

# Chapter 3 – Real equilibrium exchange rates

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*Real exchange rates often appreciate or depreciate over longer periods of time. Such behaviour may be caused by changes in their equilibrium values. This paper presents some of the popular explanations for changes in equilibrium real exchange rates and methods for calculating them. It focuses on three common approaches: the theory of purchasing power parity, the behavioural equilibrium exchange rate and the fundamental equilibrium exchange rate approaches. The paper also reviews briefly some of the recent empirical studies based on these approaches.*

## 1. Introduction

The real exchange rate expresses the terms of trade for domestic and foreign goods and services. It is defined as the nominal exchange rate adjusted by the relative price between domestic and foreign goods and services. An appreciation of the nominal exchange rate or higher price inflation at home relative to other countries may serve to strengthen the real exchange rate. This can weaken the home country's competitiveness and result in a foreign trade deficit, lower domestic activity and subdued domestic wage and price inflation. The opposite can occur if the real exchange rate depreciates.

The equilibrium real exchange rate is often used as a benchmark for assessing whether the actual real exchange rate is too strong or too weak. It can be defined as the real exchange rate level that is consistent with a state of internal and external balance. Internal balance means that all available factors of production, such as labour and capital, are being fully utilised and wage and price inflation is stable. External balance can be characterised as a state of stable net foreign assets.

However, the equilibrium real exchange rate is not an unambiguous concept in economic literature and a large number of equilibrium exchange rate concepts have been proposed, see Driver and Westaway (2001), Hinkle and Montiel (1999) and MacDonald and Stein (1999). We will, however, limit our focus to the three relatively popular approaches in economic literature. These are the theory of purchasing power parity (PPP), the behavioural equilibrium exchange

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rate (BEER) approach and the fundamental equilibrium exchange rate (FEER) approach. We will present some of the common explanations for movements in equilibrium real exchange rates and also give a brief account of recent empirical studies based on these approaches and discuss some of the main issues involved.

The article is set out as follows: the theory of purchasing power parity is presented in Section 2, followed by a brief summary of results based on Norwegian and a few international empirical studies. This approach leads primarily to a descriptive analysis of movements in the real exchange rate. Section 3 uses the Scandinavian model of inflation as a framework for explaining why a number of factors and mechanisms may contribute to trends and fluctuations in real exchange rates. Many of these factors are included in most empirical studies of real exchange rates based on the BEER approach. This approach is presented in more detail in Section 3.3. Section 4 looks more closely at the FEER approach and offers a brief overview of the main issues related to FEER-calculations. The final section (5) is a summary of the key points.

## 2. Purchasing power parity - constant equilibrium exchange rate

The theory of purchasing power parity is a usual starting point for calculations of the equilibrium exchange rate. According to this theory, domestic goods will generally cost the same as foreign goods measured in a common currency, when adjusted for international trade costs. Price differences over and above such costs will be eliminated in the long run through international trade. This means that the general terms of trade between domestic and foreign goods, measured in terms of the real exchange rate, will be in equilibrium at a certain level. Alternatively, it could be said that the equilibrium real exchange rates ( $R^*$ ) is constant:

$$R^* = \alpha. \tag{1}$$

This constant level ( $\alpha$ ) can represent conditions that entail sustained, but stable deviations from absolute purchasing power parity. Absolute purchasing power parity implies one-to-one terms of trade between domestic and foreign goods, i.e.  $R^* = 1$ . Deviations from absolute purchasing power parity may be ascribed to transportation costs and trade barriers. They may also be due to technical reasons such as different methods of calculating the general price level at home and abroad. For example, differences in consumption and production patterns between countries may result in different weights being given to individual goods when calculating the general price level.

The real exchange rate can be defined as  $R \equiv EP^f/P$ , where  $E$  is the nominal exchange rate and  $P^f/P$  is the ratio between the general price level abroad and in the home country.  $E$  denotes the

price of foreign currency in domestic currency units. A constant real exchange rate implies equal rates of price inflation at home and abroad when adjusted for changes in the nominal exchange rate.

There can be both internal and external balance when the real exchange rate is at its equilibrium level. Any deviation from the equilibrium level may give rise to internal and external imbalances. Such imbalances will, however, bring the real exchange rate back to its equilibrium level.

If, for example, the actual real exchange rate is stronger than the equilibrium real exchange rates ( $R < R^*$ ), domestic goods will cost more than foreign goods (even when trading costs are deducted). Poor competitiveness may then result in a trade deficit, low domestic activity and unemployment. The trade deficit may in turn contribute to weaken the nominal exchange rate (raise  $E$ ). At the same time, the low activity level may slow wage and price inflation at home relative to that in other countries (raise  $P^f/P$ ). The increase in the nominal exchange rate and relative price level will weaken the real exchange rate (increase  $R$ ) so that it returns to its equilibrium level.

The real exchange rate may, however, become weaker than the equilibrium level ( $R > R^*$ ) before stabilising at the equilibrium level. This can occur if the home country builds up foreign debt when the exchange rate is stronger than the equilibrium exchange rate. A depreciation in the exchange rate relative to the equilibrium exchange rate may result in a trade surplus, thereby making it easier to service and repay the accrued foreign debt. The speed with which the real exchange rate converges towards the equilibrium exchange rate will depend on how much and how quickly the nominal exchange rate and prices react to a deviation from the equilibrium level:  $(R - R^*)$ . This is determined, among other factors, by how much and how rapidly the real exchange rate affects foreign trade and the activity level, by the nominal exchange rate's response to changes in foreign trade and by how strong and fast prices respond to changes in the activity level and the nominal exchange rate. The stronger these relationships are, the faster internal and external imbalances will be eliminated and the faster the real exchange rate will stabilise at the equilibrium level, see Section 3 for further discussion.

### 2.1. Empirical evidence of purchasing power parity

An increasing number of studies show that real exchange rates have a tendency to converge towards a stable rate in the long run, see for example Rogoff (1996), Sarno and Taylor (2002), Cheung and Lai (2000) and the references therein. This tendency seems to be particularly clear in studies that observe movements in real exchange rates over longer periods and in studies of real exchange rates between countries with similar economic structures and growth rates, e.g. countries with comparable income per capita. In addition, data from small open economies with

fixed exchange rate regimes and flexible product and labour markets that have not experienced major real economic shocks also tend to provide support for stable real exchange rates, all else being equal. These features of empirical studies can be explained in light of the presentation in Sections 3 and 4.

It can, however, take a relatively long time before differences between the actual real exchange rate and the equilibrium exchange rate are eliminated. In industrial countries, it can take 3–5 years for half of a deviation to be eliminated, all else being equal. Such a long half-life is often seen as a puzzle, as it cannot be explained by lags in price changes, see Rogoff (1996). The adjustment to the equilibrium level appears to occur faster in developing countries, where the half-life is often found to be 1–2 years, see for example Cheung and Lai (2000).

Studies of Norwegian data from the past thirty years support the theory of purchasing power parity, see Akram (2000, 2002). It is particularly interesting to note that the half-life for a given deviation seems to be around 1 1/2 years, despite the fact that the Norwegian economy has experienced major real economic shocks since the 1970s. The discovery of oil and gas on the Norwegian continental shelf and revaluations of petroleum wealth as a result of substantial oil price shocks are examples of such shocks. Akram (2003) examines movements in the Norwegian real exchange rate in more detail in chapter 4 of this Occasional Paper.

Recently, a number of studies have pointed out that there may be methodological reasons for the higher estimates of half-lives and the rejection of the theory of purchasing power parity in some studies. For example, Taylor (2001) points to the use of annual data rather than quarterly or monthly data that is better able to capture price adjustments as they happen, as a possible reason; Imbs, Mumtaz, Ravn and Rey (2003) point to the use of general price indices rather than sub-indices, which can take into account possible differences in half-lives between groups of goods; and Taylor, Peel and Sarno (2001) point to the use of linear models rather than non-linear models, which can take into account whether it is unprofitable to adjust prices when there is only a small deviation from the equilibrium level.

Half-life calculations may, however, give a distorted impression of how quickly deviations from the equilibrium exchange rate are actually eliminated. The reason being that exchange rates are continuously subject to shocks, so that the real exchange rate may deviate from its equilibrium level for a longer period than expressed by partial observations, cf. Figure 1. The figure shows movements in the real trade-weighted krone exchange rate together with corresponding real exchange rates for the UK and New Zealand. The real exchange rates generally seem to fluctuate around a given level, without any persistent trends in one direction or the other. However, they do display rather long cycles. This indicates that deviations from any constant equilibrium level may last for several years. The fluctuations in the Norwegian real exchange rate are,

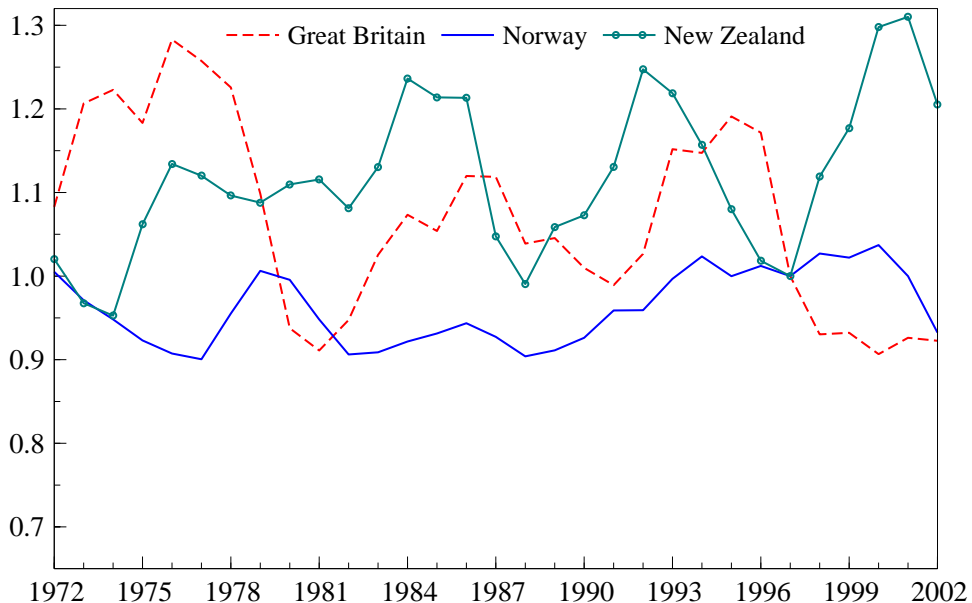


Figure 1: Trade weighted real exchange rates for New Zealand, Norway and the UK in the period 1972–2002. Annual observations; base year 1997 = 1. Source: OECD MEI.

however, relatively modest in comparison to the two other real exchange rates which support calculations of a relatively low half-life for the Norwegian real exchange rate compared with typical estimates for other industrial countries.

Figure 2 illustrates changes in some real exchange rates that appear to have experienced trend movements over a longer period of time. The figure shows that the real trade-weighted exchange rates for Australia, Canada and Sweden have become steadily weaker over time, but have experienced substantial fluctuations in periods. In the next section, we present some explanations as to why real exchange rates may show trend movements or long-term deviations from purchasing power parity.

### 3. The equilibrium real exchange rates vary

This section presents some of the usual explanations of fluctuations and trends in real exchange rates over different time horizons, using the Scandinavian model of inflation (the “main course” theory) as a framework, see Aukrust (1977) and Calmfors (1977). In this model, overall wage growth is determined in the sector exposed to international competition (hereafter c-sector),

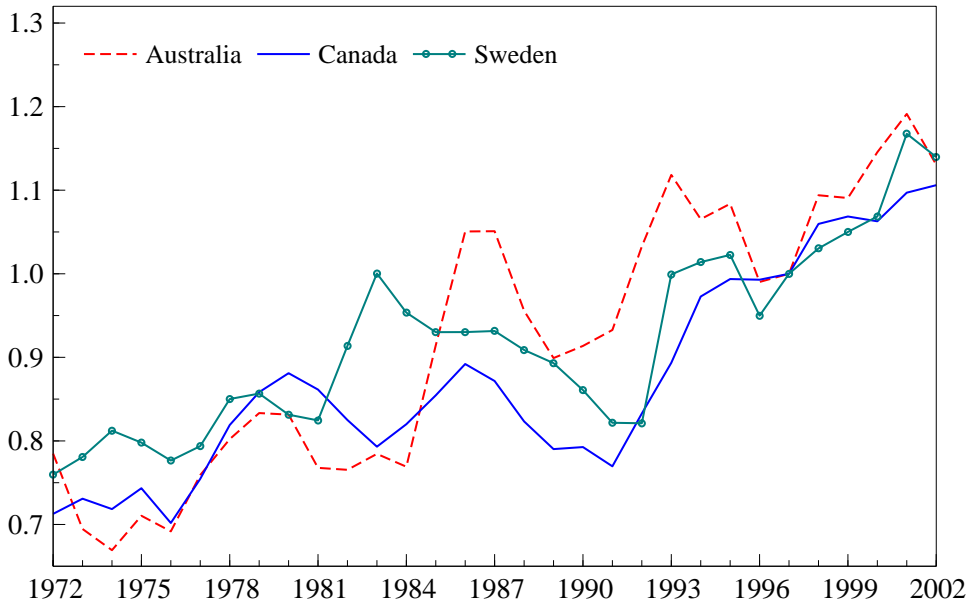


Figure 2: *Trade weighted real exchange rates for*

conditional on imported inflation and productivity growth in c-sector. Therefore, price growth in the sheltered sector (hereafter s-sector) rises with productivity growth in the c-sector, but declines with productivity growth in the s-sector itself. Thus, higher productivity growth in the c-sector relative to that in the s-sector contributes to price inflation in the home country, see Aukrust (1977) for a formal exposition.

### 3.1. In the long term

Sustained differences in relative productivity growth between the c-sector and s-sector in the home country and abroad can contribute to trend movements in the real exchange rate, see Balassa (1964) and Samuelson (1964). This effect can be explained within the framework of the main-course theory.

Relative changes in the real exchange rate ( $\Delta r$ ) can by definition be decomposed into nominal exchange rate changes ( $\Delta e$ ) and differences in inflation between the home country and abroad:

$$\Delta r = \Delta e + \Delta p^f - \Delta p. \quad (2)$$

The lower case letters  $r$ ,  $p$ ,  $p^f$  and  $e$  are the logarithms of the real exchange rate, the general price

level at home and abroad and the nominal exchange rate, respectively. The main course theory implies that domestic inflation ( $\Delta p$ ) is determined in the long run by imported inflation ( $\Delta e + \Delta p_c^f$ ) and the difference in productivity growth between the c-sector and the s-sector ( $\Delta q_c - \Delta q_s$ ):

$$\Delta p = \Delta e + \Delta p_c^f + \omega(\Delta q_c - \Delta q_s). \quad (3)$$

Here,  $\omega$  is the weight given to non-tradeable goods in the domestic price index. It is common to assume that productivity growth in the c-sector is greater than in the s-sector both at home and abroad.

Productivity growth in the c-sector ( $\Delta q_c$ ) determines wage inflation in the c-sector and the s-sector, conditional on price inflation in the world market for tradeable goods in domestic currency ( $\Delta e + \Delta p_c^f$ ). If productivity growth is higher in the c-sector than in the s-sector, the growth in prices for non-tradeable goods will increase relative to the growth in prices for tradeable goods. This will contribute to higher inflation and an appreciation of the real exchange rate (real appreciation,  $\Delta r < 0$ ). A corresponding difference in productivity growth between the c-sector and the s-sector abroad may result in higher foreign inflation, which may contribute to weaken the real exchange rate (real depreciation,  $\Delta r > 0$ ), assuming that inflation abroad is determined in the same way as in the home country:

$$\Delta p^f = \Delta p_c^f + \omega^f(\Delta q_c^f - \Delta q_s^f). \quad (4)$$

The extent to which the real exchange rate appreciates or depreciates will therefore depend on the difference in relative productivity growth between the c-sector and the s-sector at home and abroad. This can be expressed with the help of equation (5), which is obtained by inserting the expressions for  $\Delta p$  and  $\Delta p^f$  from (3) and (4) in equation (2):

$$\Delta r = \omega^f(\Delta q_c^f - \Delta q_s^f) - \omega(\Delta q_c - \Delta q_s). \quad (5)$$

This equation indicates that persistent differences in productivity between the c-sector and the s-sector at home and abroad may result in a persistent real depreciation or real appreciation, and thus contribute to trend movements in the real exchange rate.

Equation (5) indicates that trend movements can also be ascribed to differences in the size of the c-sector at home and abroad. The larger the country's c-sector is (in relation to the s-sector), the greater is its importance to inflation in the economy as a whole. All else being equal, it therefore follows that if the domestic c-sector share is larger than the foreign c-sector share, the real exchange rate will become weaker.

This can be illustrated by reformulating equation (5) as follows:

$$\Delta r = \omega \{ (\Delta q_c^f - \Delta q_s^f) - (\Delta q_c - \Delta q_s) \} + (\omega^f - \omega) (\Delta q_c^f - \Delta q_s^f). \quad (6)$$

The first expression on the right hand side represents the effects of differences in productivity growth between the c-sector and the s-sector at home and abroad, whereas the second expression represents the effect of differences in size of the c-sector at home and abroad, represented by the difference in the weights for non-tradeable goods in general price levels:  $(\omega^f - \omega)$ . Equation (6) implies that if the domestic c-sector share is greater than the foreign c-sector share,  $(\omega^f - \omega) > 0$ , the real exchange rate will depreciate, *ceteris paribus*.

### 3.2. In the short and medium term

Movements in real exchange rates in the short and medium term may be due to unsynchronised business cycles and differences in domestic and foreign monetary and fiscal policy. It has also been observed that movements in nominal exchange rates may explain a considerable part of changes in real exchange rates in the short and medium run, see for example Mark (1990). This may in part explain why real exchange rates are often more stable under fixed exchange rate regimes than under floating exchange rate regimes, see Mussa (1986). Flexible product and labour markets can also have a stabilising effect on the real exchange rate, cf. Section 2.

The importance of these factors to movements in real exchange rates can also be illustrated within the framework of the Scandinavian model of inflation by modifying some of the assumptions. Let us assume that the rates of inflation at home and abroad are both affected by the (own) activity level and that the impact of changes in the nominal exchange rate on price inflation in the c-sector is incomplete in the short or medium term. These modifications can be taken into account by including expressions for output gap (difference between actual and potential GDP) in equations for domestic and foreign price inflation and by allowing the impact of changes in the nominal exchange rate on domestic inflation to be delayed. This results in the following models for price inflation at home and abroad in the short and medium term:

$$\Delta p = \Delta e_{-1} + \Delta p_c^f + \omega (\Delta q_c - \Delta q_s) + \mu (Y - \bar{Y}), \quad (7)$$

$$\Delta p^f = \Delta p_c^f + \omega^f (\Delta q_c^f - \Delta q_s^f) + \mu^f (Y^f - \bar{Y}^f). \quad (8)$$

Here, for example,  $Y$  and  $\bar{Y}$  symbolise the actual and potential level of domestic output, whereas  $\mu$  indicates how strongly the output gap  $(Y - \bar{Y})$  affects domestic inflation.  $\Delta e_{-1}$  represents the change in the nominal exchange rate, which is assumed to have full impact on price inflation after a period. For the sake of simplicity, we have disregarded any possible lags in the impact on domestic and foreign inflation of changes in prices for tradeable goods  $(\Delta p_c^f)$ . We have also

overlooked the possibility that productivity growth can have different effects on inflation in the short and long run.

By including the equations for inflation in the short and medium term, (7) and (8), in equation (2), we get the following model for changes in the real exchange rate:

$$\Delta r = \omega^f (\Delta q_c^f - \Delta q_s^f) - \omega (\Delta q_c - \Delta q_s) + \mu^f (Y^f - \bar{Y}^f) - \mu (Y - \bar{Y}) + \Delta e - \Delta e_{-1}. \quad (9)$$

Equation (9) indicates that a period of economic expansion at home ( $Y - \bar{Y} > 0$ ), which coincides with an economic downturn abroad ( $Y^f - \bar{Y}^f < 0$ ), contributes to an appreciation of the real exchange rate ( $\Delta r < 0$ ). This can be explained by the fact that a positive output gap at home contributes to higher domestic inflation, whereas a negative output gap abroad lowers the rate of inflation abroad, all else being equal. Both effects entail a strong real exchange rate. Conversely, the real exchange rate may depreciate if there are available resources at home and pressure on resources abroad. However, if the domestic business cycle is in phase with that abroad, it is uncertain whether the real exchange rate will depreciate or appreciate. This depends on the size of the output gap and its importance to price inflation at home and abroad, i.e. on  $\mu$  and  $\mu^f$ .

The importance of flexible labour and product markets to movements in the real exchange rate can also be explained with the help of equation (9). Let us assume, for example, that the real exchange rate appreciates ( $\Delta r < 0$ ) as a result of a shock and that this translates into a lower activity level and negative output gap at home ( $Y - \bar{Y} < 0$ ). Equation (9) implies that the negative output gap would serve to reverse the appreciation of the real exchange rate. However, the speed with which the exchange rate then depreciates depends on the strength of the relationship between the activity level and price inflation, i.e. on  $\mu$ . The stronger that prices respond to the output gap (the greater  $\mu$  is), the faster the appreciation in the real exchange rate will be reversed by lower price inflation.

In the long run, the output gap at home and abroad will close (internal balance), so that the real exchange rate will be determined by conditions on the supply side such as differences in productivity growth. In the short to medium term, however, differences in the business cycle will contribute to weaken or strengthen the exchange rate over a long period, depending on the flexibility of price formation.

Differences in the business cycle may be due to asymmetric effects of shocks or asymmetric shocks. For example, a higher oil price normally has a contractionary effect on the activity level for most of Norway's trading partners that are oil importers, whereas it has an expansionary effect in Norway, which is an oil exporter. Higher oil prices may therefore lead to a stronger real exchange rate, while a fall in the oil price may result in a weaker real exchange rate, see

equation (9). Differences in monetary and fiscal policy at home and abroad can be seen as asymmetric shocks. For example, public consumption of oil revenues will generate positive demand impulses in the Norwegian economy over several years. This can contribute to pressure on prices and a stronger real exchange rate, see Torvik (2003) and Akram (2003). In the same way, interest rate differentials between the home country and abroad can effect output gaps in different directions, forcing a change in the real exchange rate. Interest rate differentials may also have a considerable effect on the nominal exchange rate and result in substantial fluctuations in the real exchange rate, as explained below.

The effect of changes in the nominal exchange rate on the real exchange rate can be explained with the help of the last expression in equation (9):  $\Delta e - \Delta e_{-1}$ . Under a floating exchange rate regime, there can be substantial movements in the nominal exchange rate, for example, as a result of interest rate differentials. This may lead to corresponding movements in the real exchange rate in the short run. In the longer run, however, changes in the nominal exchange rate may translate into price inflation so that the initial effect on the real exchange rate is cancelled out. Equation (9) shows, for example, that an appreciation of the nominal exchange rate ( $\Delta e < 0$ ) may contribute to an immediate appreciation in the real exchange rate, only to weaken it subsequently, when the nominal appreciation has impacted on price inflation. It could also be said that  $\Delta e - \Delta e_{-1} = 0$  in the longer term, so that movements in the real exchange rate are not dependent on the nominal exchange rate in the longer run. Therefore, the effect of nominal exchange rate changes under a floating exchange rate regime is the same as under a fixed exchange rate regime in the long run. A fixed rate regime can be expressed as  $\Delta e = \Delta e_{-1} = 0$ . It could thus be said that the real exchange rate is independent of the exchange rate regime in the long term.

Lags in the impact of exchange rate changes on prices are important in terms of how long the real exchange rate is affected by the nominal exchange rate. The slower the impact on prices is, the longer it will take for the real appreciation resulting from a nominal appreciation to be reversed. The impact of changes in the exchange rate on prices may also be asymmetrical. If a nominal appreciation is slower to impact than a nominal depreciation, it will take longer for a real appreciation to be reversed than for a real depreciation.

### 3.3. Behavioural equilibrium exchange rate - BEER

The BEER approach takes into account the possibility that the above mentioned macroeconomic variables may generate long swings and trend movements in the real exchange rate, see Clark and MacDonald (1998). In particular, this approach allows cross country differences in productivity growth and fiscal and monetary policies to contribute to persistent deviations from purchasing power parity, cf. sections 3.1 and 3.2. We will look at this approach in more detail

below and refer to some results from studies based on this approach. The BEER approach is based on the hypothesis of uncovered real interest parity, which can be formalised as follows:

$$r = r^e - (ir^e - ir^{e,f}), \quad (10)$$

where  $r^e$  symbolises the logarithm of the expected real exchange rate and  $(ir^e - ir^{e,f})$  represents the expected real interest rate differential between the home country and abroad. This equation indicates that an increase in the real interest rate differential coincides with a real appreciation of a corresponding size for a given expected real exchange rate. The expected real exchange rate can be determined by a set of macroeconomic variables  $z$  that characterises (relevant) aspects of the domestic and foreign economies:

$$r^e = \alpha_0 + \beta z, \quad (11)$$

where  $\alpha_0$  is constant and  $\beta$  represents the effects of  $z$ . The expected real exchange rate is assumed to be equal to the equilibrium exchange rate in the long run.

The equilibrium exchange rate in the long run (BEER,  $r^*$ ) is the real exchange rate that arises when the (effects of) macroeconomic variables are at their equilibrium levels or paths. e.g. when  $\beta z = \beta \bar{z}$ :

$$r^* \equiv \alpha_0 + \beta \bar{z}.$$

The equilibrium exchange rate  $r^*$  will reflect the development in  $\beta \bar{z}$ , i.e. in the linear combination of variables represented by  $\bar{z}$ . It follows that  $r^*$  will display a trend behaviour if  $\beta \bar{z}$  evolves as a trend, but will be a constant if  $\beta \bar{z}$  is constant. Note that  $\beta \bar{z}$  can be a constant even though the variables that are represented by  $\bar{z}$  individually display trend behaviour. It should also be noted that BEER may be equal to the PPP level of the equilibrium exchange rate if  $\beta \bar{z}$  is equal to zero and  $\alpha_0 = \ln \alpha$ , cf. equation (1).

The following model of actual movements in the real exchange rate in the short run can be derived:

$$r = \alpha_0 + \beta z - (ir - ir^f) + \varepsilon, \quad (12)$$

by inserting for  $r^e$  from equation (11) in (10) and by assuming that the actual real interest rate differential deviates from the expected one by an expectation error.  $\varepsilon$  is a stochastic add factor that represents the expectation error and the short-term effects of other factors on the real exchange rate.

The real exchange rate that is consistent with the current macroeconomic development at home and abroad can be defined as  $\widehat{r}$ . It may be calculated on the basis of an estimated version of equation (12):

$$\widehat{r} = \widehat{\alpha}_0 + \widehat{\beta}z - (ir - ir^f), \quad (13)$$

where  $\widehat{\alpha}_0$  and  $\widehat{\beta}$  denote estimates of the unknown parameters  $\alpha_0$  and  $\beta$ . Equation (12) may be estimated by taking into account the time series properties of  $r$ ,  $z$  and  $(ir - ir^f)$ .  $\widehat{r}$  may also be considered the equilibrium real exchange rate in the medium run, as it does not (explicitly) take into account short-run determinants of the real exchange rate.

The behaviour equilibrium real exchange rate in the long run ( $r^*$ ) may be estimated by making assumptions about the equilibrium level or path of  $\widehat{\beta}z$ . Estimates of  $r^*$  ( $\widehat{r}^*$ ) may then be obtained from the following relationship:

$$\widehat{r}^* = \widehat{\alpha}_0 + \widehat{\beta}\widehat{z},$$

where  $\widehat{\beta}\widehat{z}$  denotes an estimate of  $\beta\bar{z}$ .

Estimates of  $\widehat{r}$  and  $\widehat{r}^*$  may be used as a benchmark for assessing whether the real exchange rate is over or undervalued relative to current macroeconomic conditions and the long-run equilibrium rate, respectively. It should be noted, however, that such calculations are model-dependent. The size of a misalignment depends on e.g. the choice of variables that are included in  $z$ , on estimates of their effects on the real exchange rate ( $\beta$ ) and on assumptions about the equilibrium level or path  $\beta\bar{z}$ . This places considerable demands on real exchange rate models, as an apparent over or undervaluation may be due to incorrect specifications of the model. One requirement is that the effects of all variables that are important in the medium and long run are represented in the model. Another is that estimates of the equilibrium values of explanatory variables are reasonable. As these calculations are shrouded in uncertainty, it is common to assume that the deviation must be of a given magnitude before it can be deemed to be significant. Often, the difference between the actual real exchange rate and the estimated equilibrium exchange rate is required to exceed  $\pm 2$  times the estimated standard deviation before one concludes that a currency is significantly over- or undervalued.

### 3.3.1. Empirical studies based on the BEER approach

A number of variables appear to recur in empirical models of real exchange rates based on the BEER approach. In addition to the real interest rate differential, the difference between domestic and foreign productivity growth is often included, as are variables representing the terms of trade with other countries, differences between domestic and foreign public debt, and domestic net foreign assets. The relevance of several of these variables is illustrated in Subsections 3.1 and 3.2, whereas some of the variables, particularly net foreign assets, can be said to be motivated by

the FEER-approach, see Section 4. Which variables are of greatest importance naturally varies from study to study, depending on the country and the period on which estimations are based, see MacDonald and Stein (1999) for an overview.

Empirical studies of the Norwegian real exchange rate indicate that the interest rate differential with other countries, the real oil price and differences in domestic and foreign productivity growth can explain some of the fluctuations in the Norwegian real exchange rate, see for example Haldane (1997), Chaudhury and Daniel (1998), Chortareas and Driver (2001) and Bjørnland and Hungnes (2002). Akram (2003) develops an empirical model of the Norwegian real exchange rate by including all these explanatory variables in addition to the share of aggregate investment in GDP. Differences in productivity growth and the interest rate differential, however, seem to be more robust explanatory factors than the real oil price and aggregate investment.

#### 4. Fundamental equilibrium exchange rate - FEER

FEER is the real exchange rate that is explicitly consistent with internal and external balance. Internal balance means that actual production equals the potential production level, so that price inflation is stable. External balance is characterised by a sustainable level of the current account balance vis-à-vis other countries.

Calculations of FEER require an empirical model of macroeconomic variables that are influenced by the real exchange rate. Some studies use a general macroeconomic model for one or more countries, whereas others use a partial model for the balance of payments. The partial approach seems to be more popular, partly because the mechanisms that determine FEER are more transparent, but also because model development and maintenance require fewer resources.

Below, we will present a stylised version of a partial model for the balance of payments that can be used to derive FEER, see e.g. Ahlers and Hinkle (1999) for an elaboration. This model can also be used as a reference for discussing issues that may arise when calculating FEER.

Let us assume that the import volume ( $B$ ) measured in domestic product units is determined by the domestic income level ( $Y$ ) and the real exchange rate ( $R$ ). A rise in the income level has a positive effect on imports, whereas a stronger real exchange rate has a negative effect on imports. This implies the following import function:

$$B = B \left( \underset{(+)}{Y}, \underset{(-)}{R} \right). \quad (14)$$

Similarly, the domestic export volume ( $A$ ) measured in domestic product units is assumed to

depend positively on foreign income levels  $Y^f$  and the real exchange rate. This leads to the following export function:

$$A = A\left(\underset{(+)}{Y^f}, \underset{(+)}{R}\right). \quad (15)$$

The trade deficit ( $TD$ ) can then be expressed as a function of domestic and foreign income and the real exchange rate. By including import and export in the definitional equation for the trade deficit, we arrive at:

$$\begin{aligned} TD &= B(Y, R) - A(Y^f, R) \\ &= TD\left(\underset{(+)}{Y}, \underset{(-)}{Y^f}, \underset{(-)}{R}\right). \end{aligned} \quad (16)$$

The import and export functions imply that the trade deficit falls with the income level abroad and the real exchange rate, but increases with the income level at home.

The expression for the trade deficit (16) can also be used to derive the real exchange rate for given values of the trade deficit and for domestic and foreign income levels. This possibility can be expressed more explicitly by inverting equation (16) and solving it with respect to  $R$ :

$$R = R\left(\underset{(+)}{Y}, \underset{(-)}{Y^f}, \underset{(-)}{TD}\right). \quad (17)$$

This relationship indicates that the real exchange rate must depreciate when there is an isolated increase in domestic income, in order to offset the increase in the trade deficit as a result of higher import, see equations (16) and (14). Similarly, the real exchange rate must appreciate when there is an isolated increase in foreign income levels in order to avoid a fall in the trade deficit as a result of higher export, see equations (16) and (15). The net effect on the real exchange rate of income increases abroad and at home will depend on movements in income-determined import demand in relation to movements in income-determined export demand. The expression for the real exchange rate also implies that a partial increase in the trade deficit requires a stronger real exchange rate.

FEER can be defined as the level of the real exchange rate when the trade deficit and domestic and foreign income levels are at their equilibrium levels:

$$R^* = R\left(\underset{(+)}{\bar{Y}}, \underset{(-)}{\bar{Y}^f}, \underset{(-)}{\bar{TD}}\right). \quad (18)$$

Equilibrium levels for the domestic and foreign income levels ( $\bar{Y}$  and  $\bar{Y}^f$ ) can be set equal to the potential GDP levels at home and abroad. These can be assumed to rise in line with exogenously given growth rates in each period. The equilibrium level for the trade deficit ( $\bar{TD}$ )

can be determined in light of the value of net foreign assets. If the country has positive net foreign assets,  $\overline{TD}$  can be set equal to the return on these assets from abroad. In the absence of such income,  $\overline{TD}$  must be zero in the long run and import costs must be covered by export earnings. In such cases, the FEER level in the short and medium term can be determined by setting  $\overline{TD}$  equal to the level that can be funded by borrowing abroad.

The larger the sustainable trade deficit is, the stronger the real exchange rate will be. This means that all factors that increase income from abroad will entail a stronger equilibrium exchange rate, all else being equal. One factor, for example, could be a higher saving rate, which may result in positive net financial investment abroad. An improvement in the terms of trade with other countries and other conditions that could increase the trade surplus would also be positive for net financial investment abroad and provide the basis for a stronger equilibrium real exchange rate. The discovery of natural resources that can be exported would also serve to strengthen the equilibrium real exchange rate. Valuation changes for such resources would also lead to changes in the equilibrium real exchange rate. This is consistent with empirical studies that reject the hypothesis of a constant equilibrium real exchange rate, based on data from countries that have discovered natural resources, see e.g. Patel (1990).

There is a substantial difference between the FEER and BEER approaches, even though they do share some common features. Both BEER and FEER are variable equilibrium exchange rates in general. In the case of a linear model, the expression for FEER may even have the same form as in a BEER model. For example, equation (18) can be written as follows in a linear form:

$$R^* = \alpha + \beta_1 \bar{Y} - \beta_2 \bar{Y}^f - \beta_3 \overline{TD}. \quad (19)$$

The partial effects on the equilibrium exchange rate will, however, depend on the mechanisms in the macroeconomic model used to derive FEER. Thus a variable can have a different effect in a BEER and a FEER model. For example, higher domestic economic growth may strengthen the equilibrium exchange rate in a BEER model. On the other hand, in a FEER model, higher economic growth may lead to a weaker equilibrium exchange rate, if the growth contributes to a foreign trade deficit. This can be explained by the fact that the parameters in a BEER model represent the partial effects of different variables *on* the real exchange rate, while the parameters in the expression for FEER functions represent the effect *of* the real exchange rate on different variables, e.g. import and export. It could also be said that the expression of FEER in itself does not represent a theory for determining the real exchange rate. When calculating FEER, however, it is implicitly assumed that the equilibrium real exchange rate derived from a real exchange rate model, such as the BEER, will coincide with the equilibrium real exchange rate arising from a macroeconomic model in the state of internal and external balance, i.e. with the FEER.

FEER can also be constant, as in the theory of purchasing power parity, when external balance is characterised by trade balance ( $\overline{TD} = 0$ ) and the ratio between  $\beta_1 \overline{Y}$  and  $\beta_2 \overline{Y}^f$  remains unchanged, as could be the case if growth rates at home and abroad are equal and  $\beta_1 = \beta_2$ . For an illustration of this case, see for example Akram (2003).

Another special instance where FEER can be constant is if changes in the real exchange rate have an extremely strong effect on import or export, and thus on the trade deficit. In such cases, FEER will be independent of variables that define internal and external balance. The derived parameters in equation (19) will then be equal to zero. The intuition is that the trade deficit can be steered to the targeted value with slight changes in the real exchange rate if this has an extremely strong effect on import or export. This would, for example, be the case with cost-free international trade.

#### 4.1. FEER calculations in practice

The FEER approach is a widely used method for calculating equilibrium real exchange rates. Well known contributions to the literature include Williamson (1994), Barrel and Wren-Lewis (1989), Artis and Taylor (1995), Faruqee, Isard and Masson (1999) and the article collections Hinkle and Montiel (1999) and MacDonald and Stein (1999). With the exception of Kristofersen (2002) and Akram (2003), however, there do not seem to be published applications of the FEER approach on Norwegian data.

Below we will present some key issues that are recurring themes in economic literature on FEER, which will make it apparent that there is considerable uncertainty attached to FEER calculations. This indicates that one should be cautious when assessing whether a currency is over or undervalued in relation to a calculated path/level for FEER. This has also motivated calculations of the confidence interval in connection with estimations of FEER and sensitivity analyses. Such analyses look at the degree to which estimates are influenced by alternative assumptions regarding equilibrium values of variables that define internal and external balance.

One of the recurring themes in many FEER calculations is the magnitude of income and price elasticity for import and export. Most empirical studies estimate income elasticity at greater than 1 and price elasticity at less than 1 in numerical value, see for example Marquez (2002, p. 25–26) and Hinkle and Montiel (1999, p. 355 and p. 475). In some studies this is seen as unreasonable a priori, as the budget share of import goods can in theory increase over time and exceed 1 in calculations for long time horizons. This can result in unreasonable estimates for FEER particularly if import does not respond much to changes in the real exchange rate (low price elasticity). In such cases, changes in the FEER level may be deemed to be unreasonably large in the event of a partial change in the income level. In some models, income elasticity is

therefore given a value equal to 1. The disadvantage of this can be that the effect of changes in the income level on FEER in the short and medium term may be underestimated.

Another recurring theme in FEER calculations based on large macroeconomic models is the properties of the macroeconomic model. One of the requirements is that the equilibrium properties of models can be interpreted in light of economic theory, otherwise it may be difficult to have confidence in the estimate for FEER that is obtained by solving the model. It is also advantageous if the mechanisms in the model are uncomplicated and transparent so that the partial effect of a variable on FEER can be explained intuitively. The use of a partial approach is often motivated by the desire to achieve a better overview of the mechanisms behind FEER calculations. One disadvantage in doing this can be that the specifications of internal and external balance become less satisfactory, as explained below.

A third theme that is given a great deal of thought in literature is the specification of internal balance. The equilibrium level for GDP is often assumed to be equal to the potential GDP level. There are, however, different ways in which to derive this level. In calculations based on larger macroeconomic models, GDP is modelled as a function of factors of production such as labour and capital. Potential GDP is calculated as that part of GDP that can be explained by changes in factors of production. By using a partial approach, on the other hand, potential GDP is calculated rather mechanically with the help of a filter, for example a Hodrick-Prescott filter, which extracts the trend component from the time series for GDP. It is also fairly common to characterise potential GDP with the help of time trends. Alternatively, this approach is made operational by extrapolating the GDP level from a typical year with the help of average growth rates over a longer time horizon.

One argument that can be used against most studies based on a partial approach is that by estimating the equilibrium level for GDP outside the FEER model, they neglect possible feedback effects from the real exchange rate level to the GDP. Studies carried out thus far, however, suggest that estimates of FEER do not differ greatly when such interactions are taken into account, see Driver, Power and Ramsay (2001).

Another argument is that studies that estimate the equilibrium level for GDP with the help of time trends and average growth rate observations neglect the fact that (potential) GDP does not grow at a fixed rate in each period. Empirically, GDP growth is found to be variable and to a great extent unpredictable. This unpredictability may be ascribed to technological innovations and other unpredictable events. This would also indicate that caution should be used when interpreting FEER calculations.

A fourth point on which studies differ considerably is the specification of external balance, i.e.

the sustainable level of the current account balance with other countries. The simplest approach is to assume that the country does not have sustainable income from abroad, and the trade has to be balanced. Accordingly, current account balance is set at zero. However, this requirement can be too strict in the short and medium term as a deficit or surplus on the current account balance can often last for several years. This may also result in unreasonable estimates for FEER if the time perspective of interest is the short and medium term. In some studies, external balance is therefore defined as the average current account balance over a business cycle, or that the deficit or surplus on the current account balance constitutes a constant share of GDP. For countries with substantial net foreign assets or debts, the current account balance can be set equal to an estimate of income from net assets abroad. For example, in Akram (2003) income from net assets abroad is assumed to be equal to the permanent income from Norway's petroleum wealth.

Separate calculations of a sustainable level for the current account balance can also be criticised for not taking sufficient account of the saving and investment behaviour of the private and public sectors. In some more recent studies, the current account balance is viewed as an endogenous variable and modelled as a function of variables that influence public and private saving and investment behaviour, see Faruqee et al. (1999). Such variables include demographic trends and public sector budget deficits.

## 5. Summary

Equilibrium real exchange rates are used as benchmarks for assessing whether a real exchange rate is over or undervalued. In this article we have presented three of many approaches to calculating equilibrium real exchange rates: the PPP, BEER and FEER approaches. This has allowed us to point to several factors that are important to movements in the real exchange rate over different time horizons. We have also given a brief overview of empirical results and issues presented in economic literature regarding these approaches.

The theory of purchasing power parity (PPP) entails a constant equilibrium real exchange rate. This means that domestic price inflation in the long run must be equal to price inflation abroad when adjusted for nominal exchange rate movements.

An increasing number of studies show that real exchange rates have a tendency to converge towards a stable level in the long term. It has also been observed that real exchange rates in small open economies with a fixed exchange rate regime and flexible wage and price formation, which generally trade with countries with roughly the same growth rates and economic structure, often converge more rapidly towards an equilibrium level than real exchange rates in other countries. Countries that are exposed to real shocks and that do not have the above-mentioned features,

have a tendency to experience long swings and/or trend movements in the real exchange rate.

Long-term trend movements in the real exchange rate may be ascribed to persistent differences in the relative productivity growth between the traded and non-traded sectors at home and abroad. Factors that may be of importance to real exchange rates in the short and medium term include unsynchronised business cycles between the home country and abroad as a result of differences in monetary and fiscal policy, or the asymmetrical effects of shocks at home and abroad. One example of a shock that has a different effect depending on whether the country exports or imports oil, is a change in the oil price. Changes in the nominal exchange rate can also have a direct impact on the real exchange rate when changes in the exchange rate are slow to translate into price changes. Changes in the nominal exchange rate may, for example, be due to a change in the interest rate differential between the home country and other countries.

The BEER approach takes account of the fact that the above mentioned macroeconomic variables can give rise to long swings and trend movements in the real exchange rate. The equilibrium real exchange rate in the medium to long run is defined as the exchange rate level that is achieved as a result of actual values and equilibrium values for macroeconomic variables. The relationship between macroeconomic variables and the real exchange rate is estimated within an empirical model of the real exchange rate.

The FEER approach is also based on the assumption that equilibrium real exchange rates can vary over time. It is calculated on the basis of an empirical model of macroeconomic variables that are influenced by the real exchange rate. Estimates of FEER are often based on assumptions regarding potential domestic and foreign GDP and a sustainable level for the trade deficit with other countries. Conditions that increase foreign investment income result in a stronger equilibrium real exchange rate, as the sustainable level of the trade deficit rises. On the other hand, an increase in domestic GDP that pushes up the trade deficit will weaken the equilibrium exchange rate so that the trade deficit does not exceed foreign investment income.

Calculations of the equilibrium real exchange rate based on all three approaches require that the relevant empirical relationships provide satisfactory characteristics of underlying economic conditions. Estimates of BEER and FEER are also conditional on assumptions regarding the equilibrium values of a number of macroeconomic variables. There is therefore considerable uncertainty attached to estimates of equilibrium real exchange rates. An estimated misalignment should therefore be of a given magnitude before the real exchange rate can be said to be too strong or too weak.

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