shocks; and (2) the Calvo model with infinite menu cost and no idiosyncratic quality shocks. Solving the Calvo model provides me with a benchmark to compare against the CalvoPlus model and previous papers such as Coibion *et al.* (2012). The main reason I didn't use idiosyncratic shocks in the Calvo model is because price dispersion is infinite at levels of inflation target higher than 3%. The Calvo model with idiosyncratic shocks generates larger cost of inflation than with idiosyncratic shocks. Table 4 shows the model parameters.

The strategy to calibrate the model is to divide the parameters into three separate sets: (1) preference and technology; (2) menu cost and idiosyncratic shocks; and (3) Taylor rule and aggregate shocks. I externally calibrate the parameters for preferences and technology with micro-evidence of their empirical counterpart. I estimate the random menu cost and the idiosyncratic shocks processes to match micro-price facts in the data at 2% inflation target in the steady state of the CalvoPlus model. I verify ex-post that the model with business cycle reproduces similar facts. I calibrate the Taylor rule and the aggregate exogenous shocks to match their empirical counterpart during the period of the great moderation in the US economy. I find it impossible to estimate all the parameters in the CalvoPlus model since it takes one day to solve the model for a given set of parameters.

Preferences and technology: A period in the model is a month. Thus, I choose $\beta = 0.97^{1/12}(1+g)^{\sigma_{np}}$ because it implies a risk-free annual interest rate of 3%. I calibrate $g = 0.0017$ to match the annual US growth rate of 2%. The GHH preferences parameters are set to $\sigma_{np} = 1.5$ and $\chi = 0.5$ as in Greenwood, Hercowitz and Huffman (1988), which uses micro-evidence to calibrate these parameters. I set κ to match normalize labor supply to one and η_g to match the ratio of government expenditure over output equal to 0.25, the historic US average.

For the production function, I choose an elasticity between inputs γ equal to 5, an upper bound in micro–estimates.[7] For the production technology, I set the elasticity with respect to materials equal to 0.7 to match intermediate inputs over total output in the US economy. Finally, I calibrate τ_L to match an aggregate markup of 20%.

I set the Epstein-Zin parameter equal to -45 to match the cost of the business cycle representative agents model. From Alvarez and Jermann (2004) we know that there is a one-to-one mapping between the cost of business cycle and the different of rate of return of two assets: 1) an asset that pays aggregate consumption with business cycle fluctuation; and 2) an asset that pays the trend consumption. With this

[7] See Barsky, Bergen, Dutta and Levy (2003), Nevo (2001) and Chevalier, Kashyap and Rossi (2000) for micro–estimates and Burstein and Hellwig (2008) for an estimation in menu cost models.

result in mind and given a discount factor $\Lambda_{t,t+1} = \frac{Q_t P_{t+1}}{Q_{t+1} P_t}$, I compute the rate of return of the risk free and risk consumption assets

$$R_t^{rf} = \mathbb{E}_t \left[\left(\frac{(1-\beta)C_{t+1}^t + P_{t+1}^{rf}}{P_t^{rf}} \right)^{12} \right] \quad with \quad p_t^{rf} = \mathbb{E}_t \left[\Lambda_{t,t+1} \left((1-\beta)C_t^t + p_{t+1}^{rf} \right) \right] \tag{27}$$

$$R_t^{ra} = \mathbb{E}_t \left[\left(\frac{(1-\beta)C_{t+1} + P_{t+1}^{ra}}{P_t^{ra}} \right)^{12} \right] \quad with \quad p_t^{ra} = \mathbb{E}_t \left[\Lambda_{t,t+1} \left((1-\beta)C_t + p_{t+1}^{ra} \right) \right] \tag{28}$$

where $\{C_t^t, C_t\}_{t=1}^{\infty}$ are the log-linear trend of consumption and the total consumption respectively. Alvarez and Jermann (2004) estimates that $\mathbb{E}[R_t^{ra} - R_t^{rf}]100$ is in the interval $[0.19, 1.17]$. In my model, this premium is 0.24, in the lower bound of the cost of the business cycle. See online appendix for more details of this computation.

Random menu cost and quality shock stochastic processes: I estimate the random menu cost and quality shock stochastic processes to match micro-price statistics together with the physical cost of menu cost. The parameters to estimate are 5: the menu cost (θ), the probability of zero menu cost (hz), and parameters for the stochastic process of the quality shocks given by $(\{\sigma_{ai}\}_{i=1,2}, p)$. I choose these parameters by minimizing the distance between a number of moments in the steady state of the CalvoPlus model and in the data. Table 2 describes the moment in the data and in the model in the steady state with the estimated parameters. I choose the parameters so as to minimize the distance

$$\sum_{i=1}^{31} \text{weight}_i \left(\frac{\text{moment}_i^{data} - \text{moment}_i^{model}}{1 + abs(\text{moment}_i^{data})} \right)^2 \tag{29}$$

between a set of 31 moments in the model and in the data. For the moment in the data I choose the physical cost of the menu cost of 0.5% computed in Zbaracki, Ritson, Levy, Dutta and Bergen (2004) and Levy, Bergen, Dutta and Venable (1997). The physical cost of the price changes in the model is given by the model $\theta \frac{\Omega_{ss} - hz}{\eta_{ssg} + C_{ss}}$. The rest of the moments are with respect to the micro-price statistics computed with CPI price quotes.

I use monthly price quotes recollected for the consumer price index (CPI) micro dataset of the United Kingdom's Office for National Statistics (ONS). There are several advantages in this dataset. First, it is representative of the whole economy since it reflects all the prices in the consumer consumption basket. Second, it is publicly available from the time period 1996M1-2015M8. Third, it has similar micro-price facts as other low-inflation countries like the United States, Canada and the rest of the European Union countries.[8]

The ONS office surveys prices for goods and services in the United Kingdom. In total, there are 31 million price quotes in the time period between 1996 and 2015 at monthly frequency. Each product is classified by the Classification of Individual Consumption by Purpose (COICOP) at sectoral and class level.[9]

Since the price change distribution is critical to my calibration, and therefore for the cost of inflation, I apply several filters to the data to make it compatible with my model. I apply standard filters used in previous studies like Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008): I complete price quotes for temporary missing observations (less than half an year) and out-of-season with the last available price. Moreover, for the computation of the frequency of price change and moments of the price change distribution, I drop comparable and non-comparable product substitution and outliers.[10] Additionally, I drop months where there were changes in the VAT tax rate.

My model abstracts from sales and multi-sector heterogeneity; thus, I computed the micro-price facts filtering sales and sectoral heterogeneity from the data.[11] In the online appendix I report the table 2 with the different filters in the data.

- **Filter for sales:** The ONS data has a flag denoting sales for each price quote. I found that this flag does not characterize all temporal price decreases/increases where the price changes and then comes back to the same level. Therefore, first I drop all price changes with the sale flag. Then, I apply an additional filter that identifies sales: price changes where the price of the item before and after the price changes is equal.

  The algorithm of the filter consists of two steps: (1) identify the periods where a sale with T_s period starts; (2) drop price changes during a sale period. In the first step, I identify the first date for sale periods for each product:

$$\mathcal{D}^i_{T_s} = \left\{ t : \sum_{j=0}^{T_s} (p_{t+j} - p_{t-1+j}) = 0 \right\} \tag{30}$$

  The set $\mathcal{D}^i_{T_s}$ includes all dates with subsequence price changes where the price comes back to the original value. In the second step, I drop all price changes between t^* and $t^* + T_s$, where $t^* \in \mathcal{D}^i_{T_s}$.

[8] See Alvarez, Dhyne, Hoeberichts, Kwapil, Bihan, Lünnemann, Martins, Sabbatini, Stahl, Vermeulen *et al.* (2006) and Kryvtsov and Vincent (2014).

[9] See sectors and classes in online appendix.

[10] For the moments of the price change I dropped the lowest 2% and the highest 98% price changes for all the products and dates. I re-adjust the frequency of price change to take into account these price changes.

[11] See online appendix for more details.

I apply the filter for $T_s = 1, 2, 3$. Around 10% of the total price changes are identified with sales flags and another 20% of the total price changes are dropped with my filter.

- **Filter for product heterogeneity:** There is a large heterogeneity across different product. For example, the average price change across classes goes from 0.01 to 0.1 and the standard deviation of price changes across product goes from 0.07 to 0.4. In order to control for heterogeneity, I generate a normalized price change distribution denoted by $g(\Delta\tilde{p})$ from the original price change distribution $f(\Delta p)$. If Δp_t^i is the price change of item-location-shop i at time t, and with the item j, then I generate the normalize price change $\Delta\tilde{p}_t^i$ as

$$\Delta\tilde{p}_t^i = \frac{\Delta p_t^i - \mathbb{E}[\Delta p_t^i | i \in \text{item j}]}{\mathbb{S}td[\Delta p_t^i | i \in \text{item j}]}\mathbb{S}td[\Delta p_t^i] + \mathbb{E}[\Delta p_t^i] \tag{31}$$

  where $\mathbb{E}[\Delta p_t^i | i \in \text{item j}], \mathbb{S}td[\Delta p_t^i | i \in \text{item j}]$ denotes the product-level mean and standard deviation of price changes and $\mathbb{S}td[\Delta p_t^i], \mathbb{E}[\Delta p_t^i]$ denotes the aggregate mean and standard deviation of price changes. In the appendix, I show that this aggregation cleans heterogeneity in the Calvo, menu cost and Taylor model. Moreover, without this aggregation kurtosis is more the outcome of heterogeneity than the pricing model.

  By construction, this transformation does not affect the mean or the standard deviation of price changes, but it affects higher-order moments of the distribution. For example, the kurtosis of price changes without this transformation is 9, while with this transformation it is 3.7.

Each filter affects micro-price moments, and therefore the estimated parameters and the cost of inflation. Sales increases the frequency of price and the mean absolute price change. Frequency affects the cost of inflation by standard channels. A higher mean of absolute price changes increases the estimated variance of the idiosyncratic shocks, reducing the cost of inflation.

Kurtosis is key to identifying the ratio between free and not-free price adjustment. In the model, a low kurtosis implies a low ratio of free-price-adjustment over total-price-adjustment and a high kurtosis implies a higher ratio. This ratio affects the cost of inflation and therefore the optimal inflation target. The data shows a large heterogeneity across sectors and this implies a large kurtosis of 8.[12] Most of this kurtosis is due to heterogeneity across, not within, sectors in the price change distribution.

Taylor rule and aggregate shocks: I calibrate the stochastic process for aggregate productivity to reproduce business cycle statistics of linear trended output per hour computed in Cociuba, Prescott and

[12] For example, the Calvo model without fat tails generates a kurtosis of 6, see Álvarez, Le Bihan and Lippi (2014)

Ueberfeldt (2009) during the great moderation (1983Q1-2006Q4). The persistence of output per hour is 0.97 and the standard deviation is 0.0173 at quarterly frequency—I use $\rho_z = 0.985$ and $\sigma_z = 0.0055$ at monthly frequency. For the process of the government expenditure shocks, I use the estimates of Del Negro, Schorfheide, Smets and Wouters (2007) to calibrate the stochastic process of government expenditure shocks since they restrict their estimation to the great moderation, excluding the time period of large government expenditure shocks in the 1960s. I use their DSGE-VAR approach to calibrate the government expenditure shocks. The persistence of government expenditure shocks is 0.9 and the standard deviation is 0.069 at quarterly frequency—I use $\rho_g = 0.95$ and $\sigma_z = 0.0022$ at monthly frequency. Similar results can be obtained using a VAR with short run restrictions during the great moderation as in Ravn (2005).

It is difficult to find an empirical counter-part to the risk premium shocks; therefore, I partially follow Coibion *et al.* (2012) calibration strategy. I calibrate the persistence of the risk premium shock to match the routinely high persistence of the risk premium in the financial series as in Coibion *et al.* (2012). I target a quarterly persistence of the risk premium shock to 0.94—I use $\rho_q = 0.97$.

The innovation of the risk premium shocks are chosen to match the international evidence of the frequency of hitting the zero lower bound across countries. To calibrate the probability of hitting the ZLB, I construct an international database for CPI inflation and policy rates of the majority of countries around the world from the International Financial Statistics of the IMF and FRED. Appendix A describes the detail in the calculation and table 1 describes mean inflation and mean probability of hitting the zero lower bound for each country. I discard time periods before 1988Q1, since before that date central banks did not incorporate implicit or explicit inflation targeting. Then, I drop all countries with average inflation more than 4% such as Mexico and Peru. In the sample, there are countries that hit the ZLB around 30% and 55% of the time, such as Switzerland and Japan, but also countries such as South Korea, New Zealand and Australia that hadn't hit the ZLB. I compute the average inflation across countries and the probability of hitting the zero lower bound. The mean inflation is 2.2% and the probability of hitting the ZLB is 14%. I calibrate $\sigma_q = 0.0005$ to match this probability.

The parameters for the Taylor rule are taken from Del Negro *et al.* (2007).[13] These are given by $(\phi_r, \phi_\pi,) = (0.76, 1.80)$ at quarterly frequency—I use $\phi_\pi = 1.8$; and, since my model is monthly, I use $\phi_r = 0.86$. As in Del Negro *et al.* (2007), I set $\rho_r = 0$ and $\sigma_r = 0.00115$—they use $\sigma_r = 0.0018$ at quarterly

[13] I cannot reproduce their result for the Taylor rule in my model whenever I run a regression with their Taylor tule for two reasons: 1) time aggregation from monthly to quarterly frequency generates bias estimator; 2) there are differences in our definition of output gap. Despite this differences, if I run a linear regression in my model with respect to their Taylor rule I get $(\phi_r, \phi_\pi, phi_y) = (0.74, 1.84, -0.4)$ with standard errors of 0.139.

frequency. Standard formulation of DSGE models ignore price dispersion from output gap definition in the Taylor rule; thus, I use real marginal cost in the Taylor rule instead of output gap to be consistent with the previous estimates. Since aggregate output can be written as a function of real marginal cost, price dispersion, productivity shocks and government expenditure shocks—see section 7—the model has an elasticity marginal cost to output of 6.64. Therefore, I re-scale the parameters of the Taylor rule $(\phi_y, \phi_{dy}) = (0.095, 0)$ by $\mathcal{E}_{GDP,mc} \approx 6.64$ since I use real marginal cost in the Taylor rule.[14]

Whenever I compute the optimal inflation target in the Calvo model, I use indexation of price with respect to business cycle fluctuation of inflation. I set indexation to 60% to have stability in the Calvo model at 5% inflation target. Without inflation indexation, I find instability at 3% inflation target. See online appendix for model details.

6 The Model At Work: Positive Implications of the Model

This section provides compelling evidence of the positive dimensions of the model before analyzing the optimal inflation target. On the micro-side, the model is able to generate the price change distribution observed in low inflation countries, as well as the relationship between the frequency of price change and inflation variance decomposition with respect to different levels of inflation. On the macro-side, the model has similar business cycle dynamics to Calvo—the main model used by central banks.

6.1 Micro Moments of the Model

The price change distribution in low inflation economies has several characteristics that are robust across countries.[15] First, prices respond more to idiosyncratic reasons than aggregate. For example, almost one-half of the price changes are price decrease, and conditional on a price change, the average price change is around 10%. Since aggregate inflation is positive, low and with low volatility in low inflation economies, only idiosyncratic shocks can explain these facts. Second, there are a large amount of small and large price changes in the price change distribution that cannot be accounted for by an off-the-self menu cost model. For example, the 10th and 90% percentile of the absolute value of the price change are around 1-3% and 20-25% across different studies.

[14]The elasticity can be approximated by

$$\mathcal{E}_{GDP,mc} = \frac{1+\chi\alpha}{\chi(1-\alpha)} - \frac{\xi}{1-\xi mc_{ss}} \qquad with \qquad \xi = \left(\frac{\alpha\iota^{-1/(1-\alpha)}}{1-\alpha}\right)^{1-\alpha} \tag{32}$$

[15]See Klenow and Malin (2010) for a review of the studies.

These common facts across countries hold in the UK economy as table 2 describes. The mean price change in absolute value is 10% and the median is 1%. Therefore, prices respond more to idiosyncratic reasons than aggregate in UK as the rest of the countries—for example, in US Midrigan (2011) describes exactly the same numbers. Additionally, the kurtosis is 3.8; and the 10th and 90th percentiles are 1% and 23%. An off-the-self menu cost model generates prices concentrated in the Ss bands; thus it cannot reproduce this fact.

The CalvoPlus model with mixed normal distributed idiosyncratic shocks can account for the price change distribution in UK—and therefore in low inflation economies. Figure 1-panel A describes the price change distribution in the UK economy, in the Calvo and CalvoPlus model. For the data, I applied the filter described in section 5. Several features of the CalvoPlus model allow the model to match the data. The random menu cost in the model generates small price changes—almost 30% of price changes are given with zero menu cost. The mixed normal distribution generates large price changes with relative small Ss bands; without it, the model cannot generate large price changes. Finally, as I remark in section 5, since the price change distribution depends mainly on the idiosyncratic shocks, the model with business cycle dynamics generates the same price change distribution as the steady state economy—see column 2 and 3 of table 2.

The Calvo model—or any pricing model—without idiosyncratic shocks cannot replicate the price change distribution. This property is the direct consequence of the fact that aggregate shocks are relatively small in comparison to idiosyncratic shocks, and therefore this model cannot reproduce the observed price distribution as in the data.

Result 1 (Micro-price statistics at 2% Inflation Target I) *The CalvoPlus model generates the price change distribution as in the UK economy.*

The first observable feature in the model is the size of the price change, and the model is able to match it perfectly. The second observable in the model is the time between price changes. To compare the data with the model I exploit the large number of price changes for the same item in the UK database, and estimate in the data and in the model the relative hazard rate.

It is well known that heterogeneity of the frequency of price can generate a decreasing hazard rate due to the survival bias. Theoretically, it is possible to solve survival bias working with the relative stopping times I define below. Let τ_i be the time between price changes for product i. I define the relative stopping time for each item as

$$\hat{\tau}_i = \frac{\tau_i}{\mathbb{E}[\tau_i]} \tag{33}$$

The aggregate stopping time is given by

$$\hat{\tau} = \begin{cases} \hat{\tau}_1 & \text{with prob. } w_1 \\ \hat{\tau}_2 & \text{with prob. } w_2 \\ \vdots & \\ \hat{\tau}_n & \text{with prob. } w_n \end{cases} \tag{34}$$

where w_n is the relative size of each firm. In the appendix I show that this method aggregates the hazard rate without generating the survival bias in the three most used models: Calvo, Menu Cost and Taylor. The main intuition of this result is that the heterogeneity in firm's stopping times does not reflect in the distribution of the *relative stopping time*.

This method has a key challenge: the estimation of the mean of price change for a particular item. Since on average price changes have a frequency of 1 year, and the items are in the sample around 3 years, it is impossible to generate a good estimate of the expected time of price change at item level. To solve this challenge in the data, I exploit the structure of the UK database. The ONS in UK computes the aggregate inflation in the following way: they compute inflation at item level—for example, cereal box for a particular brand and a particular size—and then they aggregate using CPI weights. For this methodology to be valid, they sample the same item at different stores and different locations in UK. Under the assumption of similar pricing model across these two dimensions, it is possible to obtain a large sample of stopping times for each item; thus, it possible to apply this methodology. I replace $\mathbb{E}[\tau_i]$ to be the mean expected time across shops and locations. To estimate the expected time of price change, I only use items that have at least 100 price changes. This restriction only drops 0.25% of the sample of price changes. Moreover, I use CPI weights at item level for the weights w_i, another special feature of the CPI construction in UK.

Figure 1-panel B shows the relative hazard rate in the UK and in both models. In the Calvo model, no matter the relative distance to the mean, the probability of price change conditional of no price change is constant. In the CalvoPlus model, this probability involves weak increases until it is twice the mean, then remains constant. Notice how these two models have similar relative hazard rate. In the data there is a decreasing hazard rate. After a price change, relative to the average time between price changes,

price changes have a high probability that is decreasing with respect to age. Both models cannot match the decreasing hazard rate for the first few months. For models with decreasing hazard rate see Baley and Blanco (2013) and Bachmann and Moscarini (2011).

Result 2 (Micro-price statistics at 2% Inflation Target II) *The Calvo and the CalvoPlus models have weakly increasing relative hazard rates, while in the UK data there is decreasing relative hazard rate of price adjustment.*

Low inflation price statistics have the capacity to identify the ratio between the frequency of free adjustment and total adjustment. For example, the estimation could deliver an almost Calvo pricing model with $hz \approx 1$ and sufficient large adjustment cost. But to match a kurtosis of price change equal to 3.7, the model estimates a ratio between the frequency of free adjustment and total adjustment equal to 0.34. A question that arises is the following: is this calibration consistent with micro-pricing behavior at higher inflation environment?

I compare data and model with respect to mean of frequency of price change and inflation. First, I construct a panel data of countries with frequency of price change and inflation. Table 3 describes the set of countries from which I get the international evidence of price change and inflation. Then I regress project frequency of price change to inflation with the following model

$$freq_{ti} = \alpha_i + \beta_0 + \beta_1 inf_{ti} + \beta_1 inf_{ti}^2 \tag{35}$$

where $freq_{ti}, inf_{ti}$ are monthly frequency of price change and annual inflation. Since the mean level of frequency of price change differs from the benchmark comparison, I renormalize the figure in levels. The solid red line in figure 2 describes the fit in the intentional evidence. This relation captures a business cycle comparison between inflation and frequency, so it is not a perfect comparison.

In order to get a better comparison between model and data, I compute product level mean of frequency of price change and inflation. For each product, I constructed a frequency of price change across stores and locations and mean inflation. I then repeat the same regression as before

$$freq_i = \beta_0 + \beta_1 inf_i + \beta_1 inf_i^2 \tag{36}$$

This methodology generates a large sample of mean inflation and mean frequency of price change for each product in the UK CPI. The blue solid line describes the fit.

The steady state of the model has an almost perfect fit with the micro-data and it predicts a lower slope than the international evidence suggest. In figure 2 the black, green and yellow lines describe the mean frequency and inflation targets in the model at the steady state, and without and with zero lower bound. The frequency is decreasing in the model with business cycle since at low inflation target the mean inflation is lower than the target. The minimum frequency is around 3-4% inflation and then start to increase again. Since I cannot solve the model for high levels of inflation because Krusell-Smith starts losing predictive power at 6% inflation target, I will focus on the steady state to compare the model with the data.

Result 3 (Micro-price statistics high inflation target) *The CalvoPlus model in the steady state generates an increasing frequency of price with similar magnitude as in the micro-data and international evidence.*

6.2 Equivalence Aggregate Dynamics at 2% Inflation Target Without Zero Lower Bound Constraint Between Calvo and CalvoPlus Model

This section shows that the Calvo and CalvoPlus models have similar aggregate dynamics at 2% inflation without zero lower bound constraint on nominal interest rate. To formalize this argument, I show that an econometrician with only aggregate data could not distinguish which model generates it in a finite sample with the same length of the great moderation. Moreover, both models have a good fit with the US macroeconomic time series during the great moderation—1984Q1-2006Q4— for a medium scale DSGE model.[16]

The quantitative statement that the CalvoPlus and the Calvo models have similar business cycle dynamics extends previous findings to a medium scale general equilibrium model with endogenous monetary policy, and a richer structure for pricing behavior. This result is not obvious, since in the case of Gertler and Leahy (2008), they assume that firms cannot change their price if they don't receive an idiosyncratic shock—an assumption that I don't use. Additionally, their paper approximates firm's policy to derive the Phillips curve—a second assumption I don't use. Midrigan (2011) uses only small money shocks; in my model I have larger empirically relevant shocks for the US economy.[17] Moreover, my model doesn't have the specific structure theirs does, since I have CalvoPlus pricing and a mixed normal distribution for the idiosyncratic shocks. Furthermore, the fact that nominal interest rate depends on inflation partially mitigates different Phillips curves across models—I develop this argument below.

In order to show the claim of similar aggregate dynamics, I compute two sets of statistics: impulse-response functions with respect to the structural shocks, and business cycle statistics like standard deviation, persistence and correlations. Next I describe the impulse-response and the business cycle statistics.

Impulse-Response Function to Structural Shocks: I computed each impulse-response as an econometrician would do it; and I ask whenever these two models are different in a finite sampling of 22 years—the length of the great moderation. From random initial conditions, I simulate each model for the time period of 22 years and estimate a VAR with its respective impulse response; I repeat this procedure 5000 times. In both models, the impulse-response are random variables with their respective confidence intervals that allows me to make statistical claims as an econometrician. See the online appendix for more details on the computation of the impulse-response. Figures 3 to 6 plot the median impulse-response function with respect to the four structural shocks without zero lower bound for the

[16] In this section, I turn off business cycle inflation indexation in the Calvo model.

[17] He uses only money shocks with a quarterly persistence 0.6 and innovations of 0.0018.

Calvo and CalvoPlus model, together with the $[15, 85]$ interval confidence of the the difference of the impulse-response in the Calvo and the CalvoPlus model.

For productivity, government and monetary shocks, the median impulse-response of the Calvo and CalvoPlus models are on top of each other for output and nominal interest. As these figures show, the difference of the impulse-responses $[15, 85]$ interval confidence covers zero always. A similar pattern holds for the inflation dynamics; the $[15, 85]$ interval confidence of the difference between the two models always covers the zero, but the median has a different behavior. Inflation has a larger impact effect on the CalvoPlus model than in the Calvo model, but also lower persistence.

The fact that equilibrium dynamics are similar does not imply that the Phillips curve in both models is similar. The intuition of this result is that a steeper Phillips curve is partially offset by movement in the nominal interest rate. To explain this argument, assume a structural shock that increases the output gap. Since nominal interest rate depends on inflation, in the CalvoPlus model nominal interest will increase by more; consequently, the equilibrium output gap responds by less and therefore so does inflation. This result will break with the zero lower bound on nominal interest rate.

The result that business cycle dynamics are indistinguishable across models mitigates with respect to risk premium shocks. The risk premium shock generates a large and persistent drop in the marginal cost. A positive innovation of 1 standard deviation generates a drop in output of 0.3% with a half-life of approximately 1 years. As we can see in figure 6, inflation is significantly lower in the CalvoPlus model after a risk premium for the first two quarter, thus yielding a larger drop in nominal interest rate and lower drop in output. As I explain before, the CalvoPlus model generates lower fluctuation of output, since inflation is more sensitive in this model for large shocks and therefore the nominal interest rate.

Result 4 (Macro-dynamics in CalvoPlus and Calvo model—Result I) *The difference of impulse-response functions of the Calvo and CalvoPlus models cover zero with a* $[15, 85]$ *interval confident of the CalvoPlus model for US productivity, government expenditure and monetary shocks. For the risk premium shock, it does not cover zero for the first two quarter with respect to inflation.*

Business Cycle Statistics: Table 5 shows the business cycle statistics in the Calvo and CalvoPlus models. Additionally, I include US business cycle statistics from the great moderation as an empirical guide.[18] It is important to remark that I didn't calibrate any structural shocks to match any business cycle statistics.

[18] It is difficult to have an empirical counterpart for this exercise, since in the data there is a zero lower bound on nominal interest rate. Nevertheless, I include the data to have a guidance of the magnitude of US business cycle.

To compute business cycle statistics, I repeated a similar methodology as the impulse-response. From random initial conditions, I simulate each model for the time period of 22 years and estimate a set of business cycle statistics; I repeat this procedure 5000 times to obtain the distribution of each statistics.

The CalvoPlus generates business cycle statistics close—or even equal—to the Calvo model. As we can see in the last four columns, the [15,85] percentiles of the difference of the standard deviation, persistence and correlation in both model always includes the zero. The main difference is that the CalvoPlus model generates lower volatility of output and higher volatility of inflation; persistence and correlations with respect to output and inflation are almost identical in both models.

Result 5 (Macro-dynamics in CalvoPlus and Calvo model—Result III) *The difference of standard deviation, persistence and correlations in the Calvo and CalvoPlus models are not statistically significant at 5% confidence.*

How close are are business cycle statistics in the model to those in the data? To compare the model with the data, I computed business cycle statistics in the US economy during the period between the first quarter of 1984 and the last quarter of 2006. I focus on this time period for two reasons. First, this period has a stable inflation at a two percent trend inflation level, as Coibion and Gorodnichenko (2011) and others have shown, as in my model. From 1970 to 1984, there were movements in the trend inflation during the Burns inflationary period and the Volker stabilization period. Second, I focus on the period before 2007, where the zero lower bound was not binding, since the dynamics of the Calvo and CalvoPlus model in the ZLB are different. The description of the variables in the data is in the online appendix.

As we can see, the model is able to reproduce all business cycle statistics in the data except for the volatility of output, labor and inflation. Notice that with the assumptions over preferences and technology and pricing model, the model does not over-predict the volatility of inflation; even more, it generates a lower volatility of inflation. This is important, since it will generate a quantitative significant deflationary spiral during zero lower bound periods. Additionally, the model under-predicts on average the volatility of output and consumption variables with respect to the data. These results seem understandable since I'm comparing the model without zero lower bound constraint.

The equivalence result does not holds with either of these two features: monetary zero lower bound or positive inflation. For a detailed explanation of how the zero lower bound breaks this equivalence between aggregate dynamics between models, see the next section. Additionally, higher inflation levels change the Phillips curve in both models, and therefore the aggregate dynamics.

6.3 Business Cycle Analysis with Zero Lower Bound

This subsection focuses on understanding the macroeconomic effect of the ZLB over aggregate dynamics; more specifically, understanding a liquidity trap in a state dependent model for different levels of inflation targets. A liquidity trap is a situation where after an aggregate shock the real interest rate is too high. This leads to excessive saving, and since nominal interest rate cannot decrease due to the ZLB, this exacerbates the depression of spending and output, which in turn, creates more deflationary pressures.

Analysis of Deflationary Spiral With Non-Linear Impulse-Response: For understanding the macroeconomic dynamics during a liquidity trap I will compute conditional non-linear impulse-response of the CalvoPlus model. Positive inflation target and ZLB constraint generate non-linearities in the aggregate dynamics. In non-linear models, there are several different ways to construct the associated impulse-response. I proceed as follows: I generate 20000 random draws of the state in both models—in the case of the CalvoPlus model, the state includes the distribution of relative prices. Each draw is going to be one seed. From the initial distribution, I keep seeds that satisfy the condition that nominal interest rate is less than 30th percentile of the distribution, and generate a new random draw of 5000 new seeds that satisfy this criteria. Each of these seeds is an economy that I will shock to compute the impulse-response. I shock each economy at t=2 for all 5,000 samples with the same shocks; from then on, I continue independently simulating each economy. The impulse response is the average over the 5,000 economies for each period. Figures 7 and 8 show the impulse-response of consumption, inflation, nominal interest rate, frequency of price change, reset price and Menu Cost inflation for the Calvo and CalvoPlus model at one and four percent inflation levels. Since the main shock that triggers the ZLB is the risk premium shocks, I hit all the economies with a large positive risk premium shock given by $2\sigma_Q$ to analyze the dynamics during a binding ZLB.

At one percent inflation there is a *persistent* increase in the fraction of repricing firms due to the history of negative inflation—notice that initial inflation is lower than zero. The key feature in menu cost models is that the increase in the fraction of repricing firms is not random across firms. This new mass of repricing firms hits the upper Ss band with a large downward price adjustment.The new repricing firms decrease inflation and since the zero lower bound is binding, this initial drop in inflation increases real interest rates, even further depressing the consumption and inflation. Since the increase in repricing increases the deflationary spiral, the shadow nominal interest decreases by more; thus there is a prolonged decrement of the nominal interest rate with respect its mean level.

Result 6 (ZLB dynamics at 1 Percent Inflation—Result I) *When the economy hits the ZLB in the CalvoPlus model, there is a persistence increase in the frequency of price change of those prices selected to be large. This increases the deflationary spiral, worsening the macroeconomic effect of negative shocks.*

The key assumption to break this deflationary spiral is the elasticity of the real marginal cost with respect to real interest rates, to rule output explosive dynamics. The strong strategic complementarities and GHH preferences decrease the elasticity of the real marginal cost significantly with respect to real interest rates, ruling out explosive dynamics.

At higher inflation target the CalvoPlus model yields a much weaker deflationary spiral, to the point where it exactly equals the Calvo model (see figure 8). After a negative risk premium shock there is an immediate decrease in the frequency of price change. Since this is not persistent, it does not affect the Euler equation and therefore the dynamics of output gap. Since after this, frequency stays almost fixed—even decreases slightly—inflation dynamics and therefore aggregate dynamics are identical in both models.

Result 7 (ZLB dynamics at Three Percent Inflation—Result I) *When the economy hits the ZLB at 3 percent inflation in the CalvoPlus model, there is no persistent change in the frequency of price change. Only prices with a zero menu cost or a large idiosyncratic shock change; thus, the deflationary spiral and the output gap dynamics are the same as in Calvo.*

To get a better intuition of the previous argument, figure 9 describes the cross section distribution of relative price at zero and two percent inflation. The blue line describes the average distribution conditional of being in the zero lower bound and the black line describes the average distribution outside the zero lower bound. We can see that at zero inflation target outsize the zero lower bound, the distribution of relative price is symmetric around 1. This characteristic changes at the zero lower bound; the mode of the distribution is lower, since the reset price responds to the negative macro-economic conditions, but even with a lower reset price, the distribution of relative price is tilted toward high relative price. This asymmetry toward the upper Ss bands activates large price changes in upper Ss bands. We can see that this property does not apply at 3% inflation target, since the distribution outside the ZLB is tilted towards the lower Ss and it becomes symmetric at the zero lower bound.

A higher inflation target decreases the business cycle volatility of the output gap in the CalvoPlus for two reasons. First, and trivially, it decreases the probability of hitting the zero lower bound. Second, in the CalvoPlus menu cost models the inflation target determines the extent to which the aggregate price level reacts to aggregate shocks in periods where the ZLB is binding. At zero inflation target, the deflationary spiral is large due to the increase in the selection effect. This effect decreases with higher inflation targets.

Business Cycle Statistics with ZLB in the CalvoPlus Model: Another measure to analyze the interaction between trend inflation is to analyze business cycle statistics. Table 6 describes business cycle statistics in the model with zero lower bound in the Calvo and CalvoPlus model at 1% and 3% inflation target.

The first row describes the frequency of the zero lower bound. In both models the frequency of hitting the zero lower bound decreases significantly from 1 to 3% inflation. To understand better the zero lower bound dynamics I computed moments of the distribution of the length of zero nominal interest rate.

As we can see in table 6, the model has fat tails in the length of the zero lower bound at 1% inflation target, especially the CalvoPlus. For example, the median length is 1 years—with a mean of two and a half years. The 75th percentile and the 95th percentile are of 3 years and 9 years. At higher levels of inflation target, both the mean and the standard deviation of the zero lower bound decreases—significantly in the CalvoPlus model. As I explain with the impulse-response function, the aggregate price flexibility in periods at the zero lower bound increases significantly at low inflation targets. The endogenous deflationary spiral that occurs whenever firms optimize implies fat tails in the model, thus larger endogenous disaster risk.

The models cannot generate infrequent, but long periods of zero lower bound as in the data. This is given by the structure of aggregate shocks. The aggregate shocks in my model follow an AR(1); thus, the economy hits the zero lower bound whenever there is a large history of negative shocks. Dordal-i Carreras, Coibion, Gorodnichenko and Wieland (2016) explores the addition of fat tails in the aggregate shocks to match this feature in the data, and found that this feature increases even more the optimal inflation target.

Rows 8 to 12 describe the moments of marginal cost conditional to being in the ZLB or not. Remember that in these models marginal cost is co-linear with output gap. As we can see, higher inflation target decreases the outgap in states whenever the ZLB is binding or not binding. Interestingly, this gap becomes equal in the the Calvo and the CalvoPlus model for a sufficiently high level of inflation (3%). The zero lower bound not only changes the mean but also changes the volatility of the output gap. In the CalvoPlus model output-gap volatility is three time higher during periods when the zero lower bound is active.

Inflation behaves similarly to marginal cost whenever or not the zero lower bound is active. The first difference is the reversal of inflation volatility in the CalvoPlus model. In this model, at 3% or higher levels of inflation volatility is higher whenever the zero lower bound is not active. Intuitively, inflation is more responsive to positive shocks than negative shocks. The second difference is that higher inflation target makes inflation more volatile in the Calvo model.

7 Welfare Analysis and The Optimal Inflation Target

This section quantitatively studies the optimal inflation target in the CalvoPlus model. I also compare the results of this model with the Calvo pricing model—the standard workhorse model used for monetary policy. The first section analyses welfare though basic moments in consumption, labor supply and nominal interest. To understand better the welfare analyses, I continue the analyses through wedges of the New Keynesian model with respect to the Neoclassical Growth model. As I show later, different pricing model, different levels of inflation or even the ZLB effect welfare though these wedges. Finally, I show that the key to generate low cost of inflation is the interaction between menu cost and idiosyncratic shock.

7.1 The Optimal Inflation Target

The optimal level of inflation with zero lower bound constraint in the Calvo model is 1%; it is 3% in the CalvoPlus model. For this result, it is necessary to have the zero lower bound constraint on nominal interest rates since without it the optimal level of inflation is less than 0% in both models.

Figure 10-panels A and E describe welfare using the consumption equivalence measure with respect to zero inflation. Each point in the y-axis reads the percentage increase in the consumption to equalize the welfare at zero inflation. If we define the following recursion

$$U^{\bar{\pi}}(S,\lambda) = u\left(\left(1+\frac{\lambda}{100}\right)C(S), L(S)\right) + \beta g^{1-\sigma_{np}}\mathbb{E}_{S'}\left[U^{\bar{\pi}}(S,\lambda)^{1-\sigma_{ez}}\,\middle|\, S\right]^{\frac{1}{1-\sigma_{ez}}} \tag{37}$$

for welfare in state S with an increase in consumption given by λ at the inflation target $\bar{\pi}$, then the consumption equivalence with respect to zero inflation is given by[19]

$$\lim_{T\to\infty}\sum_{t=1}^{T}\frac{U^{\bar{\pi}}(S_t^{\bar{\pi}},\lambda(\bar{\pi}))}{T} = \mathbb{E}\left[U^{\bar{\pi}}(S,\lambda(\pi))\right] = \mathbb{E}\left[U^{0}(S,0)\right] = \lim_{T\to\infty}\sum_{t=1}^{T}\frac{U^{0}(S_t^{0},0)}{T} \tag{38}$$

Without zero lower bound, increasing the inflation target from 0 to 5% in the Calvo model is equivalent to decreasing consumption by 4%, while this number falls to 1.5% in the CalvoPlus. In this case the consumption equivalent is increasing for all levels of inflations. With zero lower bound the consumption equivalent is decreasing at low levels of inflation and then increasing. In the CalvoPlus model the optimal inflation target is 3%, with a consumption equivalent with respect to zero inflation of 2%. In the Calvo model, the optimal inflation target decreases to 1% with a consumption equivalent less than 0.1%.

[19]To compute this statistic and the ones described below, I use Monte-Carlo integration over $T = 2399980$ simulations.

Figure 10-panels B and F describe the probability of hitting the zero lower bound, with and without zero lower bound. Without zero lower bound, both models predict a similar probability of negative interest rates—slightly higher in the CalvoPlus model. With zero lower bound, the probability of hitting the zero lower bound increases with respect to a model without this restriction. The main intuition of this result, is that the deflationary spiral endogenously increases the length of periods in the zero lower bound. The deflationary spirals have a large magnitude in the CalvoPlus model; thus the the probability of hitting the zero lower in the CalvoPlus model is one-fourth larger in the CalvoPlus model than the Calvo model.

The main cost of higher inflation target is a decrease in the consumption-labor ratio, as panel C and D in figure 10 show. The consumption-labor ratio in the Calvo model falls by almost 3%, while the consumption-labor ratio in the menu cost falls by almost 1%. The first property that we can see is that the consumption-labor ratio explains a large proportion of the consumption equivalent. The second characteristic is that in the two models the consumption-labor ratio is almost flat at zero inflation target, decreasing and concave with respect to inflation targets.

The main benefit of a higher inflation is the decrease in the volatility of consumption. There are two reasons for this effect. First, the economy hits the zero lower bound less often and therefore the nominal interest rate can accommodate aggregate shocks, decreasing the volatility of consumption. Second, a higher inflation target affects the Phillips curve in the two models and through the Taylor rule and the Euler equation affects the dynamic of consumption. This effect is independent of the zero lower bound and depends mainly on the direct effect of different inflation targets in inflation dynamics in both models.

7.2 Welfare Analysis Though Wedges

Nominal rigidities generate three wedges with respect to the neoclassical growth model. The first wedge comes from firm's markups and their volatility. In the efficient allocation, firms' nominal marginal costs should be equal to the price, and therefore the real aggregate marginal cost should equal the efficient one. Monopolistic competition implies a real marginal cost less than one, since firms charge a positive markup (the markup is the inverse of the real marginal cost). Sticky prices together with monopolistic competition implies fluctuation in the aggregate markup/real marginal cost, and therefore inefficient fluctuation in the marginal cost and consumption. The distortion from the first best given by markups is the first component of output gap.

The second wedge comes from inefficient distortions in the relative prices across firms. In the first best it is optimal to have the same labor input across firms. Sticky price models create a misallocation in the input of production across firms, decreasing aggregate productivity.[20]

The third wedge is a direct consequence of resources allocated for pricing decision. For each firm, there is a physical cost with respect to labor whenever the firm changes its price. In the aggregate, the total demand of labor comes from demand of labor for production and demand of labor for repricing.

The welfare in the Calvo and the CalvoPlus model can be summarized in the stochastic process of real marginal cost and price dispersion. Inflation targets, restrictions to the monetary policy or pricing models affect welfare through the stochastic process of these two wedges. The next proposition formalizes this claim in the economy with intermediate inputs, government expenditure and TFP shocks.

Proposition 4 *Let $X = (\Delta, mc, FC, \eta_z, \eta_g)$ be the sufficiency statistics for consumption and labor and S the aggregate state of the economy. Then the welfare satisfies*

$$\begin{aligned}
KK(X) &= \frac{\alpha}{1-\alpha}\left(\frac{mc\eta_z}{\iota}\right)^{1/(1-\alpha)} \qquad L(X) = \left(\frac{(1-\alpha)(1-\tau_L)}{\kappa\alpha} KK(X)\right)^{1/\chi} \\
C(X) &= \frac{\eta_z}{\Delta}(L - FC) KK(X)^{\alpha}\left(1 - KK(X)^{1-\alpha}\frac{\Delta}{\eta_z}\right) - \eta_g(S) \\
u(S) &= (1-\sigma_{np})^{-1}\left(C(X(S)) - \kappa\,(1+\chi)^{-1} L(X(S))^{1+\chi}\right)^{1-\sigma_{np}} \\
U(S) &= u(S) + \beta g^{1-\sigma_{np}} \mathbb{E}_{S'}\left[U(S)^{1-\sigma_{ez}} \middle| S\right]^{\frac{1}{1-\sigma_{ez}}}
\end{aligned} \tag{39}$$

where $FC(S)$ in the CalvoPlus model is $\theta(\Omega(S) - hz)$ and in the Calvo model $FC(S) = 0$.

TFP losses due to different levels of inflation target is the outcome of the dispersion of relative prices and the degree of strategic complementarities in the form of intermediate inputs. Different pricing models affect the impact of inflation to the dispersion of relative prices, which is measured in Δ_t. Different degree of strategic complementarities influence the direct effect of the dispersion of relative prices to TFP losses. One of the main reasons I match the volatility of the inflation is because in my model I have a strong level of complementaries. But complementaries also increase the cost of inflation as I explained before. Therefore, ignoring strategic complementarities, as a foundation for low volatility of inflation, gives an

[20] There is a third distortion coming from the physical cost of repricing, that it is almost insignificant in my model, therefore I will ignore it.

incorrect measure of cost of inflation.[21]

There are two more quantitative properties in the model. First, fluctuation in the marginal cost affects labor supply with an elasticity of $\frac{1}{\chi(1-\alpha)} \approx 6$; that is small fluctuation of marginal cost a relatively large fluctuation of labor and consumption. Second, price dispersion has only income effect with respect to consumption and it does not affect the Hicksian labor supply elasticity. This is not a consequence of the period utility I'm using; it is a direct effect from the fact that household choice depends only on average firms' marginal cost, independent of the dispersion of relative prices.

Figure 11 describes the wedges in the Calvo and the Calvo plus model. Next I describe the economic forces for the optimal inflation target though these two wedges' first and second moments.

Mean price dispersion and marginal cost: Higher inflation target increases price dispersion in both models, but in the CalvoPlus model this increase is small compare to the Calvo model. The key to generate a small cost of inflation is to cut the large price gap implied by the random pricing decision in the Calvo model. We can see in figure 11 that the mean price dispersion in the Calvo model is 10 times more sensitive than the CalvoPlus model even if 35% of price changes in the CalvoPlus model at zero inflation target are due to zero menu cost.

As I explain before, TFP losses due to price dispersion are the outcome of strategic complementarities and the dispersion of relative prices. Notice that even if price dispersion measured in Δ_t increases at third order with respect to inflation in both models, TFP losses decrease at a second order.

Different levels of inflation targets have non-monotonic relation to marginal cost. At low levels of inflation, higher inflation targets increase marginal cost, but latter decreases marginal cost. This effect comes from discounting. After price adjustment, the firm's relative price falls over time; therefore at the time of the price adjustment the firm over-adjusts its relative price to compensate for the expected fall. If the firm does not discount the future, then the price adjustment is the same as the expected fall and therefore the equilibrium level of markups does not change. If the firm discount the future, then the firm adjusts less than the expected fall in the relative price, increasing the equilibrium marginal cost. At low inflation target this is the dominant effect.

Given that the static profit function penalizes negative price gaps more than positive price gaps, the firm always prefers a positive rather than a negative price gap with the same magnitude. If price dispersion if high, then the firm's relative price is more volatile and the firm increases the reset price as

[21] In this paper, I have strategic complementarities with the technological assumption of intermediate input, but the result that strategic complementarities affect the elasticity of TFP with respect to the dispersion of relative prices holds with fixed inputs and Kimball aggregator.

a precautionary motive to avoid negative price. Price gap is the difference between current price and the static optimal price gaps. Therefore, an increase in the fluctuation of the relative price raises the reset price, increasing the equilibrium level of markups.

Both the mean price dispersion and the mean real marginal cost can be describe at a high order of accuracy with the steady state in both models. I will use this result in the next section.

Standard deviation price dispersion and marginal cost: The main benefit of higher inflation target is given by a lower volatility of real marginal cost. As we can see 11 the volatility of marginal cost largely decreases at higher inflation target since the nominal interest rate can react more strongly with respect to the structural shocks.

The volatility of price dispersion increases with respect to inflation target by an insignificant amount (at a 4th order) with almost no effect in welfare in the CalvoPlus model. This mechanism does not hold in the Calvo model, where business cycle fluctuations of price dispersion are at the second order and therefore they are important to describe the optimal levels of inflation. This is an important property for welfare analysis. In the CalvoPlus model, price dispersion is less sensitive with respect to the mean of inflation and volatility of inflation than the Calvo model.

7.3 Understanding the Interaction Between Idiosyncratic Shocks and MenuCost

Previous work on optimal inflation target has built on the intuition that state dependent pricing generates lower cost of inflation than an otherwise time-dependent model—for example, a Calvo model. Therefore, a standard menu cost model without idiosyncratic shocks can generate lower cost of inflation; thus, a higher optimal inflation target. This section shows that this intuition is incorrect; rather, it is the interaction between menu cost and idiosyncratic shocks that generates lower cost of inflation—for example, Coibion *et al.* (2012) study the optimal inflation in the model of Dotsey *et al.* (1999) and they find a lower optimal inflation than Calvo. To show this result, I focus on the steady state price dispersion, since, as I show in previous section, the steady state is a good approximation of the mean of the business cycle. Moreover, since real wage is a function of the inflation target, I isolate the effect of inflation with respect to price dispersion with a contingent tax on labor input as in section 3. Thus, real wages are constant for all levels of inflation. See online appendix for the details in the computation in this section.

To explain the results let us assume the limit of the random menu cost whenever this model converges to a simple menu cost—$hz \to 0$—and analyze price dispersion with and without idiosyncratic shocks. This limit implies an unique stationary ergodic distribution of relative for the menu cost with no idiosyncratic shocks. Figure 13 shows price dispersion, firm's Ss bands length and frequency of price change at different level of inflation for the Calvo model, and the menu cost model with and without idiosyncratic shocks.[22] We can see several properties: 1) price dispersion in the menu cost with no idiosyncratic shocks increases faster than the Calvo model at low levels of inflation; thus it has higher cost of inflation; 2) menu cost with idiosyncratic shocks has lower elasticity of price dispersion with respect to inflation, but the highest price dispersion at low inflation targets; 3) price dispersion in the menu cost with idiosyncratic shocks and in the Calvo model is flat at zero inflation target, increasing and convex; 4) the length of the Ss bands in the menu cost without idiosyncratic shocks has larger sensitivity with respect to inflation than in the model with idiosyncratic shocks.

The three model have a minimum price dispersion at zero inflation target, since at this level there is no drift in the relative prices. The critical property in the Calvo model is that it is differentiable at zero and therefore it has a zero order effect with respect to inflation. This mean that the effect of inflation to price dispersion is second order. Next proposition shows this result. The proof is in the online appendix.

Proposition 5 (Price Dispersion in the Calvo Model) *Let $\Delta_{ss}(\Pi_{ss})$ be the price dispersion in the Calvo model at a level of inflation Π. Then $\Delta_{ss}(\Pi_{ss})$ is continuous, with $\frac{d\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}}\Big|_{\Pi_{ss}=1} = 0$ and $\frac{d^2\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}^2}\Big|_{\Pi_{ss}=1} > 0$.*

Previous proposition shows that even if the Calvo model has a large cost of inflation, it is large second order effect; thus, inflation has a significant effect on misallocation of input at levels of inflation more than 1 or 2%. See Blanco (2015) for more properties of steady state price dispersion.

The continuity of price dispersion with respect to inflation in the Calvo model, does not hold in the menu cost model with no idiosyncratic shocks. In this model, price dispersion jumps from zero at zero inflation to a positive number at any positive level of inflation. To understand this result first notice that the Ss bands are positive at all levels of inflation. As we can see in figure 13, the length of the Ss bands at zero inflation are positive and increasing with respect to inflation. The main intuition of this result at zero inflation is that the benefit of changing the price is discrete, since it is given by $\max_x x^{-\gamma}(x - mc_{ss}) + \frac{\theta w_{ss}}{Y_{ss}}$, but the cost is continuous, since it is given by $\tilde{p}^{-\gamma}(\tilde{p} - mc_{ss})$. Since length of the Ss bands are positive at all levels of inflation, the distribution of relative prices jumps from a probability atom at the optimal price at zero inflation target, to an uniform distribution at any positive level of inflation. Figure 12 shows this property of the distribution of relative prices at 0%, 0.3% and 4% inflation target.

[22] For this figure, I use the benchmark calibration with $hz = 0.001$.

The discontinuity of price dispersion at zero inflation target in the menu cost with idiosyncratic shocks is the first property that makes the cost of inflation large in model with only menu cost cost. The second reason is that the length of the Ss bands have a large elasticity with respect to inflation, and therefore the support of the distribution of relative prices increases with inflation. This generates a second order effect of inflation to price dispersion at positive levels of inflation. It is difficult to compare this second order effect with Calvo, but from the figure 13 it seems higher at levels of inflation less than 1.5% of inflation.

Both in the Calvo model and in the menu cost without idiosyncratic shocks price dispersion start at zero, but it is higher in the menu cost given the two argument explained before. Thus, we can see clearly the next result:

Result 8 (Cost of Inflation in Menu Cost with No Idiosyncratic Shocks) *Price dispersion In the menu cost without idiosyncratic shocks and the Calvo model is zero at zero inflation target. There exist a positive* $\bar{\Pi}$*, s.t. for all levels of inflation between* $(0, \bar{\Pi}]$ *price dispersion is higher in the menu cost without idiosyncratic shocks.*

As we see in figure 13, menu cost with idiosyncratic shocks has a relative large price dispersion at zero inflation, due to idiosyncratic shocks; and a low elasticity of price dispersion with respect to inflation. There are two reasons for this result: 1) Ss are almost insensitive with respect to inflation at low inflation levels, since firms' react mainly to idiosyncratic shokcs; and 2) the distribution of relative prices is almost to low inflation levels. The first effect is a direct consequence of large idiosyncratic shocks. This result does not depends in an increase in the frequency of price change; even more, the frequency of price is almost constant as in the Calvo model. The intuition why price dispersion is insensitive with respect to inflation comes directly from the symmetry of the distribution of relative prices at zero inflation target. To see this notice that firms are hit by a sequence of productivity shocks with positive or negative accumulated sum in a symmetric way. At zero inflation, an increase in the inflation levels increases the price dispersion of those firms with positive shocks, but also decreases price dispersion of those firms with negative shocks. At zero inflation, these two effects cancel each other with the direct implication of a low cost of inflation.

To understand the argument, let $\Delta(\Pi)$ be the price dispersion Π, $g(x, \Pi)$ be the distribution of log-relative prices, and $S, s(\Pi)$ the upper and lower Ss bands in the log-scale, where I indexed each object be the level of inflation Π. Assuming that $S(1) = -s(1)$, $S'(1) = s'(1) = 0$ and that $g_\Pi(x, 1) = -g_\Pi(-x, 1)$—

all these assumption are true in a continuous time setting, see Alvarez, Gonzalez-Rozada, Neumeyer and Beraja (2011)—, then doing a Taylor approximation on $\Delta(\Pi)$ with respect to Π we have

$$
\begin{aligned}
\Delta(\Pi) &\approx \int_{s(\Pi)}^{S(\Pi)} x^2 g(x,\Pi)dx \qquad \text{since mean log-rel. prices are zero} \\
&= \Delta(1) + \left(\int_{s(1)}^{S(1)} x^2 g_\Pi(x,1)dx + (S(1)^2 g(S(1),1)S'(1) - s(1)^2 g(s(1),1)s'(1))\right)(\Pi-1) + o((\Pi-1)^2) \\
&= \Delta(1) + \left(\int_{0}^{S(1)} x^2 (g_\Pi(x,1) + g_\Pi(-x,1))dx\right)(\Pi-1) + o((\Pi-1)^2) \\
&= \Delta(1) + o((\Pi-1)^2) \qquad (40)
\end{aligned}
$$

where in the first step we used the standard log-approximation and in the second step we took the the first order derivative. Notice that since $s'(1) = S'(1) = 0$, the term $(S(1)^2 g(S(1),1)S'(1) - s(1)^2 g(s(1),1)s'(1)) = 0$; and because $g_\Pi(x,1) = -g_\Pi(-x,1)$, the term $\int_{s(1)}^{S(1)} x^2 g_\Pi(x,1)dx = 0$ is also zero. These results implies

Result 9 (Cost of Inflation in Menu Cost with Idiosyncratic Shocks) *In the menu cost with idiosyncratic shocks, price dispersion is continuous at zero inflation, with zero first order effect.*

With the last result it is easy to see that menu cost with idiosyncratic shocks is model that generates the lowest cost of inflation, followed by Calvo and then the menu cost without idiosyncratic shocks, at low levels of inflation. Since the Calvo model with idiosyncratic shocks generates higher cost of inflation than the menu cost, it is the interaction between menu cost and idiosyncratic shocks that generates a low cost of inflation.

Finally, as figure 13 shows, the menu cost model with idiosyncratic shocks does not generates low cost of inflation due to an increase of frequency of price change. We can see that at low inflation levels the frequency is almost constant as price dispersion. When calibrated to match micro-price statistics, idiosyncratic shocks are so large that inflation has almost no effect on either price dispersion or frequency at low levels of inflation.

All the results hold in my calibration with random menu cost. Figure 14 shows price dispersion, firm's policy and frequency of price change at different level of inflation for the Calvo model, and the CalvoPlus model with and without idiosyncratic shocks. There is only a change in the level in the frequency of price change in the CalvoPlus model with and without idiosyncratic, but not in the cost of inflation or firms policy.

8 Conclusion

As Golosov and Lucas (2007) observe, firms change their prices once a year, with an average size of 10% and half of these changes are being downward. This cannot be rationalized in a model where firms react to aggregate inflation only, but it can be rationalized in a model with menu cost and idiosyncratic shocks. This paper used this framework, founded on micro data, to find an optimal inflation target of 2%, twice as high as the leading sticky price model Calvo—0.66%. In order to do this, I extend a random menu cost model with idiosyncratic shocks to a standard New Keynesian framework with a Taylor rule subject to a ZLB constraint and rich aggregate dynamics given by aggregate, government expenditure, money and risk premium shocks. The main reason for this result is that price dispersion—the main cost in sticky price model—has low sensitivity with respect to inflation target. Moreover, the likelihood of hitting the ZLB constraint and deflationary spirals during periods when the ZLB is binding are decreasing with the inflation target.

Going forward, it would be important to further explore the optimal inflation in different environment. For example, many countries with inflation targets are small open economies; does this feature affect the optimal inflation target? Are the structural shocks hitting these economies different? Second, this paper focuses on price rigidities assuming a flexible wages. How does the optimal inflation change with sticky price and sticky wages? Does downward wage rigidity affect the optimal inflation target? Answering these questions in a framework similar to this paper and different from Calvo seems the correct environment to think about these questions.

In this paper, I didn't focus on the transition dynamics from the current inflation target to the optimal one and I use a Taylor rule to described monetary policy. These two assumptions could change the optimal inflation target. With respect to the transition dynamics, the distributional consequences of changing the inflation target should be taken into account before changing the current inflation target.

With respect to the modeling choice for Central Bank behavior, an interesting extension of this paper would be to compute the optimal inflation target in a model where there is an optimality behavior with respect to the Central Bank that matches observed Central Banks behavior. This path has two important problems. First, there are no tools to solve optimal policy in a heterogeneous agents model with non-convexity at aggregate and idiosyncratic levels. Second, it is well know that optimal monetary policy under commitment generates unrealistic time series of nominal interest rate—see Clarida, Gali and Gertler (1999). Therefore, in order to approach the optimal policy methodology it would be important to have a set of frictions that generates a realistic path for nominal time series.

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A Frequency of Hitting the Zero Lower Bound Across Countries

I constructed a quarterly panel data across countries of inflation and nominal interest rate from the IMF-IFS database and FRED. For all the countries, I use FRED, except for Argentina, Peru and Singapore—for which I use IMF-IFS data. Data is not seasonal adjusted. For the interest rates I use monthly average to compute the quarterly interest rate. For inflation, I use consumer price index—CPI; for nominal interest rate, I use a combination of policy rates—whenever this one is available—and call rates—overnight interbank borrowing rate. The countries that I use call rates or policy rates are:

- **Policy rate:** Poland, South Africa, Denmark, Finland, Austria, United Kingdom, Czech Republic, Canada, United States, Japan.
- **Call rate:** Sweden, Italy, Australia, Germany, Peru, New Zealand, Portugal, Slovenia, Brazil, Israel, Switzerland, Belgium, France, Mexico, Iceland, Slovakia, Norway, Luxembourg, Netherlands, Chile, Korea, Republic of., Turkey, Ireland, Spain, Poland.

Table 1 describes the data for each country, where I compute the average probability hitting the ZLB and the average inflation for each country. I define the event of hitting the ZLB whenever the nominal rate is less than 0.71. I drop countries with average inflation in the period between 1998-2016 more than 4% or countries with less than 20 years of data—this is the last column. The last two row computes the average of inflation and frequency of ZLB taking Europe as different countries—average 1— or taking Europe as a country—average 2.

Table 1: Data Description

Country	Part of EU	Start Date	End Date	Historical Freq. ZLB	Historical Mean Inf.	After 1988 ZLB Freq.	After 1988 Mean Inf.	in/out
Argentina	NOT EU	2014q1	2015q2	0	16.21	0	16.21	out
Australia	NOT EU	1990q3	2015q1	0	2.53	0	2.53	in
Austria	EU	1958q1	2014q4	.1	3.27	.2	2.18	in
Belgium	EU	1999q1	2015q1	.34	1.95	.34	1.95	out
Brazil	NOT EU	1996q4	2014q4	0	6.05	0	6.05	out
Canada	NOT EU	1955q1	2015q1	.02	3.64	.05	2.17	in
Chile	NOT EU	1996q1	2015q1	.05	3.49	.05	3.49	out
Czech Republic	EU	1992q1	2015q1	.26	4.59	.26	4.59	out
Denmark	EU	1967q1	2015q1	.05	4.61	.08	2.13	in
Finland	EU	1958q4	2015q1	.03	4.68	.06	2.13	in
France	EU	1955q1	2015q1	.09	4.33	.2	1.74	in
Germany	EU	1960q1	2015q1	.1	2.67	.2	1.91	in
Iceland	NOT EU	1998q2	2015q1	0	4.98	0	4.98	out
Ireland	EU	1999q1	2015q1	.34	2.21	.34	2.21	out
Israel	NOT EU	1992q4	2015q1	.04	3.88	.04	3.88	in
Italy	EU	1999q1	2015q1	.34	1.98	.34	1.98	out
Japan	NOT EU	1955q1	2014q4	.3	2.96	.66	.54	in
Luxembourg	EU	1999q1	2015q1	.34	2.17	.34	2.17	out
Mexico	NOT EU	1976q1	2015q1	0	21.06	0	11.65	out
Netherlands	EU	1999q1	2015q1	.34	1.93	.34	1.93	out
New Zealand	NOT EU	1985q1	2015q1	0	3.37	0	2.4	in
Norway	NOT EU	1982q1	2015q1	0	3.14	0	2.31	in
Peru	NOT EU	1995q4	2016q1	0	3.56	0	3.56	in
Poland	EU	1989q4	2015q1	0	15.41	0	15.41	out
Portugal	EU	1999q1	2015q1	.34	2.1	.34	2.1	out
Singapore	NOT EU	1987q3	2013q4	.28	2.01	.29	2.02	in
Slovakia	EU	1995q3	2015q1	.28	4.59	.28	4.59	out
Slovenia	EU	2004q1	2015q1	.49	2.12	.49	2.12	out
South Africa	NOT EU	1957q1	2014q4	0	7.52	0	7.2	out
Spain	EU	1973q2	2015q1	.13	6.65	.2	3.2	in
Sweden	EU	1955q1	2015q1	.03	4.37	.07	2.22	in
Switzerland	NOT EU	1972q1	2015q1	.29	2.27	.36	1.31	in
Turkey	NOT EU	1986q2	2014q4	0	33.1	0	32.97	out
United Kingdom	NOT EU	1955q1	2015q1	.1	4.98	.22	2.65	in
United States	NOT EU	1955q1	2015q1	.11	3.62	.24	2.61	in
Europe		1955q1	2015q1	.08	4.37	.15	2.22	
Average 1		1955q1	2016q1	.09	3.69	.15	2.37	
Average 2		1955q1	2016q1	.09	3.37	.15	2.44	

Part of EU describes if the country is part of the European union; **Start and End Date** denotes the initial and final quarter; **Freq. ZLB** denotes denotes the frequency of hitting the zero lower bound defined as the rate less than 0.75 ; **Mean Inf.** denotes mean inflation over the sample; **Historical** denotes the average over the entire sample; **After 1988** denotes the average after 1988.

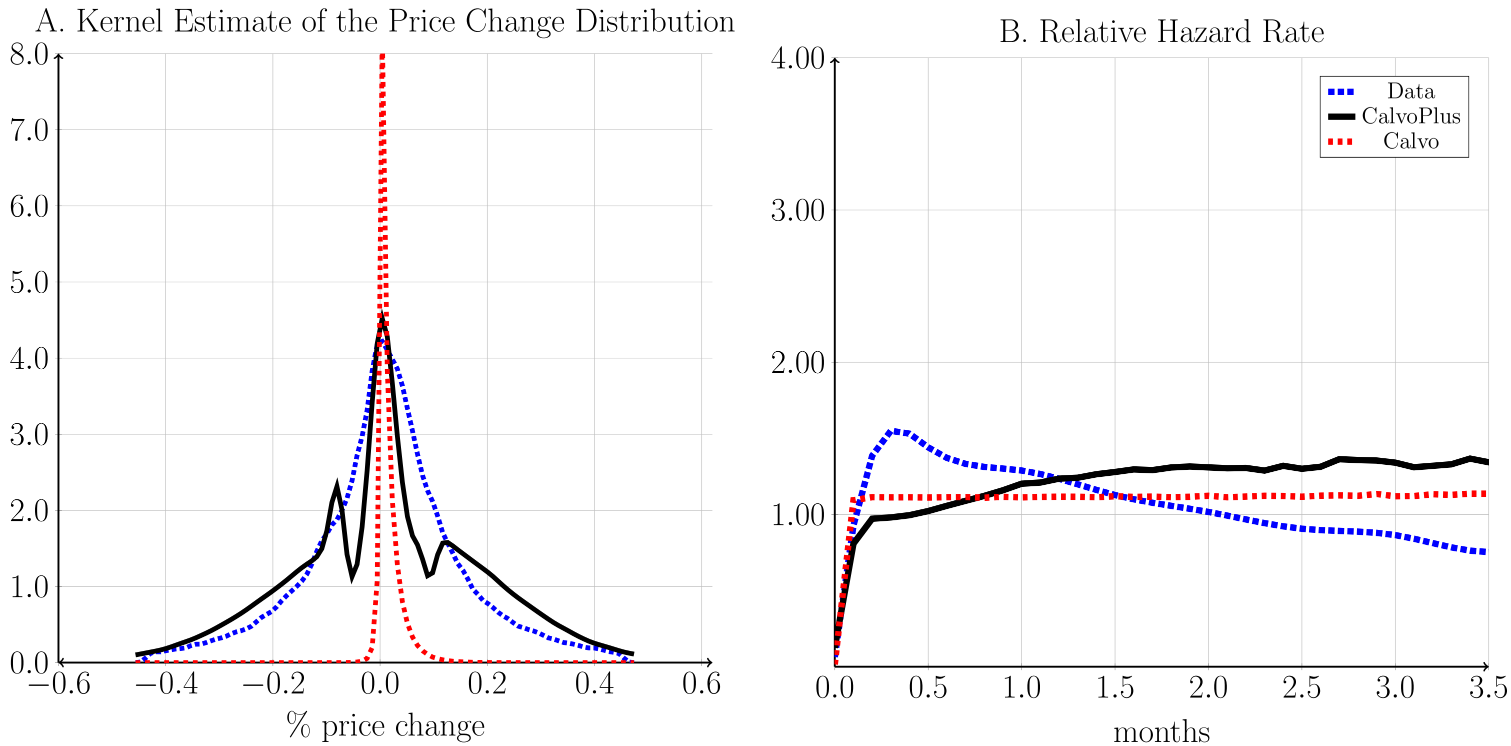
B Micro-Price Statistics Model-Data

Table 2: Micro-price Moments data-model at 2 Percent Inflation

moments	Data	Model SS	Model BC
Absolute Value of Price Change Moments			
mean	0.101	0.129	0.133
Standar Deviation	0.094	0.115	0.116
Skewness	1.274	1.270	1.216
Kurtosis	4.022	4.504	4.397
5th percentile	0.005	0.005	0.005
10th percentile	0.010	0.010	0.010
25th percentile	0.029	0.039	0.038
50th percentile	0.070	0.089	0.095
75th percentile	0.144	0.190	0.196
90th percentile	0.244	0.299	0.301
95th percentile	0.304	0.361	0.367
Price Change Moments			
mean	0.010	0.017	0.013
Standar Deviation	0.137	0.172	0.176
Skewness	-0.012	-0.144	-0.082
Kurtosis	3.849	3.860	3.715
5th percentile	-0.230	-0.286	-0.291
10th percentile	-0.157	-0.212	-0.215
25th percentile	-0.061	-0.080	-0.0851
50th percentile	0.010	0.022	0.015
75th percentile	0.079	0.095	0.101
90th percentile	0.182	0.228	0.233
95th percentile	0.256	0.306	0.310
Frequency	0.103	0.101	0.101
Cost of Price Adjustment	0.0047	0.0031	-
hz/Ω_t	-	0.3415	-

See online appendix for the computation of the moments in the data. Moments in the model in the steady state (third column) are computed in the steady state at 2 % inflation target. Model with business cycle are computed in the model wth ZLB at 2% inflation target.

Figure 1: Micro-Price Statistics 2% Target Inflation Model-Data

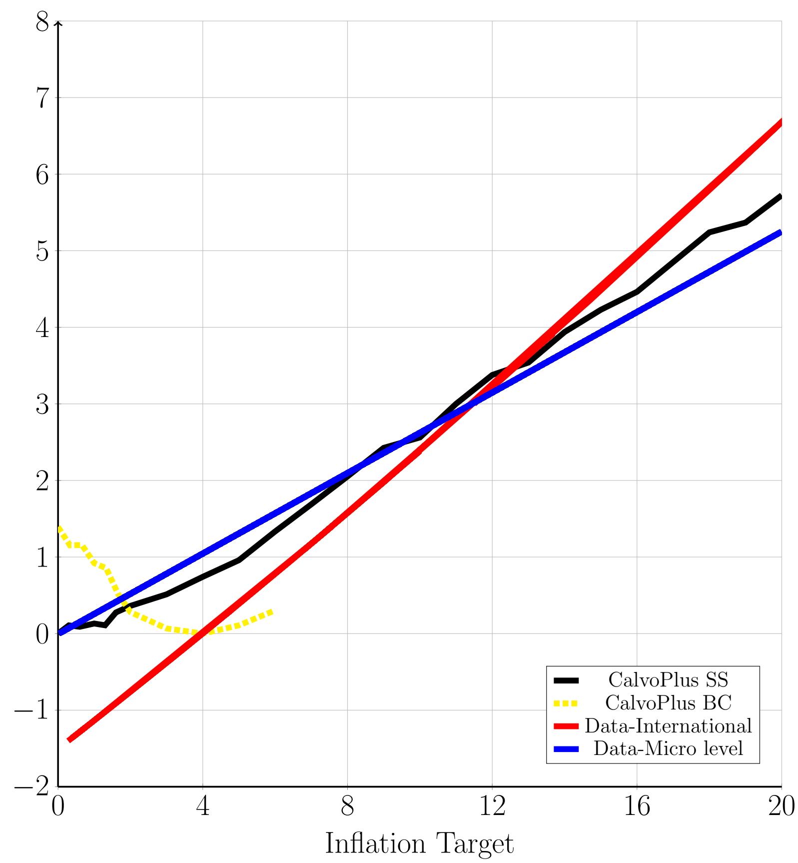


Panel A describes the price change distribution in the CPI UK data,in the CalvoPlus model and in the Calvo models. Panel B describes their respective hazard rates. The dotted blue line is the data, the solid black line is the CalvoPlus model and the dashed red line is the Calvo model. The Calvo model price change distribution is re-scaled to fit in the same scale as the data. All price change distribution are centered in zero.

Table 3: Frequency of price change and inflation across countries

country	source	initial year	end year	mean inflation	mean frequency
Argentina	Alvarez et al (2015)	1991	1997	99202.1	42.6819
Brazil	Barros et al. (2009) Monthly	1996	2013	.423402	37.2333
Israel	Baharad and Eden (2004)	1991	1992	12.7513	26.2051
Mexico	Gagnon (2009)	1994	2002	14.3003	29.395
Norway	Wulfsberg(2016)	1975	2004	5.37058	22.1043
US	Nakamura at el. (2016)	1978	2014	3.69253	10.7967

Figure 2: FREQUENCY OF PRICE CHANGE AT DIFFERENT INFLATION LEVELS MODEL-DATA



This figure describes the frequency of price change in the model and in the data for annual inflation targets between 0 and 20%. The y-axis on the left describes the frequency normalize to zero. The solid lines are the frequency in model at the steady state, in the international data and in the product level data.

C Calibration

Table 4: Model parameters and targets

Parameter	Value	Target
	Preferences and Technology	
β	$0.97^{1/12}(1+g)^{\sigma_{np}}$	Interest Rate 4%
σ_{ez}	-45	Cost of Business Cycle (3%)
σ_{np}	1.5	Greenwood *et al.* (1988)
χ	0.5	Greenwood *et al.* (1988)
γ	5	Micro-estimate (4-6)
α	0.7	intermediate inputs/total output of 0.7
g	0.0017	2 % growth rate
τ_L	0.87	Aggregate Markup 10%
κ	180	Normalize labor supply to 1
	Random Menu Cost and Quality shock processes	
Parameter	Value	Target
(σ_a^i)	$(0.196, 0.011)$	GMM
p	0.077	GMM
hz	0.035	GMM
θ	0.121	GMM
	Taylor Rule and Aggregate Shocks	
Parameter	Value	Target
$(\phi_r, \phi_\pi, \phi_{mc}, \phi_{dmc})$	$(0.86, 1.80, 0.63, 0.00)$	Del Negro *et al.* (2007)
$(\rho_Z, \sigma_Z 100)$	$(0.98, 0.00055)$	Cociuba *et al.* (2009)
$(\rho_G, \sigma_G 100)$	$(0.95, 0.0022)$	Del Negro *et al.* (2007)
$(\rho_R, \sigma_R 100)$	$(0, 0.00125)$	Del Negro *et al.* (2007)
$(\rho_Q, \sigma_Q 100)$	$(0.97, 0.0005)$	International ZLB frequency and Coibion *et al.* (2012)
	Additional Calvo Parameters	
Parameter	Value	Target
λ	0.6	Stability in Calvo at 5% inflation target

D Business Cycle Statistics With No ZLB and 2% Inflation Target

This section describes the computation the linear impulse-response for the Calvo and CalvoPLus model, together with the business cycle statistics

D.1 Business Cycle Statistics

This section describes the computation business cycle moments for each model. Let $m_x^{T_s,M}$ denote the moment m, x is the variable, T_s the sampling length and the M the model. The moments are standard deviation, persistence and correlations. Next, I describe the steps for computing $M_x^{T,m}$ in the two models.

1. Simulate the model for a large T. Let $\{X_t\}_{t=0}^T$ the time-series of the aggregate variables.
2. Generate a random sequence of dates $\{t_i\}_{i=1}^N$ and draw $\{\{X_t\}_{t_i}^{t_i+T_s}\}_{i=1}^N$ samples.
3. For each $i = 1, 2, ..., N$:

    i. For a random sample, compute $M_x^{T_s,m}$

Table 5: Business Cycle Moments at 2 Percent Inflation and no ZLB

moment	GDP	C	L	R	w	Π
Data						
Standard Dev.	1.61	1.49	1.98	0.38	2.44	0.21
Autocorrelation	0.95	0.94	0.97	0.95	0.96	0.64
Corr. with output	1.00	0.90	0.75	0.55	0.14	0.03
Corr. with inflation	0.14	0.09	-0.20	0.26	0.29	1.00
CalvoPlus Business Cycle Statistics—Median						
Standard Dev.	1.78	2.42	1.20	0.26	0.60	0.09
Autocorrelation	0.91	0.91	0.88	0.81	0.88	0.87
Corr. with output	1.00	1.00	0.98	-0.41	0.98	-0.17
Corr. with inflation	0.98	-0.18	0.01	0.49	0.01	1.00
Calvo Business Cycle Statistics—Median						
Standard Dev.	1.85	2.45	1.25	0.26	0.63	0.06
Autocorrelation	0.90	0.90	0.88	0.80	0.88	0.92
Corr. with output	1.00	1.00	0.97	-0.39	0.97	-0.27
Corr. with inflation	0.97	-0.27	-0.02	0.58	-0.02	1.00
Difference Between Calvo and CalvoPlus Models IC(15,85)						
Standard Dev.	(-0.72,0.61)	(-0.90,0.85)	(-0.46,0.35)	(-0.06,0.06)	(-0.23,0.17)	(-0.00,0.05)
Autocorrelation	(-0.06,0.07)	(-0.06,0.07)	(-0.07,0.08)	(-0.08,0.10)	(-0.07,0.08)	(-0.11,0.02)
Corr. with output	(-0.00,0.00)	(-0.00,0.00)	(-0.01,0.04)	(-0.36,0.34)	(-0.01,0.04)	(-0.32,0.53)
Corr. with inflation	(-0.01,0.04)	(-0.33,0.53)	(-0.40,0.50)	(-0.34,0.17)	(-0.40,0.50)	(-0.00,0.00)

GDP=Gross Domestic Output Per Capital; L=Total Labor Supply; R= quarterly Interest Rate; w= Real Wage; Π= Quarterly Inflation. The description of the variables in the data are in the Online Appendix. The variables in the data are linear detrended from 1984:Q1 to 2007:Q1. The moments in the Calvo and CalvoPlus models are the 50 percentiles of each statistics. The difference are the 25 and 75 percentile. All statistics are over 5000 simulation over 22 years.

D.2 Linear Impulse Response Functions

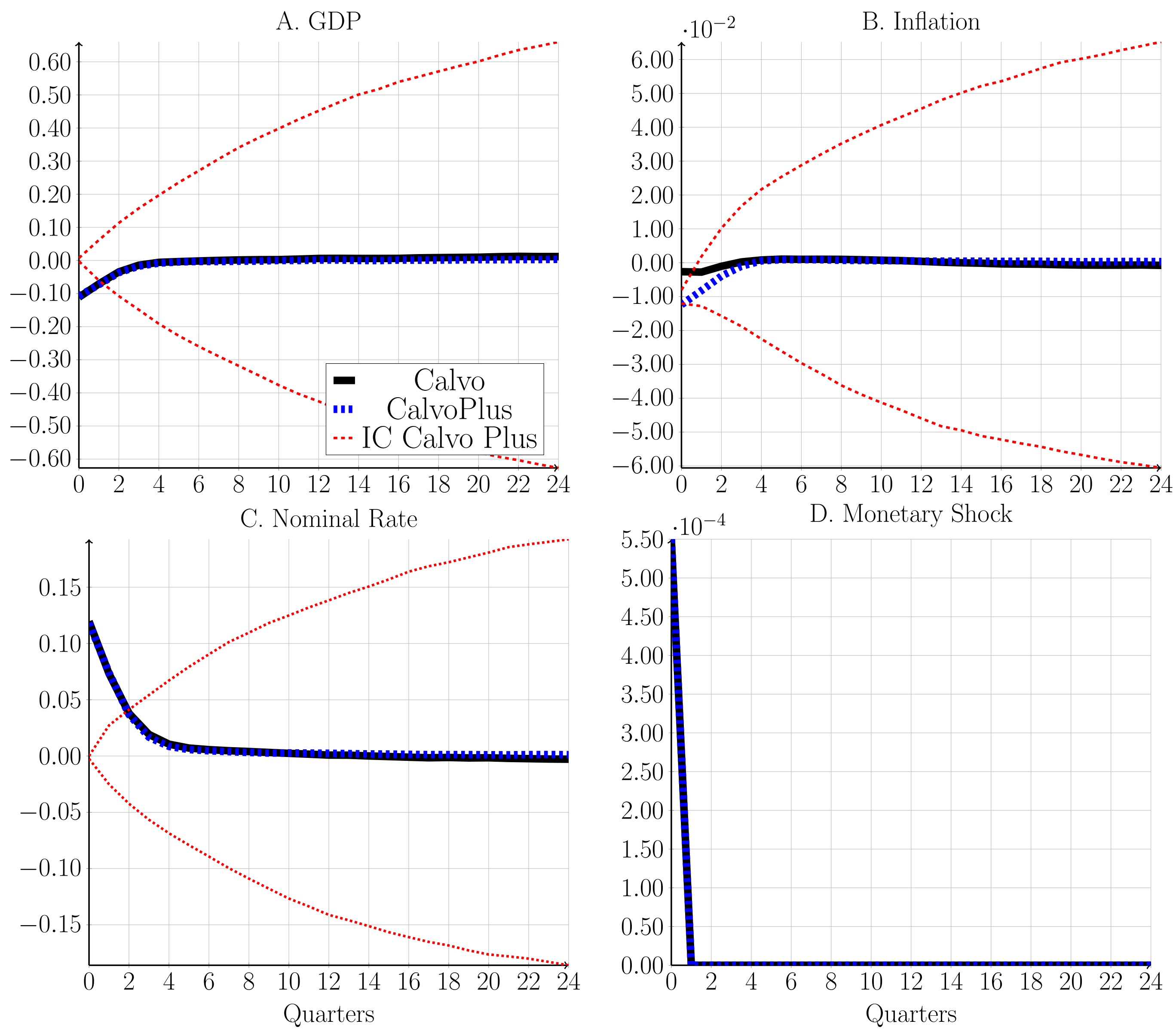
Let IR^{M}_{tx} be the linear impulse-response in the model M, at time t, of the structural shock $x \in \{z, g, r, q\}$. Let $\hat{IR}^{MT}_{tx}$ be the estimate in a sample of length T. Next I describe the steps to generate the random variable $\hat{IR}^{MT}_{tx}$ using Monte Carlo methods.

1. Simulate the model for a large T. Let $\{X_t\}_{t=0}^{T}$ be the time-series of the aggregate variables, S^X_t the vector that includes real marginal cost, price dispersion, nominal interest rate and the exogenous variables at time t, and let S^Y_t the vector that includes gdp, inflation and consumption.
2. Generate a random i.i.d. sequence of dates $\{t_i\}_{i=1}^{N}$ and draw $\{\{X_t\}_{t_i}^{t_i+T_s}\}_{i=1}^{N}$ samples.
3. For each $i = 1, 2, ..., N$:
   i. For a random sample, estimate the state space model:

$$S^X_{t+1} = \beta_x S^X_t + \Omega_x \epsilon^x_{t+1} \quad ; \quad S^Y_t = \beta_y S^X_t + \Omega_y \epsilon^y_t \tag{41}$$

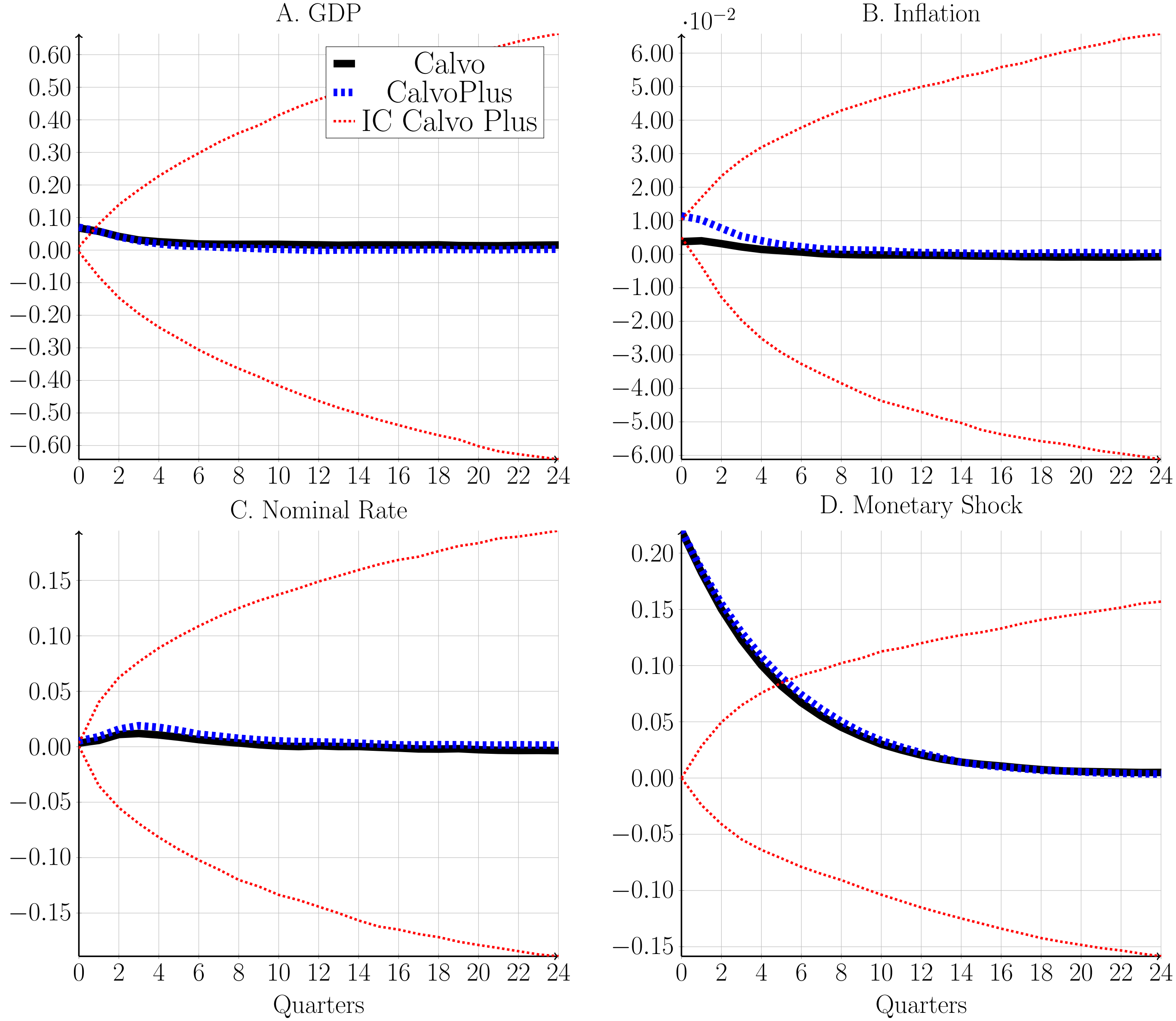
   ii. Compute the impulse-response with respect to σ_x aggregate shock, where x denotes a variable. Compute the impulse-response $IR_x(t)^i$.
4. $\{IR_x(t)\}_{i=1}^{N}$ is a random sample from $\hat{IR}^{MT}_{tx}$.

Figure 3: Impulse-Response to a Monetary Shock



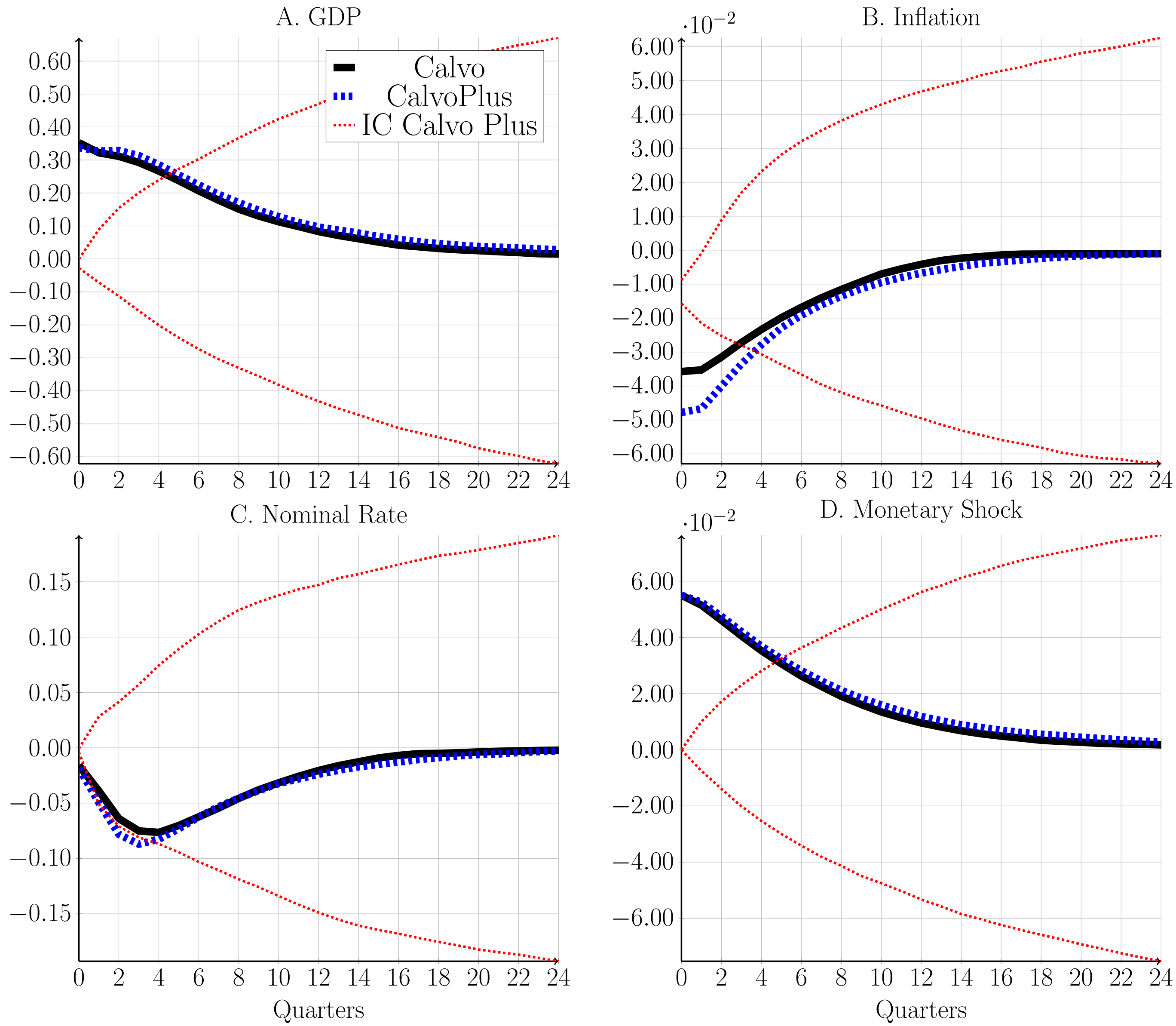
Panel A to D describe the impulse response functions of the output, inflation and nominal interest rate to a monetary shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the blue dashed line describes the median impulse-response functions of the CalvoPlus model; the red dotted lines describe the 25 and 75 percentiles of the difference in the impulse-response functions between the two models over 450 simulations of 22 years.

Figure 4: Impulse-Response to a Government Expenditure Shock



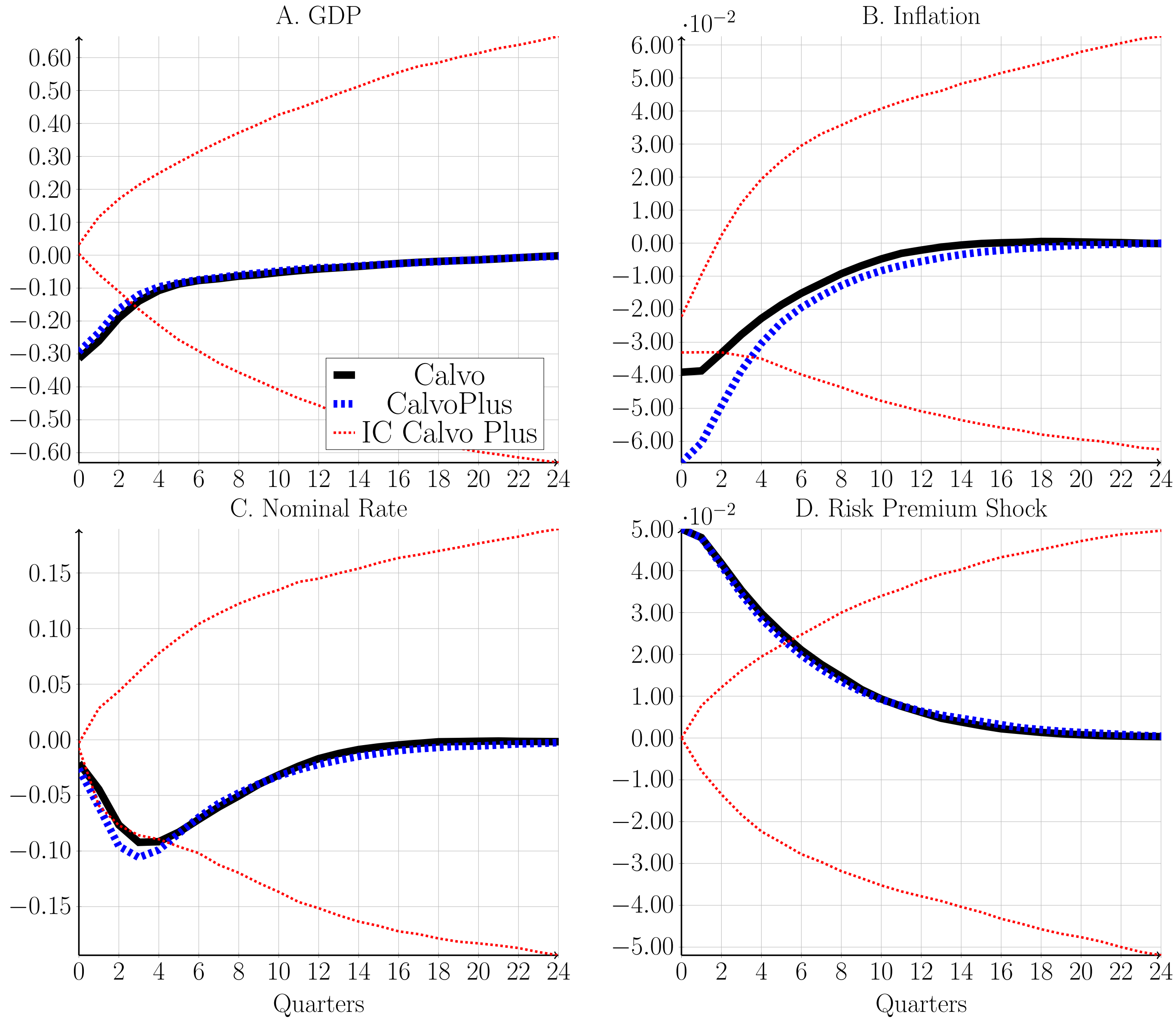
Panel A to D describe the impulse response functions of the output, inflation and nominal interest rate to a government expenditure shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the blue dashed line describes the median impulse-response functions of the CalvoPlus model; the red dotted lines describe the 25 and 75 percentiles of the difference in the impulse-response functions between the two models over 450 simulations of 22 years.

Figure 5: Impulse-Response to a Productivity Shock



Panel A to D describe the impulse response functions of the output, inflation and nominal interest rate to a productivity shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the blue dashed line describes the median impulse-response functions of the CalvoPlus model; the red dotted lines describe the 25 and 75 percentiles of the difference in the impulse-response functions between the two models over 450 simulations of 22 years.

Figure 6: Impulse-Response to a Risk Premium Shock



Panel A to D describe the impulse response functions of the output, inflation and nominal interest rate to a risk premium shock at 2% inflation target in the models without zero lower bound constraint to the nominal interest rate. The black solid line describes the median impulse-response functions of the Calvo model; the blue dashed line describes the median impulse-response functions of the CalvoPlus model; the red dotted lines describe the 25 and 75 percentiles of the difference in the impulse-response functions between the two models over 450 simulations of 22 years.

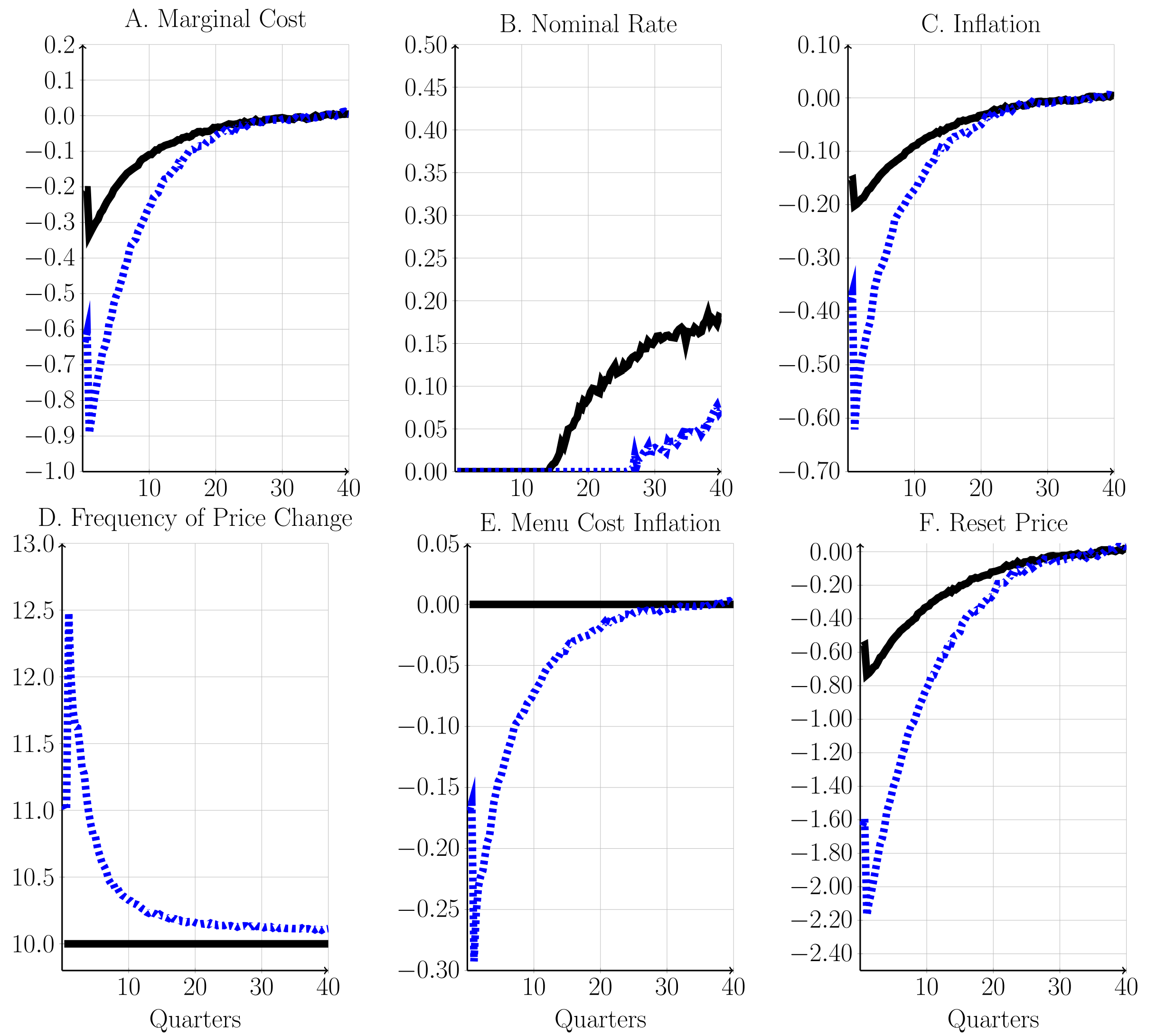
E Business Cycle Statistics With ZLB

Table 6: Business Cycle Moments with ZLB Calvo and Menu Costs

	CalvoPlus Model		Calvo Model	
moments	1 IT	3 IT	1 IT	3 IT
Frequency of ZLB	0.43	0.05	0.31	0.13
Lenght of the zero lower bound				
Mean	9.41	2.16	5.38	3.64
Std	13.00	1.67	7.32	4.29
25 perc.	2.00	1.00	1.00	1.00
50 perc.	4.00	2.00	3.00	2.00
75 perc.	12.00	3.00	6.00	4.00
95 perc.	36.00	5.70	19.00	12.00
Marginal Cost—Output Gap—moments				
Mean if $R > 1$	0.21	0.01	0.08	0.03
Mean if $R < 1$	-0.28	-0.16	-0.17	-0.17
Std if $R > 1[100]$	0.17	0.19	0.16	0.16
Std if $R < 1[100]$	0.43	0.19	0.27	0.20
Inflation moments				
Mean if $R > 1$	0.27	0.01	0.06	0.03
Mean if $R < 1$	-0.36	-0.09	-0.13	-0.19
Std if $R > 1[100]$	0.13	0.09	0.11	0.12
Std if $R < 1[100]$	1.21	0.07	0.11	0.12

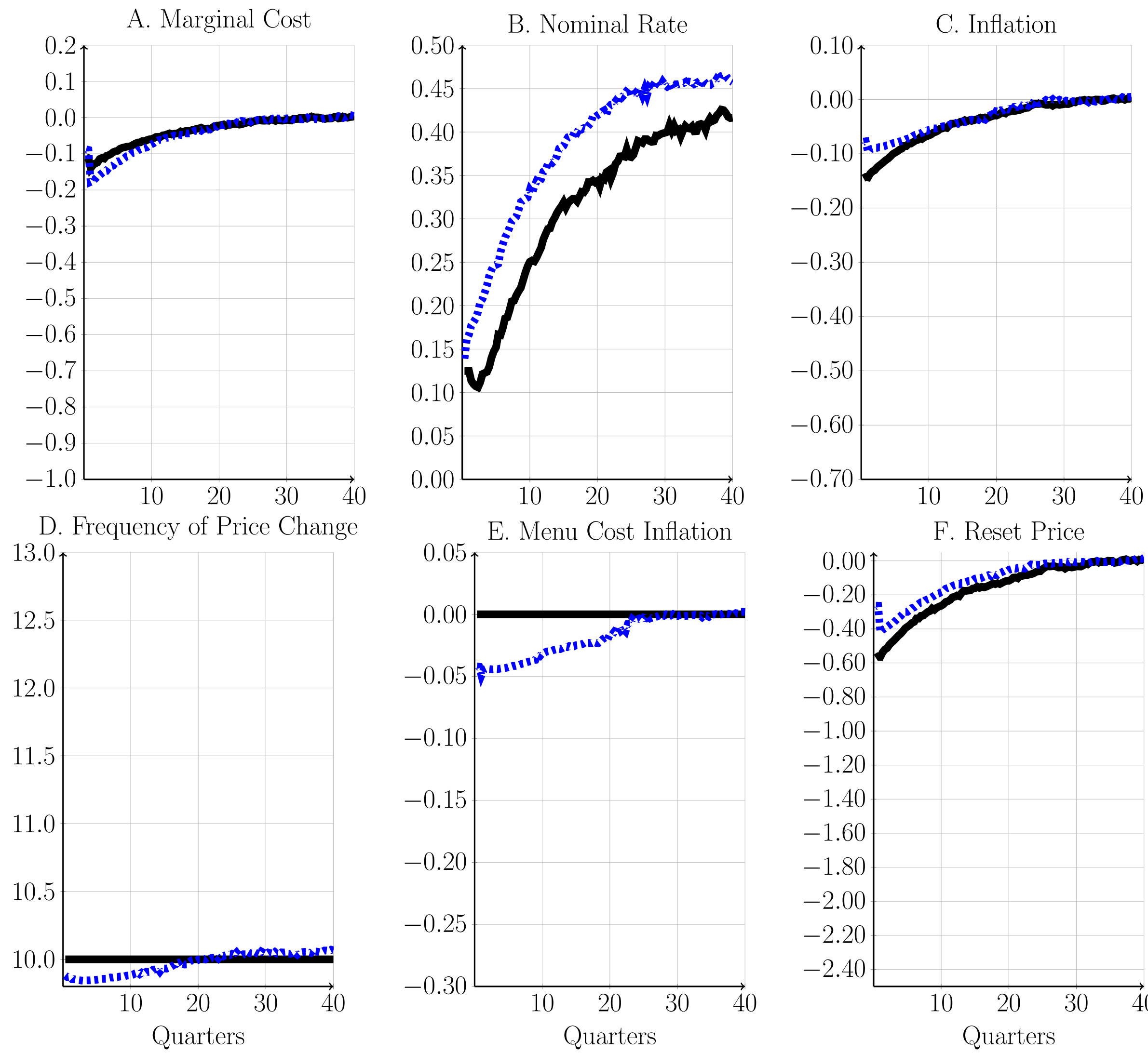
Rows 1 to 5 describe moments of the nominal interest rate with respect to the zero lower bound. All the moments are computed with the model aggregate at quarterly frequency.

Figure 7: Non-Linear Impulse Response for Risk Premium Shock at one Percent Inflation Target



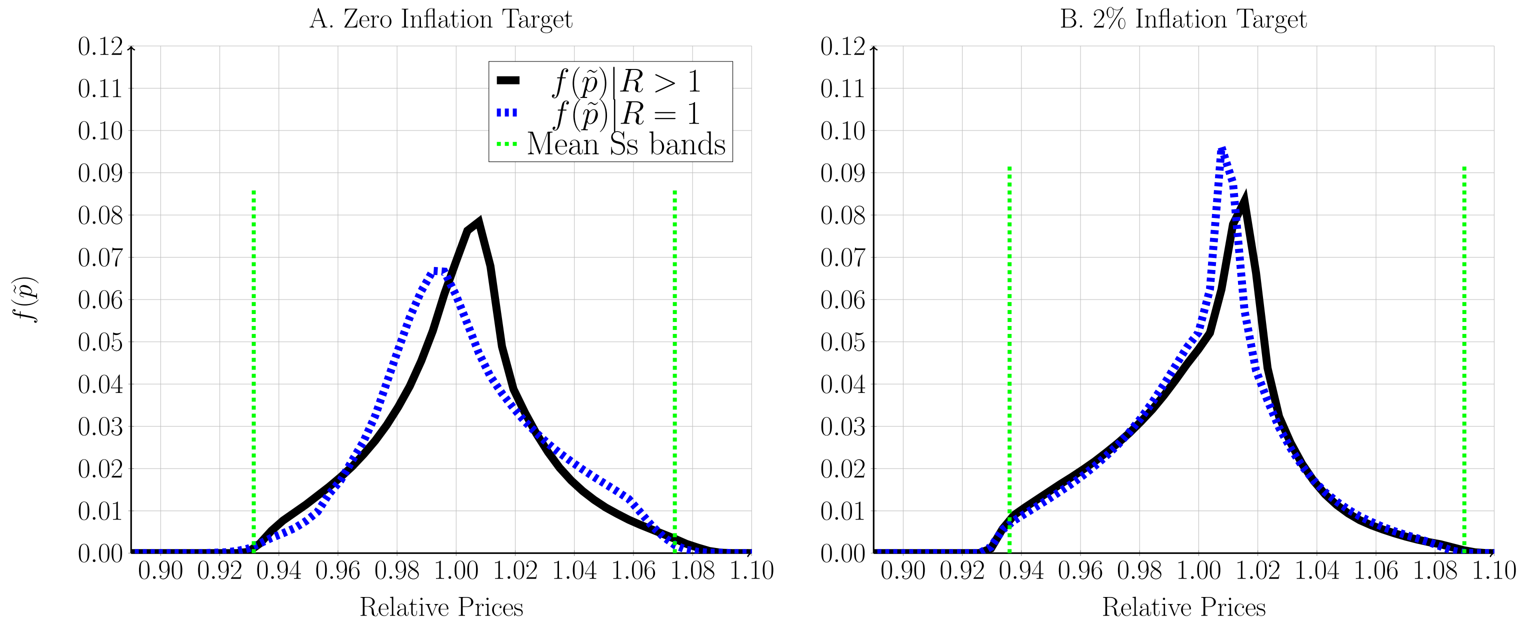
Panel A to F describe the non-linear median impulse response in the CalvoPlus model and in the Calvo model. Panel A, C, E and F are in percentage deviation from the terminal value. Interest rate and inflation are monthly rates.

Figure 8: Non-Linear Impulse Response for Risk Premium Shock at Three Percent Percent Inflation Target



Panel A to F describe the non-linear median impulse response in the CalvoPlus model and in the Calvo model. Panel A, C, E and F are in percentage deviation from the terminal value. Interest rate and inflation are monthly rates.

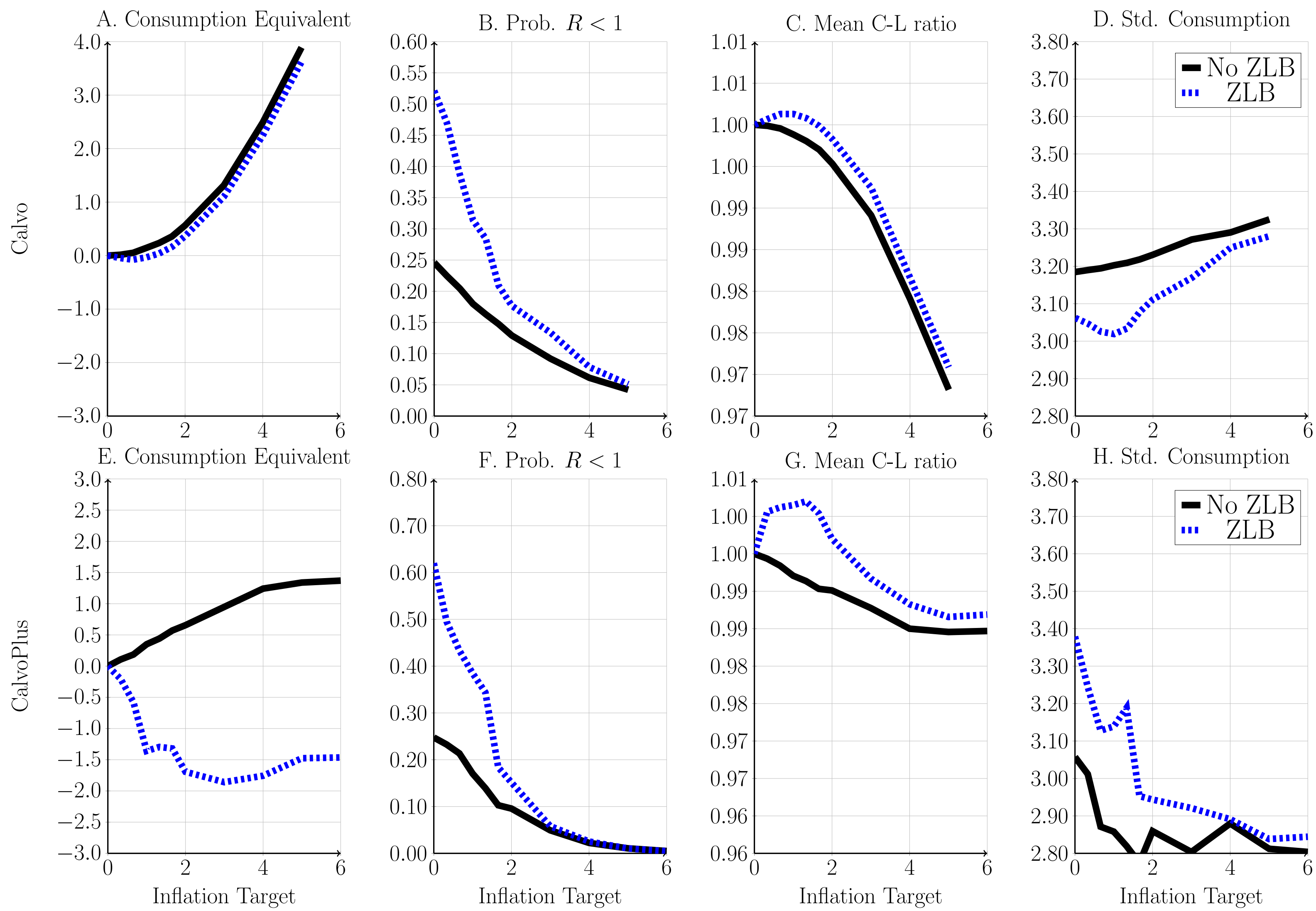
Figure 9: Distribution of Relative Prices at 1% and 3% Inflation Target



Panel A to B describe the distribution of relative prices conditional of being at the zero lower bound or not, together with the average Ss bands in the simulation. Panel A describes the distributions at 0% inflation target and panel B describes the distribution at 2% inflation target. All the conditional ergodic means are given by $\frac{\sum_t f_t(\tilde{p}) I(X_t)}{\sum_t I(X_t)}$ where $X_t = \{R_t \geq 1, R_t = 1\}$

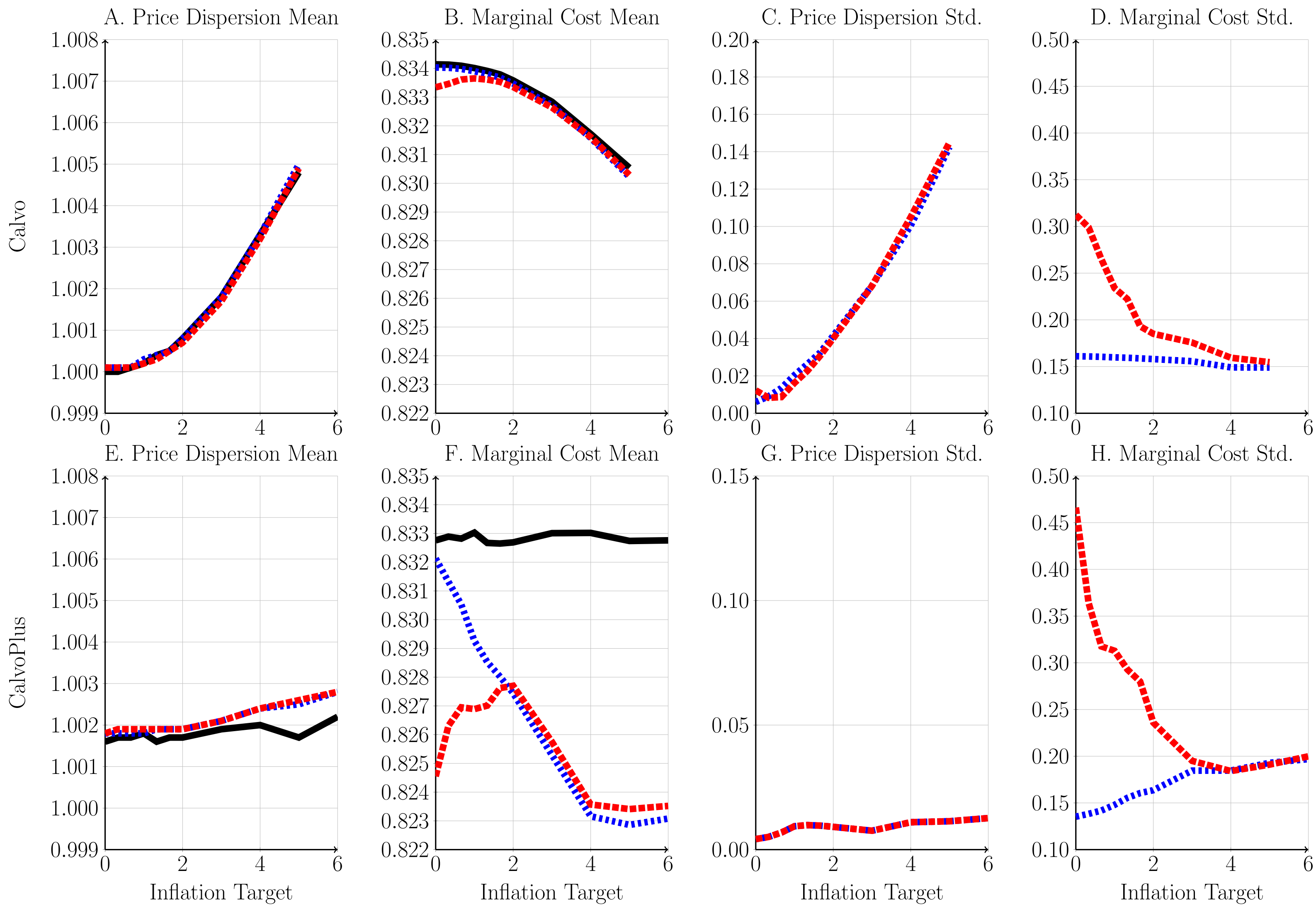
F Figures

Figure 10: Main Moments For Welfare in the Calvo and Calvo Plus Model



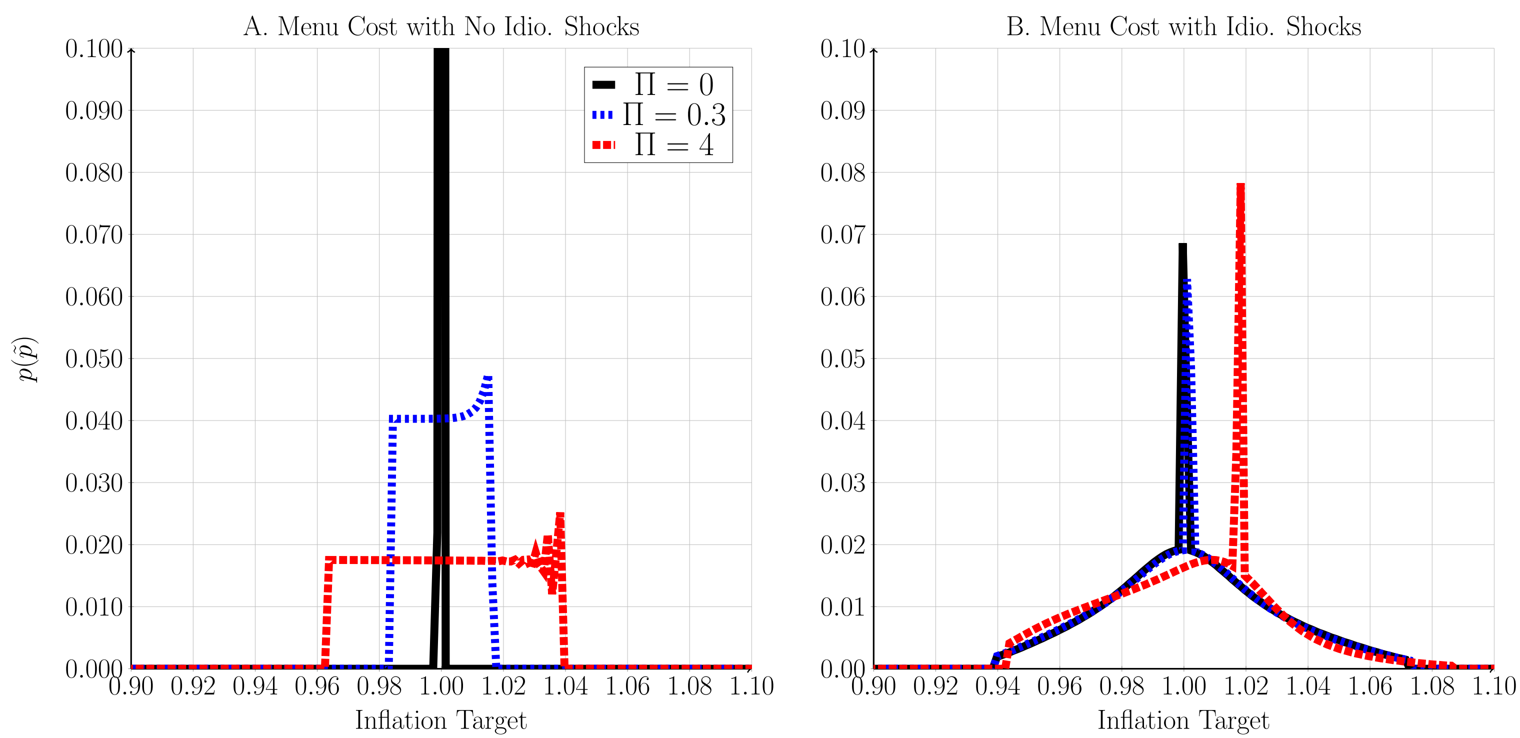
Panel A to D describe the consumption equivalent, the probability of zero interest rate, the consumption-labor supply ratio and the standard deviation of consumption in the Calvo Model. Panel E to H describe the the same variables in the CalvoPlus Model. The black solid line plots the moments without zero lower bound and the blue dashed line the moments with zero lower bound.

Figure 11: First Two Moments in the Price Dispersion and Marginal Cost



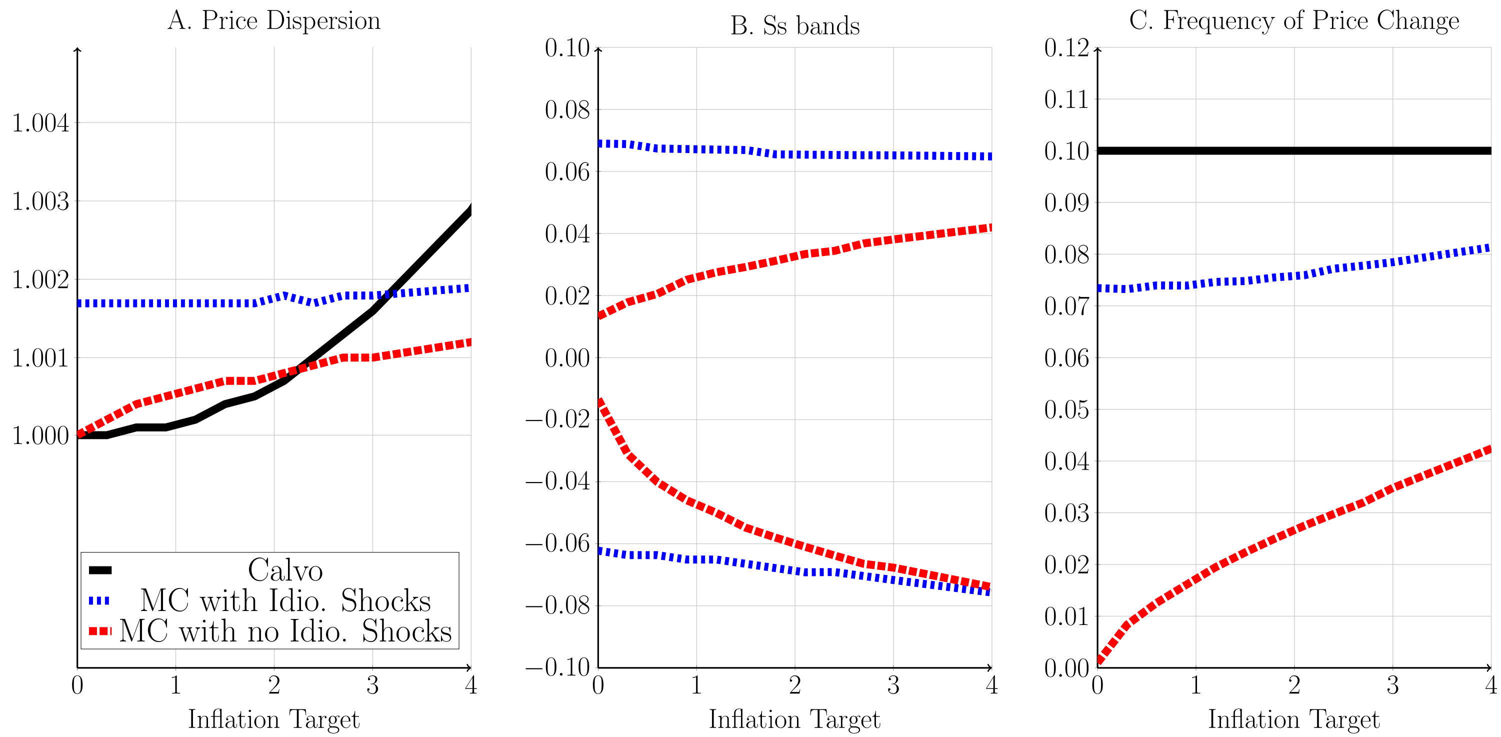
Panel A to D describe the mean and standard deviation of the price dispersion and marginal cost of the Calvo Model. Panel E to H describe the mean and standard deviation of the price dispersion and marginal cost of the CalvoPlus Model. The black line plots the steady state moments, the blue dashed line the moments without zero lower bound and the read dotted lines plots the moment in the models with zero lower bound.

Figure 12: Steady State Distribution of Rel. Prices in the Menu Cost with and without Idiosyncratic Shocks



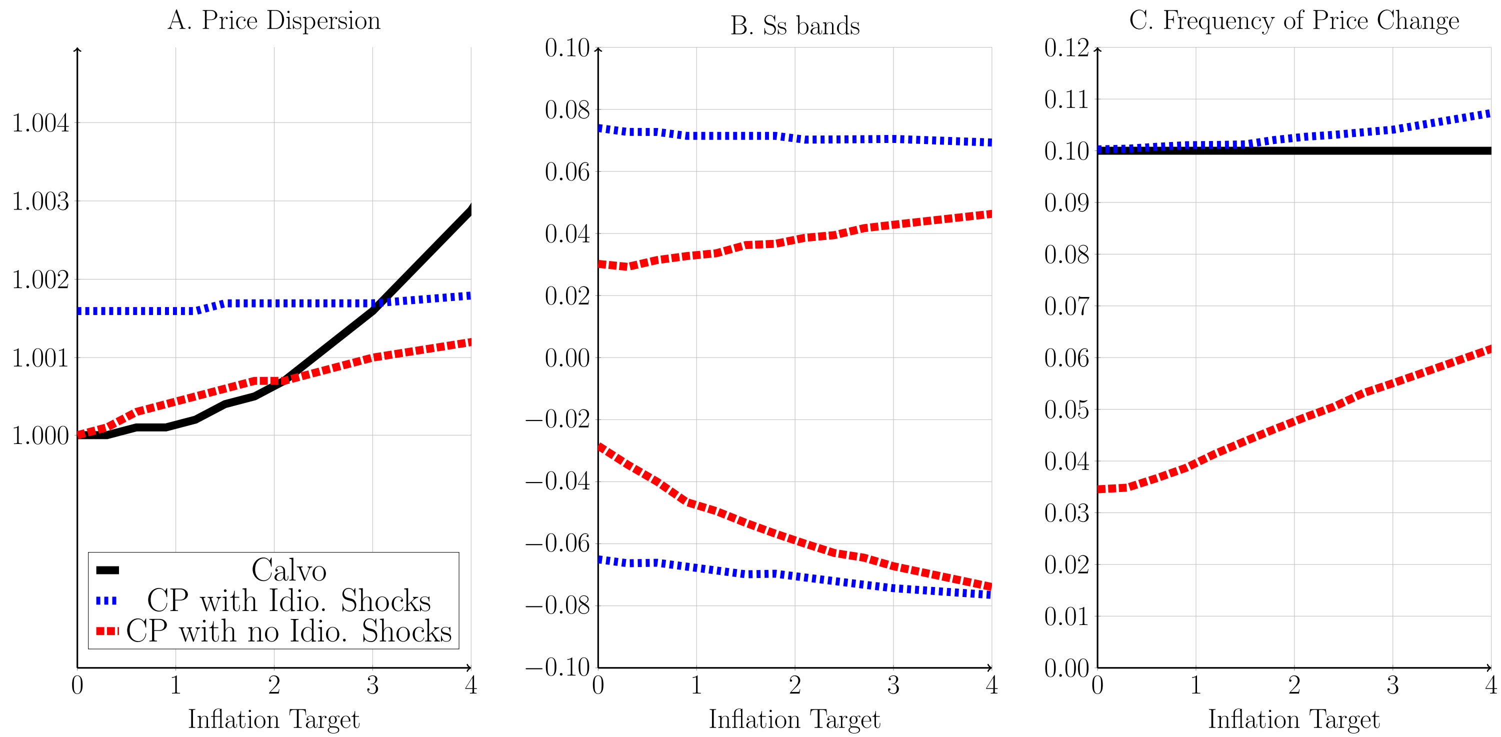
Panel A and B describe the distribution of relative price at 0, 0.3 and 4% inflation inflation target

Figure 13: Steady State Statistics in the Calvo, and Menu Cost with and without Idiosyncratic Shocks



Panel A describes the price dispersion in the Calvo, Menu Costs with idiosyncratic shocks and without idiosyncratic shocks. Panel B describes the log-distance between the upper and lower Ss bands with respect to the reset price. Panel C describes the frequency of price in the three models.

Figure 14: Steady State Statistics in the Calvo, and CalvoPlus with and without Idiosyncratic Shocks



Panel A describes the price dispersion in the Calvo, CalvoPlus with and without idiosyncratic shocks. Panel B describes the log-distance between the upper and lower Ss bands with respect to the reset price. Panel C describes the frequency of price in the three models.

G Numerical Appendix

This numerical appendix describes the equilibrium conditions of the CalvoPlus Model together with the evaluation of Krusell-Smith.

G.1 Recursive Equilibrium conditions

I divide the equilibrium condition in 4 blocks: household optimality conditions, firm's optimality conditions, monetary policy and aggregate feasibility, Krusell-Smith cross-equation approximation and exogenous shocks. For simplicity, I denote with X the detrended variable of $\tilde{X}$ and S the aggregate state of the economy given by $S = \Big(mc_{-}, \Delta_{-}, \tilde{R}_{-}, \eta_z, \eta_g, \eta_q\Big)$.

- **Household optimality conditions:**

$$mu(S) = \beta(1+g)^{-\sigma_n}\eta_{q-}(S)R(S)\mathbb{E}_{S'}\left[\left(\frac{U(S')/U_{ss}}{\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu(S')}{\Pi(S')}\middle| S\right] \tag{42}$$

$$\Sigma(S) = \mathbb{E}_{S'}\left[\left(\frac{U(S')}{U_{ss}}\right)^{1-\sigma_{ez}}\middle| S\right] \tag{43}$$

$$mu(S) = \bar{u}\left(C(S) - \kappa\frac{L(S)^{1+\chi}}{1+\chi}\right)^{-\sigma_{np}} \tag{44}$$

$$\kappa L(S)^{\chi} = w(S) \tag{45}$$

$$u(S) = \bar{u}\frac{\left(C(S) - \kappa\frac{L(S)^{1+\chi}}{1+\chi}\right)^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{46}$$

$$U(S) = (1-\beta)u(S) + \beta(1+g)^{1-\sigma_{np}}U_{ss}\Sigma(S)^{\frac{1}{1-\sigma_{ez}}} \tag{47}$$

  - Where the original variable can be obtain as:

$$\tilde{C}_t = C(S_t)(1+g)^t \quad ; \quad \tilde{mu}_t = mu(S_t)(1+g)^{-\sigma_{np}t} \quad ; \quad \tilde{L}_t = L_t \quad ; \quad \tilde{\Pi}_t = \Pi(S)$$
$$\tilde{u}_t = u(S_t)(1+g)^{(1-\sigma_{np})t} \quad ; \quad \tilde{U}_t = U(S_t)(1+g)^{(1-\sigma_{np})t} \quad ; \quad \tilde{w}_t = w(S_t)(1+g)^t \tag{48}$$

- **Firm's optimality conditions:**

$$v(\tilde{p},S)=\mathbb{E}_{S,x}\left[\left(\frac{U(S')}{U_{ss}\Sigma(S)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\left((1-hz)\max_{c,nc}\left\{v^c(S)-\bar{\theta}(S'),v(\tilde{p}'(\tilde{p}),S')\right\}+hzv^c(S)\right)\right] \tag{49}$$

$$P^*(S)=\arg\max_{\tilde{p}}\left\{\Phi(\tilde{p},S)+\beta(1+g)^{1-\sigma_{np}}v(\tilde{p},S)\right\} \tag{50}$$

$$v^c(S)=\Phi\left(P^*(S),S\right)+\beta(1+g)^{1-\sigma_{np}}v\left(P^*(S),S\right) \tag{51}$$

$$v^{nc}(\tilde{p},S)=\Phi\left(\tilde{p},S\right)+\beta(1+g)^{1-\sigma_{np}}v\left(\tilde{p},S\right) \tag{52}$$

$$\Phi(\tilde{p},S)=mu(S)Y(S)\tilde{p}^{-\gamma}\left(\tilde{p}-mc(S)\right) \tag{53}$$

$$\tilde{p}'(\tilde{p})=\begin{cases}\frac{\tilde{p}e^{\Delta a_l}}{\Pi(S')} & \text{with prob. } p\\ \frac{\tilde{p}e^{\Delta a_h}}{\Pi(S')} & \text{with prob. } 1-p\end{cases}\quad;\quad\bar{\theta}(S)=\theta w(S')mu(S')\quad;\quad\Delta a\sim N(0,\sigma_a) \tag{54}$$

- **Monetary policy and aggregate feasibility:**

$$\Pi(S)=\left(\frac{1-\Omega(S)}{1-\Omega(S)(P^*(S))^{1-\gamma}}\right)^{\frac{1}{1-\gamma}}\varphi(S)$$

$$mc(S)=\iota\frac{((1-\tau_L)w(S))^{1-\alpha}}{\eta_{z-}(S)} \tag{55}$$

$$R^*(S)=\tilde{R}_-(S)\left(\frac{1+\bar{\pi}}{\beta g^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi(S)}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{mc(S)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{mc(S)}{mc_-(S)}\right)^{\phi_{dy}} \tag{56}$$

$$\tilde{R}(S')=\left(\left(\frac{\beta(1+g)^{-\sigma_{np}}}{1+\bar{\pi}}\right)R^*(S)\right)^{\phi_r}\eta_r' \tag{57}$$

$$R(S)=\max\{1,R^*(S)\} \tag{58}$$

$$(L(S)-\theta(\Omega(S)-hz))=Y(S)\left(\frac{(1-\alpha)}{\alpha(1-\tau_L)w(S)}\right)^{\alpha}\frac{\Delta(S)}{\eta_{z-}(S)} \tag{59}$$

$$\eta_{g-}(S)+C(S)=Y(S)\left(1-\left(\frac{(1-\tau_L)w(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta(S)}{\eta_{z-}(S)}\right) \tag{60}$$

  - Where the original variable can be obtain as:

$$\tilde{mc}_t=mc(S_t)\quad;\quad\tilde{Y}_t=Y(S_t)(1+g)^t \tag{61}$$

- **Krusell-Smith cross-equation approximation:** let $P^2(S)$ be a second order polynomial with respect to the aggregate state

$$\log\left(\Delta(S)\right) = P^2(\log(S)) \quad ; \quad \log\left(\Omega(S)\right) = P^2(\log(S)) \quad ; \quad \log\left(\varphi(S)\right) = P^2(\log(S)) \tag{62}$$

- **Exogenous shocks:**

$$\begin{aligned}
\log\left(\eta_g(S')\right) &= (1-\rho_g)\log(\eta_g^*) + \rho_g \log(\eta_g(S)) + \sigma_g \epsilon_g' && (63)\\
\log\left(\eta_z(S')\right) &= (1-\rho_Z)\log(\eta_z^*) + \rho_z \log(\eta_z(S)) + \sigma_z \epsilon_z' && (64)\\
\log\left(\eta_r)S')\right) &= (1-\rho_r)\log(\eta_r^*) + \rho_r \log(\eta_r(S)) + \sigma_r \epsilon_r' && (65)\\
\log\left(\eta_q(S')\right) &= (1-\rho_q)\log(\eta_q^*) + \rho_q \log(\eta_{q(S)}) + \sigma_q \epsilon_q && (66)\\
& && (67)
\end{aligned}$$

G.2 Krusell-Smith Evaluation

I evaluate Krusell-Smith in the simulation. The first step is to construct a time series of simulated inflation, price dispersion and frequency of price change, together with simulated marginal cost, interest rate and structural shocks. Let

$$\left\{mc^(S_t^s), \Delta_{t-1}^s, \tilde{R}_-(S_t^s), \eta_z(S_t^s), \eta_g(S_t^s), \eta_q(S_t^s), \Pi_t^s, \Omega_t^s\right\}_t \tag{68}$$

the simulated state. Let

$$\{mc(S_t^s), mu(S_t^s), R(S_t^s), R^*(S_t^s), Y(S_t^s), C(S_t^s), L(S_t^s), P^*(S_t^s, w(S_t^s))\}_t \tag{69}$$

the model implied equilibrium functions that comes from solving the equilibrium conditions and

$$\left\{\hat{mc}_t^s, \hat{mu}_t^s, \hat{R}_t, \hat{R}_t^*, \hat{Y}_t, \hat{C}_t^s, \hat{L}_t^s, \hat{P}_t^{*,s}, w_t^s\right\}_t \tag{70}$$

the solution of the equilibrium equations in the simulation. Next, I describe the construction of each function.

- **Construction of $\left\{\hat{mc}_t^s, \hat{R}_t, \hat{R}_t^*, \hat{Y}_t, \hat{C}_t^s, \hat{L}_t^s, \hat{P}_t^{*,s}, w_t^s\right\}_t$ using of Backward Looking Equations and $\hat{mu}_t^s$:** I construct $\left\{\hat{mc}_t^s, \hat{R}_t^s, \hat{R}_t^{*,s}, \hat{Y}_t^s, \hat{C}_t^s, \hat{L}_t^s, \hat{P}_t^{*,s}, w_t^s\right\}_t$ solving the static system of equations

$$\hat{R}_t^{*,s} = \tilde{R}_-(S_t^s)\left(\frac{1+\bar{\pi}}{\beta(1+g)^{-\sigma_{np}}}\right)\left(\left(\frac{\Pi_t^s}{(1+\bar{\pi})}\right)^{\phi_\pi}\left(\frac{\hat{mc}_t^s)}{mc_{ss}}\right)^{\phi_y}\right)^{1-\phi_r}\left(\frac{\hat{mc}_t^s}{mc_-(S_t^s)}\right)^{\phi_{dy}} \tag{71}$$

$$\hat{R}_t^s = \max\{1, \hat{R}_t^{*,s}\} \tag{72}$$

$$\hat{mu}_t^s = (\hat{C}_t^s - \kappa\frac{(\hat{L}_t^s)^{1+\chi}}{1+\chi})^{-\sigma_{np}} \tag{73}$$

$$\kappa(\hat{L}_t^s)^\chi = \hat{w}_t^s \tag{74}$$

$$\eta_z(S_t^s)(\hat{L}_t^s - \theta(\Omega_t^s - hz)) = \hat{Y}_t^s\left(\frac{(1-\alpha)}{\alpha(1-\tau_L)\hat{w}_t^s}\right)^\alpha \Delta_t^s \tag{75}$$

$$\eta_{g-}(S) + \hat{C}_t^s = \hat{Y}_t^s\left(1 - \left(\frac{(1-\tau_L)\hat{w}_t^s\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_t^s}{\eta_z(S_t^s)}\right) \tag{76}$$

$$\tag{77}$$

$$\hat{mc}_t^s = \iota\frac{((1-\tau_L)\hat{w}_t^s)^{1-\alpha}}{\eta_z(S_t^s)} \tag{78}$$

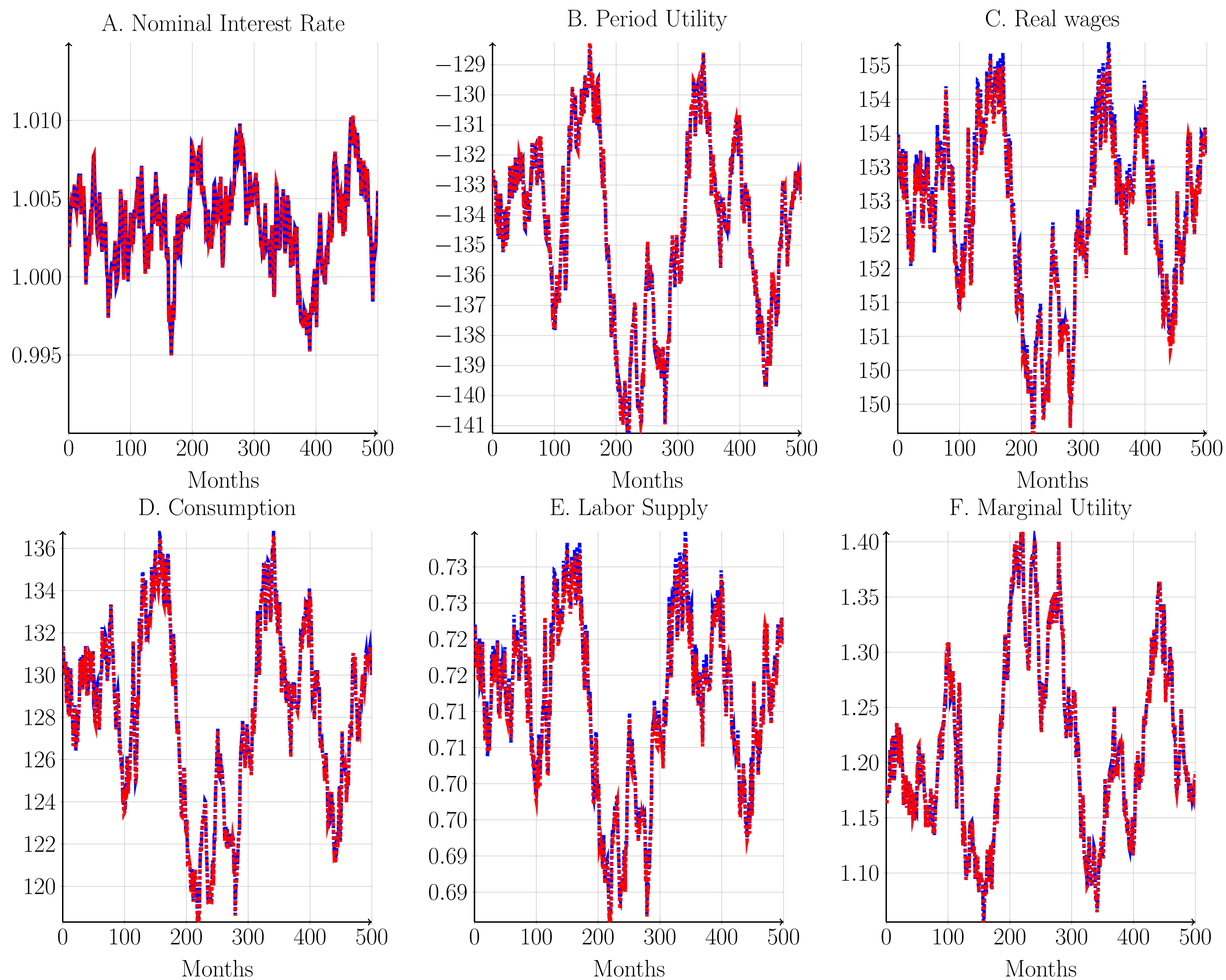
where $\hat{mu}_t^s$ is an input in this system of equations.

- **Verification of Forward Looking Equations:** I construct $\hat{mu}_t^s$ using household Bellman equatiion. To verify this conditions I project the realized inflation with respect to the state; I define this function as $\tilde{\Pi}(S)$ and I use it to check the forward looking equations. In the case of the Euler equation, I construct marginal utility in the simulation as

$$\hat{mu}_t^s = \beta(1+g)^{-\sigma_n}\eta_{q-}(S_t^s)\hat{R}_t(S^s)\mathbb{E}_{S'}\left[\left(\frac{U(S')/U_{ss}}{\Sigma(S_t^s)^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\frac{mu(S')}{\tilde{\Pi}(S')}\middle| S_t^s\right] \tag{79}$$

$$\tag{80}$$

Figure 15: Predicted and Simulated Aggregate Time Series



Panel A to F describe the macroeconomic time series for one simulation at zero inflation. The blue line describes the model implied aggregate variable with the Krusell-Smith projections and the red line describes the implied macroeconomic time series with simulated inflation, frequency of price change and price dispersion.

Table 7: Krusell-Smith Evaluation with No ZLB

Annual Inflation Target

moment	0	1	2	3	4	5	6
Nominal interest rate	0.023	0.032	0.041	0.054	0.059	0.093	0.141
Period Utility	0.006	0.008	0.006	0.008	0.009	0.013	0.011
Real wage	0.232	0.290	0.254	0.289	0.274	0.448	0.519
Consumption	0.028	0.029	0.028	0.030	0.027	0.046	0.056
Labor supply	0.232	0.290	0.254	0.290	0.274	0.449	0.519
Marginal utility	0.001	0.001	0.001	0.001	0.000	0.001	0.001

This table describes the ratio between the variance of the predicted error and the total variance of the same variable for nominal rates, reset price, real wages, consumption, labor supply and marginal utility. I construct each column in the following way: let X_t be the aggregate variable that comes from using Krusell-Smith projections and $\hat{X}_t$ is the same variable that solves the equilibrium equation with the simulated inflation, price dispersion and frequency of price change, then each element in the table is given by $\frac{Var(log(X)-log(\hat{X}))}{Var(log(X))}100$ for $X \in \{R, P, w, C, L, mu\}$ and annual inflation targets of 0,2,4,6. See appendix G for the method to construct $\hat{X}_t$ for each aggregate variable.

Table 8: Krusell-Smith Evaluation with ZLB

Annual Inflation Target

moment	0	1	2	3	4	5	6
Nominal interest rate	0.353	0.609	0.175	0.138	0.073	0.079	0.181
Period Utility	0.139	0.266	0.007	0.009	0.007	0.010	0.011
Real wage	0.992	2.196	0.311	0.326	0.207	0.321	0.528
Consumption	0.188	0.314	0.041	0.035	0.020	0.030	0.057
Labor supply	0.991	2.174	0.312	0.327	0.207	0.321	0.528
Marginal utility	0.093	0.128	0.004	0.001	0.000	0.000	0.002

This table describes the ratio between the variance of the predicted error and the total variance of the same variable for nominal rates, reset price, real wages, consumption, labor supply and marginal utility. I construct each column in the following way: let X_t be the aggregate variable that comes from using Krusell-Smith projections and $\hat{X}_t$ is the same variable that solves the equilibrium equation with the simulated inflation, price dispersion and frequency of price change, then each element in the table is given by $\frac{Var(log(X)-log(\hat{X}))}{Var(log(X))}100$ for $X \in \{R, P, w, C, L, mu\}$ and annual inflation targets of 0,2,4,6. See appendix G for the method to construct $\hat{X}_t$ for each aggregate variable.