Private Cards and the Bypass of Payment Systems by Merchants

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Abstract

This paper studies the incentives of a merchant to bypass a payment platform by issuing private cards. In our model, a payment platform organizes the interactions between a monopolistic issuer and a monopolistic acquirer by choosing a level of interchange fee. We study how the level of interchange fee impacts a merchant's decision to issue private cards. In a basic setting in which there are no strategic interactions between merchants, we show that there can be inefficient bypass. Then, we allow for strategic interactions between merchants by giving an Hotelling structure to the product market. We show that if a merchant decides to issue private cards, he sets a very agressive price for its payment service, to compete with the issuer, and to steal consumers from the other merchant. When it is possible to deter entry, we prove that the payment platform can prevent the merchant from issuing private cards by lowering the level of the interchange fee.

JEL Codes: G21, L31, L42.

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1 Introduction

In the United-States, in 2006, payment card transactions cost merchants nearly $57 billion.[1] The costs of card payments is a major source of conflict between banks and merchants. Merchants have to pay a fee (the "merchant fee") to their bank (the "Acquirer") each time a consumer pays by card, which they claim to be excessive.[2] In 2005, in the United-States, the usual amount of the merchant fee ranged from 1% to 2.7% of the transaction. Merchants argue that they cannot pass through to consumers the cost of a payment card transaction, since surcharges are forbidden by most payment card associations (like Visa). Also, they contend that it has become impossible to refuse a payment instrument which is now widely used by consumers.

This explains why merchants have thought about strategies to reduce the costs of payment card transactions. One of these strategies, which has been implemented by large retailers such as Wal-Mart or JC Penney and Macy's, has been to start issuing "private cards". Unlike payment cards issued by banks, which are members of payment card associations, private cards can only be used at the retailer's shop. The private card enables the merchant to save the cost of the merchant fee, if it is issued without the support of a financial institution. The detention and usage of private cards have become widespread over the last ten years. According to the International Card Manufacturer's Association (IMCA), 5.6 billion private cards have been sold or delivered worldwide in 2004. Private cards account for 42.9% of the cards issued.

The purpose of this paper is to analyse merchants' incentives to issue their private cards, and to characterise the possible reactions of the payment card association.

Payment card networks are often managed by a payment association, such as Visa, or MasterCard, which organises the interactions between the bank of the cardholder, the "Issuer", and the bank of the merchant, the "Acquirer". Such payment card associations entail several benefits for the bank members. For instance, an Issuer is ensured that his payment card will be accepted by all the merchants that are affiliated with the association, while an Acquirer knows that the Issuer will respect the rules for security and processing that are designed by the association. Hence, banks benefit from network effects of membership, and from a reduction in the asymmetry of information when they proceed to payment transactions. Payment card associations also enable banks to allocate optimally the total cost of a payment card transaction between each other, by choosing an "interchange" fee, that is paid by the Acquirer to the Issuer,

[1] Source: Nilson Report, Issue 877 (2007).

[2] See for instance www.nationalgrocers.org, "Of further concern is the grocery industry trend toward both higher interchange rates and higher volumes of electronic transactions, with a number of companies reporting more than 50 percent of their purchases being made with credit and debit cards". See also the Visa Wal-Mart case (2003).

each time a consumer pays by card. The effect of the interchange fee is to reduce the marginal cost of the Issuer and to increase the marginal cost of the Acquirer. This is a way for the payment association of subsidizing the consumers' side, by allowing the Issuers to choose a lower price for the payment card, to the detriment of the merchants' side. Hence, though interchange fees stimulate the demand for card payments, their effect on merchants' side may provide large retailers with incentives to bypass the payment card association.

This paper aims also at analysing the impact of private cards on the level of the interchange fee that is chosen by the payment association. We try to determine if the payment platform can use the interchange fee to deter merchants from entering the market for payment card transactions.

The possibility to bypass the payment association by issuing private cards has never been studied in the literature on payment card systems.[3] Among others, Rochet and Tirole (2002) and Wright (2004) show that the optimal level of interchange fee depends on the nature of competition between merchants. An interesting insight, provided by Rochet and Tirole (2002), is that merchants are ready to accept higher merchant fees to avoid losing market share if they refuse cards. But no paper takes into account the fact that merchants can compete with the payment association by providing their own payment services.

We model the payment card association as a two-sided platform which organises the interactions between a monopolistic Issuer and a monopolistic Acquirer. To analyse a merchant's incentives to issue private cards, we build two different settings. We start by a basic model in which merchants are local monopolists. This framework enables us to study the conditions under which a merchant issues private cards, if there are no strategic interactions with other merchants, and if the payment platform does not react to this decision. We show that there can be some inefficient bypass because of three different effects. By internalising the acquisition of payments, the merchant can retain its benefit of being paid by card, instead of transferring this surplus entirely to the monopolistic Acquirer. This effect is reinforced if the merchant is more efficient than the payment platform, since he sets a lower fee for card payments, which increases the volume of transactions that are paid by card. Furthermore, by issuing private cards, the merchant can extract some surplus from the card users on the issuing side.

Then we give a different structure to the product market to allow for strategic interactions between merchants. There are two merchants that are differentiated à la Hotelling, and positioned exogenously at the two extremes of a linear city of length one. One merchant provides a good of higher quality than the other. Merchants are homogenous as to their card acceptance

[3]For a review of the literature, see Rochet (2003).

benefit and accept payment cards if the merchant fee is not too high. The merchant which produces the good of higher quality can choose to issue its private card. The private card cannot be used by consumers to pay at the other merchant's shop. To issue the private card, the merchant has to pay a fixed cost. We assume that consumers differ across their card usage benefit, which is the same for a given consumer if he pays by card or if he uses the private card.

We start by showing that, in this setting, if the merchant decides to issue its private card, he chooses a transaction fee equal to zero, such that, if consumers come to his shop, they always prefer the private card to the bank-issued payment card. The intuition is that the merchant is active on the market for card transactions and on the product market, and that his incentives on each of these two markets are to set a very low price for the private card. On the market for payment transactions, we show that the merchant has an incentive to undercut the price that is set by the issuer for the payment card. Also, the merchant chooses a low price for the private card because he obtains a higher benefit per transaction if his consumers pay with the private card than if they pay cash. On the product market, the merchant has an incentive to set a low price for the private card, because he obtains a higher market share, by stealing consumers from his competitor.

We prove that, if one merchant issues private cards, the other merchant becomes less resistant to card acceptance than in the benchmark case, in which no merchant issues private cards. The threat of losing consumers on the product market raises the maximum merchant fee that he is willing to pay to accept payment cards. Since the monopolistic Acquirer chooses the maximum merchant fee compatible with merchants' acceptance of payment cards, the effect of the private card is to increase the merchant fee. On the other hand, the Issuer charges a lower card fee, to compete with the very aggressive price that is set by the merchant for the private card. Therefore, the competition with the private card changes the structure of prices.

Then, we derive the impact of the interchange fee on the merchant's incentives to enter the market for payment card transactions by issuing private cards. We show that there are two effects. A higher interchange fee reduces the card fee, which toughens the competition with the Issuer. This effect lowers the merchant's incentives to issue private cards. On the other hand, a higher interchange fee increases the merchant fee, and hence the costs of the rival merchant, which raises the benefits of issuing private cards. In our setting, the first effect is dominant if the card fee is sufficiently low, and the incentives to issue private cards decrease with the interchange fee. Otherwise, if the card fee is high, the second effect becomes dominant. If the card fee is low, the payment platform cannot deter entry, since it already sets the maximum interchange fee compatible with positive profits for the acquirer when the merchant does not issue private cards. Hence, at the equilibrium, there is either blockaded entry or entry accommodation. The payment platform can only deter entry if the card fee is sufficiently high by reducing the level of the interchange fee. Therefore, if entry is not blockaded, we show that the threat of the private card could lead the payment platform to reduce its interchange fee.

We also consider other forms of market structures on the banking retail markets. With perfect competition on the acquisition side and a monopoly on the issuing side, we show that the payment system may set a low interchange fee to deter entry. If both sides of the market are perfectly competitive, the payment platform minimises the probability of entry by choosing Baxter's interchange fee. In an extension of our model, we also study the reaction of the payment platform if the second merchant can follow its competitor and issue private cards.

The rest of this paper is organised as follows. In section two, we start by presenting a basic model of bypass by a local monopolist. In section three, we study the bypass of the payment association by a strategic merchant. In section four, we extend the model presented in section three by assuming other market structures on banking retail markets, and by studying the possible reaction of the other merchant. Finally, we conclude.

2 A model of bypass by a local monopolist

In this section, we introduce a basic model to understand the merchants' incentives to bypass the payment association. We determine the conditions under which a merchant issues private cards, if it has no strategic interaction with other merchants, and if the payment platform does not react to its bypass decision.

2.1 The model

We build a model in which merchants are local monopolists. In our setting, if one merchant chooses to bypass the payment association, this decision does not impact the other merchants' choice to accept cards, and the prices chosen by the banks.

Merchants: A continuum of local monopolists of mass one sells an identical good to the consumers at a price p. The marginal cost of selling the good is the same for all merchants and equal to c. Each merchant may decide to accept payment cards that are issued by the members of the payment association. We assume that, if a merchant is indifferent between accepting and refusing payment cards, he accepts them. We suppose that merchants are homogeneous as regards to their card acceptance benefit, which we denote by b_S, with $b_S \geq 0$.

We consider that one merchant, which we denote by M_0, can decide to issue a private card at a fixed cost F. If a consumer decides to use the private card when he goes to this shop, he has to pay a transaction fee, f^{PC}. The merchant incurs a cost $c_M \in [0, 1]$ for each transaction paid by the private card. This cost corresponds to the costs of issuing and acquiring a transaction paid with the private card.

Consumers: A consumer has to choose whether or not to buy the good, and which payment instrument to use. In his wallet, each consumer always holds cash and a payment card issued by his bank.[4] He can always use cash at no cost[5] to pay for his expenses. If he decides to use a payment card, he has to pay a transaction fee, f, to the issuer of the card (a bank or the merchant M_0), provided that the card is accepted.

Each local monopoly faces a mass one of consumers. A consumer is characterised by his benefit, b_B, of using a card rather than cash. We assume that the benefit b_B is the same whether the card is issued by the bank or by the merchant, and that b_B is uniformly distributed over $[0, 1]$. One interpretation is that consumers may attach different values to the convenience of using a card rather than cash.

A consumer, whose card usage benefit is b_B, enjoys a net utility of

$$U = v - p + b_B - f,$$

if he uses his card and pays the transaction fee f, and a net utility of

$$U = v - p,$$

if he pays cash, where v represents a fixed utility obtained from consuming the good.

Banks: The Issuer (I) and the Acquirer (A) are monopolists. For each transaction, the Issuer charges card users with a fee, f^C, and the Acquirer charges merchants with a fee, $m \geq 0$. The Acquirer pays to the Issuer a per-transaction interchange fee, denoted by a^P, with $a^P \geq 0$. Banks' have constant marginal costs c_i per transaction, for $i = I, A$, and profits are denoted by Π_I and Π_A. If no merchant accepts cards, banks make no profits, i.e., $\Pi_i = 0$ for $i = I, A$.

Payment system: The payment system (S) chooses the interchange fee, a^P, which maximises the sum of banks' profits, $\Pi_S = \Pi_I + \Pi_A$. We assume that the Non-Discrimination Rule (NDR)

[4] In the model, we consider cardholding decisions as exogenous, and focus on the choice of the payment instrument at the point of sales.

[5] The costs and the benefits of using cash are normalised to zero.

holds, which means that merchants are forbidden to charge different prices according to the payment instrument used for the transaction.

Other assumptions: We assume that $v - c \geq 2$. This assumption ensures that each monopolistic merchant chooses a price such that the market is covered. We also assume that $0 \leq b_S \leq c_I + c_A \leq 1$. This assumption ensures that some consumers but not all use their payment cards.

Timing The timing of the game is as follows:

1. The payment platform chooses the interchange fee, a^P, which maximises the joint profits of the banks.
2. The monopolistic merchant M_0 decides whether or not to issue private cards.
3. Banks choose simultaneously and non-cooperatively their transaction fees, f^C and m, and the merchant M_0 decides simultaneously on the private card transaction fee, f^{PC}.
4. Each monopolistic merchant chooses its price and whether or not to accept payment cards.
5. Consumers decide whether or not to buy the good and which payment instrument to use (cash, payment card or private card).

With this timing, we assume that merchant M_0 decides whether or not to issue a private card, once the interchange fee has been set. Indeed, in practice, payment platforms do not adjust the level of the interchange fee very frequently. Besides, we choose to focus on the effect of the interchange fee on a merchant's incentives to bypass the payment system.

We look for the subgame perfect equilibrium, and solve the game by backward induction.

2.2 Stage 4 and 5: card acceptance decisions and prices

We study the prices that are chosen by a merchant whether he issues private cards or not.

If the merchant does not issue private cards: The following Lemma gives the price chosen by a local monopolist, and the card acceptance condition.

Lemma 1 *If $m \leq b_S$, each monopolistic merchant accepts payment cards, and extracts all the surplus of the cash users by choosing $p^* = v$.*
Each merchant makes profit $\pi_m^C = v - c + (b_S - m)(1 - f^C)$. Otherwise, if $m > b_S$, the merchants refuse payment cards.

Proof. See Appendix A-1. ■

If the merchant issues private cards: We now determine the price that is chosen by merchant M_0 if he issues private cards. We focus on the case in which $f^{PC} \leq f^C$. Otherwise, the private card would never used by the consumers. Since the merchant has to pay the fixed cost of issuing private cards, the strategy of issuing private cards would be clearly dominated.

Lemma 2 *If merchant M_0 issues private cards, he extracts all the surplus from cash users by choosing $p^* = v$, and he makes profit $\pi_{M_0}^{PC} = v - c + (b_S - c_M + f^{PC})(1 - f^{PC}) - F$.*

Proof. See Appendix A-2. ■

In both lemmas, as the consumers' surplus of buying the good is sufficiently high by assumption, a local monopoly maximises its profit by choosing a price such that the market is covered. In absence of strategic interactions with other merchants, a merchant accepts payment cards if its benefit per transaction is higher than the fee he pays to the Acquirer.

2.3 Stage 3: transaction fees

We now determine the transaction fees that are chosen by the Issuer, the Acquirer, and by the merchant M_0 if the latter has issued private cards. Since there is a continuum of local monopolists in our model, the decision of the merchant M_0 to issue private cards does not influence the prices that are chosen by the banks. Let D^C denotes the demand of card users. The Issuer and the Acquirer choose the transaction fees, f^C and m respectively, that maximise their profits,

$$\Pi_I = D^C(f^C + a^P - c_I),$$

and

$$\Pi_A = D^C(m - a^P - c_A),$$

subject to the card acceptance condition $m - b_S \leq 0$,[6] and the conditions $\Pi_I \geq 0$ and $\Pi_A \geq 0$. According to Lemma 1, a monopolistic merchant that accepts cards chooses a price such that the market is covered. Hence, if merchants accept cards, $D^C = 1 - f^C$.

Since Π_A increases with m, the Acquirer maximises its profit by choosing the maximal merchant fee compatible with merchants' acceptance of payment cards. Therefore, we have $m^* = b_S$. For this value of the merchant fee, each merchant is indifferent between accepting and refusing payment cards, and from Lemma 1, the profit of a merchant is $\pi_m^C = v - c$. The transaction fee that maximises the Issuer's profit is $(f^C)^* = \left(1 - a^P + c_I\right)/2$.

[6] If merchants do not accept payment cards, we have $\Pi_I = \Pi_A = 0$.

With the following Lemma, we give the optimal private card fee chosen by merchant M_0 if he issues private cards.

Lemma 3 *If merchant M_0 issues private cards, he sets $\left(f^{PC}\right)^* = \min\left\{(1 - b_S + c_M)/2, (f^C)^*\right\}$. If $c_M - b_S \leq c_I - a^P$, he obtains a profit of $\pi_{M_0}^{PC} = v - c + \frac{1}{4}(1 + b_S - c_M)^2 - F$; otherwise, he obtains a profit of $\pi_{M_0}^{PC} = v - c + (b_S - c_M + (f^C)^*)\left(1 - (f^C)^*\right) - F$.*

Proof. See Appendix A-3. ■

Notice that $\left(f^{PC}\right)^* < (f^C)^*$ if and only if $c_M - b_S < c_I - a^P$. This condition means that the private card fee is strictly lower than the payment card fee if, for each transaction, the net cost of issuing private cards is strictly lower than the net cost born by the issuer.

2.4 Stage 2: the bypass condition

Merchant M_0 has an incentive to issue private cards if and only if $\pi_{M_0}^{PC} \geq \pi_m^C$. This bypass condition can be rewritten as

$$\frac{(1 + b_S - c_M)^2}{4} \geq F,$$

if $c_M - b_S < c_I - a^P$, and

$$(b_S - c_M + \frac{1 - a^P + c_I}{2})\left(\frac{1 + a^P - c_I}{2}\right) \geq F,$$

otherwise.

2.5 Stage 1: the optimal interchange fee

The payment platform chooses the interchange fee a^P that maximises banks' joint profits,

$$\begin{aligned} \Pi_I + \Pi_A &= \left((f^C)^* + m^* - c_I - c_A\right)\left(1 - (f^C)^*\right) \\ &= \left(b_S - a^P - c_A + \frac{1 + a^P - c_I}{2}\right)\left(\frac{1 + a^P - c_I}{2}\right), \end{aligned} \quad (1)$$

subject to $\Pi_I \geq 0$ and $\Pi_A \geq 0$. Maximising (1) with respect to a^P gives an unconstrained optimum $a^P = b_S - c_A$.[7] Since for this value of the interchange fee, we have $\Pi_I \geq 0$ and $\Pi_A = 0$, then the optimal interchange fee is

$$\left(a^P\right)^* = b_S - c_A.$$

[7] Since $\Pi_I + \Pi_A$ is concave in a^P, the second order condition is verified.

The optimal interchange fee chosen by the payment platform is equal to Baxter's interchange fee. The following Proposition gives the conditions under which merchant M_0 chooses to bypass the payment association.

Proposition 1 *If $c_M \leq c_I + c_A$, merchant M_0 chooses to bypass the payment association if and only if $(1 + b_S - c_M)^2 / 4 \geq F$. Otherwise, if $c_M > c_I + c_A$, he chooses to bypass if and only if $(1 + b_S - c_M - c_M + c_I + c_A)(1 + b_S - c_I - c_A) / 4 \geq F$.*

Proof. This result is obtained by replacing for $\left(a^P\right)^*$ in the bypass condition given at stage 2. ■

As a corollary, consider the limit case $F = 0$. If merchant M_0 is at least as efficient as the payment system (that is, $c_M \leq c_I + c_A$), there is always bypass, since $|b_S - c_M| \leq 1$. If merchant M_0 is less efficient than the payment system (that is, $c_M > c_I + c_A$), then there can be bypass if $b_S - c_M$ is sufficiently high. In other words, there can be inefficient bypass.

The incentives to bypass are driven by three different effects. To begin with, by internalising the acquisition of payments, merchant M_0 can retain its benefit of being paid by card rather than cash, b_S, instead of transferring this surplus entirely to the monopolistic Acquirer through the merchant fee. This effect is reinforced if merchant M_0 is more efficient than the payment system, since the lower fee for card payments (as $\left(f^{PC}\right)^* < (f^C)^*$) increases the volume of card transactions. Finally, by issuing private cards, merchant M_0 can extract some surplus from the card users on the issuing side.

We checked the robustness of these bypass conditions by assuming different market structures on the banking retail markets. If there is perfect competition on the acquisition side, and a monopolistic issuer, the bypass conditions remain the same, as the card fee that is chosen by the issuer and the interchange fee that is set by the payment platform are identical. If there is perfect competition on the issuing side, and a monopolistic acquirer, the bypass condition is also the same, though the optimal interchange fee is different (See Appendix A-4).

3 A model of bypass by strategic merchants

In this section, we modify our basic model to account for the strategic interactions between merchants on the one hand, and between the bypassing merchant and the banks, on the other hand.

3.1 The model

We use the framework that we developed in the previous section, except that we now give an "Hotelling" structure to the product market. Our setting is modified as follows.[8]

Merchants Two merchants, denoted by 1 and 2, are located at the extremities of a linear city of length one. Merchant 1's shop is located at point 0 and merchant 2's shop at point 1. Each merchant $i \in \{1, 2\}$ sells the same good at price p_i, and the marginal costs are the same and equal to c. We assume that merchant 1 can issue private cards at a fixed cost F, whereas merchant 2 cannot issue private cards.[9] The merchants are homogeneous as regards to their card acceptance benefit, which is denoted by b_S. Merchant 1 obtains the same card acceptance benefit whether the transaction is paid by a bank's card or the private card.

Consumers Consumers are uniformly located along the linear city. They incur a linear transportation cost t when they travel to shop either at merchant 1's or merchant 2's shop. When it decides to shop at merchant i's, a consumer purchases zero or one unit of the good.

In his wallet, each consumer always holds cash and a payment card issued by his bank. The payment card issued by the bank can be used either to buy from merchant 1 or merchant 2, provided it is accepted at the point of sales. A consumer may also hold a private card, issued by merchant 1, which can only be used to purchase a good at merchant 1's shop.

Each consumer is characterised by his benefit, b_B, of using a card rather than cash. We assume that the benefit b_B is the same whether the card is issued by the bank or by merchant 1, and that b_B is uniformly distributed over $[0, 1]$.

A consumer located at x, whose card usage benefit is b_B, and who buys from merchant i located at x_i, enjoys a net utility of:

$$U = v + t\,|x - x_i| - p_i + b_B - f,$$

if he uses his card, and pays the transaction fee f, and a net utility of

$$U = v + t\,|x - x_i| - p_i,$$

if he pays cash, where v represents a fixed utility obtained from consuming the good. We assume

[8] In Section 5, we will discuss how the market structure on the acquisition side affects our results.

[9] For instance, we could assume that the fixed cost of introducing private cards for merchant 2 is prohibitively high. In the extension section, we will discuss the possible reaction of merchant 2.

that v is sufficiently high, such that the market is covered.[10]

Other assumptions We also make the following assumptions.

Assumption 1. $t \geq 11/3$.

This assumption ensures that Π_I is concave with respect to f^C.[11]

Assumption 2. $0 \leq c_M \leq b_S \leq c_I + c_A < 1$

The fact that $b_S \geq c_M$ implies that merchant 1 makes a net benefit for each transaction paid with the private card. We also assume that $c_M \leq c_I + c_A$, which means that merchant 1 is at least as efficient as the association of the Issuer and the Acquirer. Finally, since $b_S \leq c_I + c_A < 1$, it is socially optimal that some consumers but not all pay with their payment cards.

Timing: The timing of the game is the same as in the previous section. At stage 1, the payment platform chooses the interchange fee that maximises banks' joint profit, a^P. At stage 2, merchant 1 decides whether or not to issue a private card. At stage 3, merchant 1 decides on the level of the transaction fee for the private card, f^{PC}, while banks choose simultaneously and non-cooperatively their transaction fees, f^C and m. At stage 4, merchants choose their prices p_1 and p_2, and whether or not to accept cards. At stage 5, the consumers decide which payment instrument to use (cash, payment card or private card), and which merchant to buy from.

We look for the subgame perfect equilibrium, and solve the game by backward induction.

3.2 A benchmark: no private card

We start by analysing a benchmark, in which we assume that it is too costly for merchant 1 to issue private cards.[12] We determine the condition under which both merchants accept payment cards, and the optimal interchange fee chosen by the payment card system. This benchmark case is close to Rochet and Tirole (2002). But, in our setting, we assume that banks on each side of the payment platform are monopolies, whereas, in Rochet and Tirole, there is perfect competition in the acquisition market and imperfect competition in the issuing market.

We focus on the equilibrium in which both merchants accept cards.[13] Let $(a^P)^B$, $(f^C)^B$ and

[10] For instance, if no merchant issues private cards, and if both merchants accept cards, the market is covered if $v \geq c + 3t/2$.

[11] See Appendix E1.

[12] This benchmark also corresponds to the subgame in which merchant 1 does not issue a private card.

[13] There might also be "high resistance" equilibria as in Rochet and Tirole (2002), in which no merchant accepts cards.

$(m)^B$ denote the equilibrium interchange fee, transaction fee and merchant fee, respectively. We denote by $\pi_i^B\left((f^C)^B, m^B\right)$ the equilibrium profit of merchant i.

Proposition 2 *If merchant* 1 *cannot issue private cards, both merchants accept payment cards if* $m \leq b_S + \left(1 - f^C\right)/2$. *The optimal interchange fee is* $(a^P)^B = (4(b_S - c_A) + 1 - c_I)/3$, *and the optimal transaction fees are* $(f^C)^B = (1 + 2(c_I + c_A - b_S))/3$, *for the Issuer, and* $(m)^B = (4b_S + 1 - (c_I + c_A))/3$, *for the Acquirer.*

Proof. See Appendix B. ■

As in Rochet and Tirole (2002, 2006), we find that strategic merchants are ready to pay for a higher merchant fee, to attract consumers to their stores. They internalise a fraction of the cardholders' benefit of using their cards. If merchants were not strategic, the maximum merchant fee compatible with card acceptance would be b_S, and the optimal interchange fee would be $a^P = b_S - c_A$, as shown in Section 2.

3.3 The equilibrium with private cards

In this Section, we assume that merchant 1 can issue private cards[14] and we determine the equilibrium of the game, starting from the last stage.

3.3.1 Stage 5 and 4: card acceptance decisions and prices

If merchant 1 does not issue private cards, the analysis is similar to the benchmark case. From now on, we assume that merchant 1 issues private cards, and we determine the demands for merchants 1 and 2. We denote $\Delta f = f^C - f^{PC}$ and we assume that consumers who shop at merchant 1's and are indifferent between the payment card and the private card use the private card.

At stage 5, consumers take into account the price of the good in their decision to shop either at merchant 1's or merchant 2's, as well as the availability of each payment instrument. As each merchant can either accept or refuse cards, we have four possible cases, depending on the merchants' acceptance decisions. We denote by $\pi_i^{\delta_1,\delta_2}$ the profit of merchant i, where δ_i denotes the card acceptance decision of merchant i. We set $\delta_i = NC$ if merchant i refuses payment cards and $\delta_i = C$ if he accepts cards. At stage 4, merchant i chooses the price p_i that maximises his profit,

$$\pi_i^{\delta_1,\delta_2} = \left(D_i^{PC} + D_i^C + D_i^{Cash}\right)(p_i - c) + (f^{PC} + b_S - c_M)D_i^{PC} + (b_S - m)D_i^C,$$

[14] Or, equivalently, that the fixed cost of a private card system is not prohibitively high.

where D_i^{PC}, D_i^C, and D_i^{Cash} denote the demand of consumers who shop at merchant i's and pay with the private card, the payment card and cash, respectively. Notice that $D_2^{PC} = 0$ as, by assumption, merchant 2 does not issue private cards.

We determine below the equilibrium of stages 4 and 5 in each of the four possible cases, for $(\delta_1, \delta_2) \in \{NC, C\}^2$.

Both merchants accept payment cards. We start by analyzing consumers' decisions at stage 5. If $f^{PC} > f^C$, consumers who shop at merchant 1's always use their payment card instead of the private card, as their net utility from using the payment card, $b_B - f^C$, is strictly greater than their net utility of using the private card, $b_B - f^{PC}$. Therefore, the demands for merchant 1 and merchant 2 are identical to their demands in the benchmark case, if they both accept payment cards, and can be found in Appendix B.

If $f^{PC} \leq f^C$, consumers who shop at merchant 1's prefer the private card to the payment card. Consumers such that $b_B < f^{PC} \leq f^C$ always pay cash, as their net utility from a payment by card is negative. A standard Hotelling analysis shows that each merchant i obtains a share w_i of these consumers, where

$$w_i = \frac{1}{2} + \frac{1}{2t}(p_j - p_i),$$

for $(i; j) \in \{1; 2\}^2$ and $i \neq j$. By integrating for $b_B \in \left[0, f^{PC}\right]$, we obtain that the demand from cash users is equal to $f^{PC} w_i$ for merchant i.

Consumers such that $b_B \in \left[f^{PC}, f^C\right)$ trade off between purchasing from merchant 1 and paying with the private card and purchasing from merchant 2 and paying cash, as their net utility from a payment by card $(b_B - f^C)$ is negative, whereas their net utility from a payment with the private card $(b_B - f^{PC})$ is positive. The marginal consumer is given by

$$v - p_1 - tx + b_B - f^{PC} = v - p_2 - t\left(1 - x\right),$$

that is,

$$x\left(b_B\right) = \frac{1}{2} + \frac{p_2 - p_1 + b_B - f^{PC}}{2t}.$$

Aggregating for $b_B \in \left[f^{PC}, f^C\right)$, the demand for merchant 1 from these consumers is

$$\int_{f^{PC}}^{f^C} x\left(b_B\right) db_B = \frac{p_2 - p_1}{2t}\Delta f + \frac{(\Delta f)^2}{4t} + \frac{\Delta f}{2},$$

whereas the demand for merchant 2 from these consumers is

$$\int_{f^{PC}}^{f^{C}} (1 - x(b_B))\, db_B = \frac{p_1 - p_2}{2t}\Delta f + \frac{\Delta f}{2} - \frac{(\Delta f)^2}{4t}.$$

Consumers such that $b_B \geq f^C$ trade off between purchasing from merchant 1 and paying with the private card and purchasing from merchant 2 and paying with the payment card, since their net utility of paying by card is positive. The marginal consumer is given by

$$v - p_1 - tx + b_B - f^{PC} = v - p_2 - t(1-x) + b_B - f^C,$$

that is,

$$x = \frac{1}{2} + \frac{p_2 - p_1 + \Delta f}{2t},$$

therefore, aggregating over $b_B \in [f^C, 1]$, the demand for merchant 1 from these consumers is

$$(1 - f^C)\left[\frac{p_2 - p_1 + \Delta f}{2t} + \frac{1}{2}\right],$$

whereas the demand for merchant 2 from these consumers is

$$(1 - f^C)\left[\frac{1}{2} - \frac{p_2 - p_1 + \Delta f}{2t}\right].$$

To sum up, we find that the demand of cash users is

$$D_1^{Cash} = f^{PC} w_1,$$

and

$$D_2^{Cash} = f^C w_2 - \frac{(\Delta f)^2}{4t},$$

for merchant 1 and merchant 2, respectively. Compared to the benchmark case, the demand of cash users for merchant 1 is determined by the price of the private card, which plays the same role as the payment card. If the price of the private card is lower than the price of the payment card, merchant 2 loses some of its cash users who prefer to shop at merchant 1's and pay with the private card. This corresponds to the second term in D_2^{Cash}.

The demand of card users is the demand of private card users for merchant 1,

$$D_1^{PC} = (1 - f^{PC})w_1 + \frac{(1 - f^C)\Delta f}{2t} + \frac{(\Delta f)^2}{4t}, \tag{2}$$

and the demand of payment card users for merchant 2,

$$D_2^C = (1 - f^C)w_2 - \frac{(1 - f^C)\Delta f}{2t}. \tag{3}$$

If the private card is less expensive than the payment card, merchant 1 attracts some cash users and some card users from merchant 2. The number of cash users who switch from merchant 2 to merchant 1 is given by the third term in (2), that is, $(\Delta f)^2/(4t)$. The number of card users who switch from merchant 2 to merchant 1 is given by the second term in (2), that is, $(1 - f^C)\Delta f/(2t)$.

We now turn to stage 4 of our game. Merchant 1 makes profit

$$\pi_1^{C,C} = \left(D_1^{Cash} + D_1^{PC}\right)(p_1 - c) + (b_S + f^{PC} - c_M)D_1^{PC},$$

whereas merchant 2 makes profit

$$\pi_2^{C,C} = \left(D_2^{Cash} + D_2^{C}\right)(p_2 - c) + (b_S - m)D_2^{C}.$$

When merchants decide on their prices, they take into account both their net revenues from product sales (the first term in the profit functions) and the costs or benefits associated to card payments (the second term in the profit functions). Replacing for the expressions of demands in π_1 and π_2, and solving for the first order conditions,[15] we find that the equilibrium prices are

$$p_1 = c + t + \frac{1}{3}\left(\frac{(\Delta f)^2}{2} + (\Delta f)(1 - f^C) + (m - b_S)(1 - f^C) - 2\left(f^{PC} + b_S - c_M\right)(1 - f^{PC})\right),$$

and

$$p_2 = c + t + \frac{1}{3}\left(-\frac{(\Delta f)^2}{2} - (\Delta f)(1 - f^C) + 2(m - b_S)(1 - f^C) - \left(f^{PC} + b_S - c_M\right)(1 - f^{PC})\right).$$

A higher fee for the private card has two opposite effects on equilibrium prices. First, a higher f^{PC} decreases merchant 1's perceived marginal cost for the transactions paid by the private card, which tends to reduce merchants' prices. Second, a higher f^{PC} reduces the volume of transactions paid by the private card, hence, leads to a higher *average* perceived marginal cost for merchant 1. This is because the perceived marginal cost for transactions paid cash, c, is higher than the perceived marginal cost for transactions paid by the private card, $c - \left(b_S + f^{PC} - c_M\right)$, since $b_S > c_M$. For sufficiently low values of f^{PC}, the first effect dominates the second effect,

[15] The second order condition is verified.

and prices decrease with the private card fee. On the contrary, for sufficiently high values of f^{PC}, prices increase with the private card fee.

Replacing for the equilibrium values of p_1 and p_2 in $\pi_1^{C,C}$ and $\pi_2^{C,C}$, we obtain the equilibrium profits, which can be found in Appendix C.

Merchant 1 does not accept payment cards, while merchant 2 accepts them. If $f^{PC} \leq f^C$, whether merchant 1 accepts payment cards or not, his card consumers will always use the private card as it is cheaper. Therefore, whether he accepts cards or not, merchant 1 will face the same demand, and the equilibrium prices and profits are identical to the previous case. If $f^{PC} > f^C$, a similar analysis as in the previous section shows that the demand of cash users is

$$D_1^{Cash} = f^{PC} w_1 - \frac{(\Delta f)^2}{4t},$$

and

$$D_2^{Cash} = f^C w_2,$$

for merchant 1 and merchant 2, respectively. The second term in D_1^{Cash} represents the cash consumers of merchant 1 who decide to purchase from merchant 2 and pay by card. The demand of card users is the demand of private card users for merchant 1,

$$D_1^{PC} = (1 - f^{PC})w_1 + \frac{(1 - f^{PC})\Delta f}{2t},$$

and the demand of payment card users for merchant 2,

$$D_2^C = (1 - f^C)w_2 - \frac{(1 - f^{PC})\Delta f}{2t} + \frac{(\Delta f)^2}{4t}.$$

The second term in D_1^{PC} is negative (as $\Delta f < 0$) and represents the private card users who prefer to shop at merchant 2's and pay with the payment card. This term corresponds to the second term in D_2^C. The last term in D_2^C corresponds to the cash consumers of merchant 1 who decide to shop at merchant 2's and pay by card. The equilibrium prices are

$$p_1 = c + t + \frac{1}{3}\left(-\frac{(\Delta f)^2}{2} + (\Delta f)(1 - f^{PC}) + (m - b_S)(1 - f^C) - 2(f^{PC} + b_S - c_M)(1 - f^{PC})\right),$$

and

$$p_2 = c + t + \frac{1}{3}\left(\frac{(\Delta f)^2}{2} + -(\Delta f)(1 - f^{PC}) + 2(m - b_S)(1 - f^C) - (f^{PC} + b_S - c_M)(1 - f^{PC})\right).$$

The effect of f^{PC} on prices is similar to the previous case. Equilibrium profits, $\pi_i^{NC,C}$, can be found in Appendix C.

Merchant 1 accepts all payment cards, while merchant 2 refuses them. If $f^{PC} > f^C$, private cards are never used by consumers. This case is identical to the benchmark case, in which merchant 2 does not accept cards, while merchant 1 accepts them. If $f^{PC} \leq f^C$, payment cards are never used by consumers, as merchant 2 does not accept cards, and consumers prefer to use the private card when they shop at merchant 1's. Using the same analysis as in the previous cases, we find that the demands of cash users for merchant 1 and merchant 2 are

$$D_1^{Cash} = f^{PC} w_1,$$

and

$$D_2^{Cash} = w_2 - \frac{(1 - f^{PC})^2}{4t},$$

respectively. The demand of private card users for merchant 1 is

$$D_1^{PC} = (1 - f^{PC})w_1 + \frac{(1 - f^{PC})^2}{4t}.$$

The second term in D_1^{PC} represents the cash users of merchant 2 who decide to shop at merchant 1's and pay with the private card. The equilibrium prices are

$$p_1 = c + t + \frac{1}{3}\left(\frac{(1 - f^{PC})^2}{2} - 2(f^{PC} + b_S - c_M)(1 - f^{PC})\right),$$

and

$$p_2 = c + t + \frac{1}{3}\left(-\frac{(1 - f^{PC})^2}{2} - (f^{PC} + b_S - c_M)(1 - f^{PC})\right),$$

and the equilibrium profits, $\pi_i^{C,NC}$, can be found in Appendix C. The effect of f^{PC} on prices is similar to the previous cases.

Both merchants refuse payment cards. As consumers trade off between the private card and cash at merchant 1's and can only pay cash at merchant 2's, the demands are identical to the previous case in which merchant 2 refuses cards but not merchant 1, and $f^{PC} \leq f^C$. Equilibrium prices and equilibrium profits, $\pi_i^{NC,NC}$, are also identical.

Card acceptance conditions. At stage 4, simultaneously with setting prices, the merchants decide whether or not to accept cards. The situation in which both merchants accept cards constitutes a Nash equilibrium if and only if

$$\pi_1^{C,C}\left(m, f^C, f^{PC}\right) \geq \pi_1^{NC,C}\left(m, f^C, f^{PC}\right),$$

and

$$\pi_2^{C,C}\left(m, f^C, f^{PC}\right) \geq \pi_2^{C,NC}\left(m, f^C, f^{PC}\right).$$

The first condition means that merchant 1 has no incentive to deviate to the equilibrium in which merchant 2 is the only one who accepts cards. The second condition means that merchant 2 makes more profit if both merchants accept cards than in a situation where merchant 1 is the only one who accepts cards. The card acceptance decisions depend on the transaction fees, m, f^C and f^{PC}, which are set at stage 3 of the game.

3.3.2 Stage 3: choice of transaction fees

In this section, we assume that merchant 1 issues private cards, and we determine the transaction fees chosen by the banks and merchant 1.[16] We show that there exists an equilibrium in which both merchants accept payment cards, and that in this equilibrium, merchant 1 sets $f^{PC} = 0$.

We start by analyzing the decision of merchant 1. For given m and f^C, merchant 1 chooses the private card fee, f^{PC}, so as to maximise his profit,

$$\pi_1^{x_1,x_2} = \left(D_1^{PC} + D_1^C + D_1^{Cash}\right)(p_1 - c) + (f^{PC} + b_S - c_M)D_1^{PC} + (b_S - m)\,D_1^C. \qquad (4)$$

The following lemma shows that merchant 1's best response has a remarkable property.

Lemma 4 *If merchant* 1 *issues the private card, for any* m *and* f^C*, his best response is to choose a transaction fee equal to zero, that is,* $f^{PC} = 0$.

Proof. In Appendix D1, we show that if $f^{PC} < f^C$, merchant 1's profit decreases with the price of the private card, f^{PC}. Consequently, in this case, for any m, his best response is to set $f^{PC} = 0$. In Appendix D2, we show that merchant 1 always makes more profit if he undercuts f^C by choosing $f^{PC} < f^C$. Therefore, merchant 1's best response is to choose a transaction fee which is equal to zero. ■

Lemma 4 shows that merchant 1 sets a very aggressive price for his private card, which is below cost. In Section 2, we proved that, in absence of strategic interactions with the payment association and with other merchants, the bypassing monopolistic merchant chose a strictly

[16] If merchant 1 does not issue private cards, this is the benchmark case, that we have analyzed in Section 3.2.

positive card fee. Therefore, the intuition of Lemma is that the strategic interactions on the market for card transactions on the one hand, and the strategic interactions on the product market on the other hand, provide merchant 1 with strong incentives to set a low fee for the private card.

On the market for card transactions, merchant 1 competes in prices with bank I. Since the payment card and the private card are perfect substitutes, if f^C is sufficiently high, then the Bertrand logic applies, and merchant 1 has an incentive to undercut the price of the payment card. Indeed, from the analysis of stage 4 and 5, we know that if merchant 1 accepts payment cards and if he undercuts the Issuer by setting a slightly lower fee, that is, $f^{PC} = f^C - \epsilon$, with ϵ small, then the demand of card payments (either with a payment or a private card) remains unchanged. Consequently, merchant 1 has an incentive to undercut bank I if $f^C + b_S - c_M \geq b_S - m$ (see term (II) in (4)). Apart from this competitive effect on the market for card transactions, merchant 1 also has an incentive to lower his private card fee to encourage consumers to pay with the private card instead of cash, as he earns a higher benefit with the private card.

The private card fee has also an impact on competition in the product market. First, merchant 1 has an incentive to set a low fee to attract consumers of merchant 2, who prefer to shop at merchant 1's for lower transaction costs. Second, a lower f^{PC} softens competition on the product market because it increases the perceived marginal cost of merchant 1 for card transactions.

The effects of f^{PC} on the profits made on the product market and the market for payment transactions go in the same direction, and provide merchant 1 with strong incentives to set a very low private card fee.

We have proved that $f^{PC} = 0$ constitutes a dominant strategy for merchant 1. Therefore, from this point, we analyse the decisions of bank I and bank A for $f^{PC} = 0$. As $f^{PC} = 0$, for any f^C and m, the consumers of merchant 1 always prefer the private card to the payment card. Hence, the payment card may only be used by consumers of merchant 2. If merchant 2 refuses the payment card, banks do not make any profit. Therefore, bank I and A choose f^C and m, under the constraint that merchant 2 accepts cards. We show that this is the case for sufficiently low values of m.

Lemma 5 *There exists* $\widetilde{m}(f^C) \in \left(b_S + (1 - f^C)/2; b_S + 3(1 - f^C)/4\right)$, *such that merchant* 2 *accept payment cards for* $m \leq \widetilde{m}(f^C)$. *Merchant* 1 *is indifferent between accepting and refusing payment cards.*

Proof. See Appendix E. ■

As in the benchmark case, if the merchant fee is sufficiently low, there is an equilibrium in which both merchants accept payment cards. Since $f^{PC} = 0$, consumers always choose the private card to pay at merchant 1's. Hence, merchant 1 is indifferent between accepting and refusing payment cards. Merchant 2 accepts payment cards for sufficiently low values of m.

Corollary 1 *For a given payment card fee, f^C, the merchants are less resistant to card acceptance if merchant 1 issues private cards than if it does not.*

Proof. Indeed, in the benchmark case, the card acceptance condition was $m \leq b_S + (1 - f^C)/2$, while we have $\widetilde{m}(f^C) \geq b_S + (1 - f^C)/2$. ■

Merchant 2's incentive to deviate from the equilibrium in which both merchants accept cards is equal to the difference between his profit in case of deviation and his statu-quo profit.

In the benchmark case, as in Rochet and Tirole (2002), merchant 2's decision to refuse cards has two effects on his profit, a "perceived marginal cost effect", and a "market share effect". First, merchants' perceived marginal costs change if merchant 2 refuses cards, as he saves the merchant fee, m, net of the benefit of being paid by card, b_S. Therefore, his perceived marginal cost decreases if $m - b_S > 0$, and increases otherwise. Besides, when merchant 2 refuses cards, the proportion of card users at merchant 1's increases. Hence, merchant 1's average perceived marginal cost increases if $m - b_S > 0$, and decreases otherwise. Consequently, the higher m, the higher the benefits of deviation for merchant 2. Second, if he decides to refuse cards, merchant 2 may lose market share, as some of his card users may decide to switch to merchant 1. This market share effect makes deviation less profitable for merchant 2. Its magnitude is higher when the payment card fee is lower.

If merchant 1 issues a private card, merchant 2's incentives to refuse cards also depend on a perceived marginal cost effect and a market share effect. The market share effect is comparable to the one observed in the benchmark case, except that its magnitude is higher because merchant 1 sets a private card fee equal to zero. This reduces merchant 2's incentives to deviate, in comparison to the benchmark case. The perceived marginal cost effect has a different impact on merchant 1, since consumers always prefer the private card to the payment card when they shop at merchant 1's. For merchant 1, the perceived marginal cost of private card payments is negative (equal to $c_M - b_S$). Hence, when merchant 2 deviates and refuses cards, the proportion of private card users at merchant 1's increases, which reduces the average perceived marginal cost of merchant 1 (even if $m > b_S$). Therefore, merchant 2's incentives to deviate are lower compared to the benchmark case.

This explains why merchant 2 is less resistant to card acceptance. A direct consequence is that, for a given f^C, the Acquirer can set a higher merchant fee if merchant 1 issues private cards.

We now determine the transaction fees, f^C and m, that maximise the profits of the Issuer and the Acquirer, respectively, for $m \leq \widetilde{m}(f^C)$, that is,

$$\Pi_I = \left(f^C + a^P - c_I\right) D_2^C,$$

and

$$\Pi_A = (m - a^P - c_A) D_2^C,$$

where, from Appendix F,

$$D_2^C = \frac{1}{2t}(1 - f^C)\left[t + \frac{1}{3}\left(-(b_S + 1 + f^C)f^C + c_M - m(1 - f^C)\right)\right]. \quad (5)$$

The Issuer and the Acquirer trade off between a higher margin and a higher volume of card transactions. Notice, from equation (5), that the volume of card transactions is decreasing with the merchant fee, m. This is because the merchant fee is passed to consumers through merchant 2's perceived marginal cost.

The following proposition shows that there exists a unique equilibrium in which both merchants accept cards, and that the Acquirer chooses the maximum merchant fee compatible with merchant acceptance.

Proposition 3 *There exists a unique equilibrium, such that merchant* 1 *sets* $f^{PC} = 0$*, the Acquirer chooses the maximum merchant fee compatible with merchant* 2*'s card acceptance, and the issuer chooses a strictly positive card fee.*

Proof. See Appendix F. ■

The optimal merchant fee must be compatible with merchant 2's non deviation condition, as the Acquirer makes zero profit if merchant 2 deviates from the equilibrium in which the merchants accept cards. In Appendix F, we show that the merchant fee that maximises the Acquirer's profit does not satisfy the non deviation condition. Hence, since Π_A is concave in m,[17] the optimal merchant fee is equal to $\widetilde{m}(f^C)$.

Proposition 4 *The merchant fee is higher in the presence of a private card, while the transaction fee chosen by the Issuer for the payment card is lower, that is, we have* $m^* > m^B$ *and* $(f^C)^* < (f^C)^B$*.*

[17] This is proved in Appendix F.1.

Proof. See Appendix G. ■

When merchant 1 issues private cards and sets a very aggressive private card fee, the Issuer reacts by setting a lower payment card fee than in the benchmark case. Notice, however, that the Issuer's reaction cannot be explained only by the competition with the private card on the market for payment transactions, since f^{PC} is set to zero and $(f^C)^* > 0$.

The Issuer's reaction is also related to the product market. By setting $f^{PC} = 0$, merchant 1 obtains what could be interpreted as a quality advantage over merchant 2, which reduces the demand of merchant 2, including the demand from card users. The Issuer has an incentive to reduce merchant 2's quality disadvantage by lowering the payment card fee; in other words, the Issuer internalises the effect of the payment card fee on competition in the product market. As the payment card fee is reduced, the Acquirer can increase his merchant fee, since $\widetilde{m}(f^C)$ is decreasing in f^C.

Hence, the effect of the private card is to reinforce the market power of the Acquirer, as it makes merchants less resistant to card acceptance. On the contrary, the private card reduces the market power of the Issuer, because the latter has to lower the payment card fee to stimulate the demand of card users at merchant 2's. A consequence is that the price structure of the payment platform changes because of the competition with the private card. Unfortunately, it proves difficult to determine analytically the effect of the introduction of private cards on the total price that is charged by the payment platform, $f^C + m$. Therefore, we have to revert to numerical simulations.

We define $a^{\max}$ as the highest value of the interchange fee, a^P, such that the Acquirer's margin, $\widetilde{m}\left(\left(f^C\right)^*\left(a^P\right)\right) - a^P - c_A$, is positive.[18] We ran our simulations for the following values of the parameters: $t \in \{4, 4.1, ..., 10.0\}$, $c_A \in \{0.2, 0.3, 0.4\}$, $c_I \in \{c_A, c_A + 0.05, c_A + 0.1\}$, $c_M \in \{c_I + c_A - 0.2, c_I + c_A - 0.15, ..., c_I + c_A - 0.05\}$, $b_S \in \{c_M + 0.01, c_M + 0.02, ..., c_I + c_A - 0.01\}$, and $a^P \in \left\{0, 0.01, ..., \min\left\{a^{\max}, (a^P)^B\right\}\right\}$. The simulations show that for all t, c_I, c_A, c_M, and b_S, there exists a threshold value of a^P, which we denote by $\widetilde{a}^P$, such that the total price charged by the platform, $f^C + m$, is higher with the private card than in the benchmark case if $a \leq \widetilde{a}^P$, and the reverse is true otherwise.[19] This results shows that the introduction of private cards can lead to an *increase* of the total price charged by the platform, due to the reinforcement of the market power of the Acquirer.

[18] From Lemma 6, we know that when the interchange fee increases, the Acquirer charges a higher merchant fee. However, from Lemma 5, the equilibrium merchant fee is bounded from above. Since $\widetilde{m}\left(\left(f^C\right)^*\left(a^P\right)\right) - a^P - c_A \geq 0$, the interchange fee is also bounded from above.

[19] For each value of the parameter vector (t, c_I, c_A, c_M, b_S), we computed $\widetilde{a}^P$. Then, we tested whether $\left(m^* + (f^C)^*\right)\left(a^P\right) \geq \left(m + f^C\right)^B\left(a^P\right)$ if and only if $a^P \leq \widetilde{a}^P$, by testing this statement for each value of $a^P \in \left\{0, 0.01, ..., \min\left\{a^{\max}, (a^P)^B\right\}\right\}$.

3.3.3 Stage 2: decision to issue a private card

Merchant 1 decides to issue private cards if and only if

$$\pi_1^{C,C}\left((f^{PC})^*, (f^C)^*, m^*\right) - F \geq \pi_1^B\left((f^C)^B, m^B\right). \tag{6}$$

Notice that this corresponds to a vertical integration decision, except that it takes place in a two-sided market, that is, merchant 1 has to decide whether or not to create his own payment platform.

3.3.4 Stage 1: choice of the interchange fee

In this section, we start by conducting some comparative statics with respect to the interchange fee, if merchant 1 issues private cards. Then, we determine the optimal level of the interchange fee, that we compare with the one obtained in the benchmark case.

Comparative statics We assume that merchant 1 issues private cards, that is, condition (6) is satisfied. We analyse the effect of the interchange fee on the optimal transaction fees chosen by the Issuer and the Acquirer.

Lemma 6 *The transaction fee chosen by the Issuer for the payment card is decreasing with* a^P*, while the merchant fee chosen by the Acquirer is increasing with* a^P*, that is, we have* $d(f^C)^*/da^P < 0$ *and* $d(m)^*/da^P > 0$*.*

Proof. See Appendix H. ■

The interchange fee, a^P, has a direct and a strategic effect on the transaction fees, f^C and m. First, a higher a^P implies a lower perceived marginal cost for bank I and a higher perceived marginal cost for bank A. Therefore, bank I has incentives to decrease f^C, while bank A is willing to increase m. Second, m and f^C are strategic substitutes.[20] Therefore, a higher a^P implies a lower f^C, which in turn implies a higher m. Similarly, a higher a^P implies a higher m, hence a lower f^C. As the direct effect and the strategic effect have the same sign, we find that the payment card fee decreases with a^P, whereas the merchant fee increases with a^P.

We now study the impact of the interchange fee on entry. The entry condition, given by (6), can be rewritten as $EC(a^P) \geq 0$, where

$$EC(a^P) = \Psi^2 - 2tF - t^2,$$

[20] This result is shown in Appendix F-4.

and $\Psi = t + \frac{1}{3}(-c_M + \left((f^C)^* + m^*\right)(1-(f^C)^*) + \frac{((f^C)^*)^2}{2} + b_S(f^C)^*)$. Taking the derivative of EC with respect to a^P, we obtain

$$(EC)^{'}(a^P) = \frac{2}{3}\Psi \times \left[\underbrace{(b_S + 1 - (f^C)^* - m^*)\frac{d(f^C)^*(a^P)}{da^P}}_{\text{(I)}} + \underbrace{(1-(f^C)^*)\frac{dm^*(a^P)}{da^P}}_{\text{(II)}}\right].$$

Assumption 1 implies that $\Psi \geq 0$. Since $b_S + 1 - (f^C)^* - m^* > 0$ from Lemma 5, and $d(f^C)^*/da^P < 0$ from Lemma 6, then term (I) is negative. Term (II) is positive as $dm^*/da^P > 0$, from Lemma 6. This shows that the interchange fee impacts merchant 1's incentives to issue private cards in two opposite ways. If the interchange fee increases, the perceived marginal cost of merchant 2 rises through the payment of the merchant fee. Therefore, merchant 1 benefits from a reduction of the demand of merchant 2, since the latter is forced to increase its price. Merchant 1's incentives to issue its private card become higher, because it gives him the opportunity of increasing its market share, while saving the cost of the merchant fee, which has increased. At the same time, if the interchange fee increases, this triggers a reduction of the payment card transaction fee, which yields a higher demand for merchant 2 if merchant 1 offers a private, hence lowers the incentives of merchant 1 to issue its private card.

The following Proposition shows that, in our setting, the first effect dominates the second effect, that is, $EC(a^P)$ is decreasing with a^P, if $(f^C)^*$ is sufficiently low.

Proposition 5 *Merchant* 1*'s incentives to issue private cards decrease with the interchange fee,* a^P*, if* $(f^C)^*\left(a^P\right) \leq 1/2$*, whereas they increase with* a^P *if* $(f^C)^*\left(a^P\right) \in (0.516; 1]$*. If* $(f^C)^*\left(a^P\right) \in (0.5; 0; 516)$*, the incentives to issue private cards are increasing with* a^P *if* t *is sufficiently high.*

Proof. See Appendix J. ■

Optimal interchange fee Now, we determine the interchange fee that maximises banks' joint profits, that we denote by $\left(a^P\right)^*$. Whether merchant 1 issues private cards or not depends on the sign of $EC(a^P)$. Therefore, the banks' joint profits can be written as

$$(\Pi_I + \Pi_A)(a^P) = \begin{cases} (\Pi_I + \Pi_A)^B(a^P) & \text{if} \quad EC(a^P) < 0 \\ (\Pi_I + \Pi_A)^{PC}(a^P) & \text{if} \quad EC(a^P) \geq 0, \end{cases}.$$

where $(\Pi_I + \Pi_A)^{PC}(a^P) = ((f^C)^* + m^* - c_I - c_A)D_2^C(a^P)$. In the following Proposition, we characterise the possible equilibrium outcomes.

Proposition 6 *The equilibrium can be characterised by either:*

(i) entry accommodation: the payment system chooses the interchange fee that maximises its profit conditional on the fact that merchant 1 *issues private cards,* $\left(a^P\right)^* = \left(a^P\right)^{PC}$ *.*

(ii) blockaded entry: the payment system sets $\left(a^P\right)^* = \left(a^P\right)^B$ *and there is no entry.*

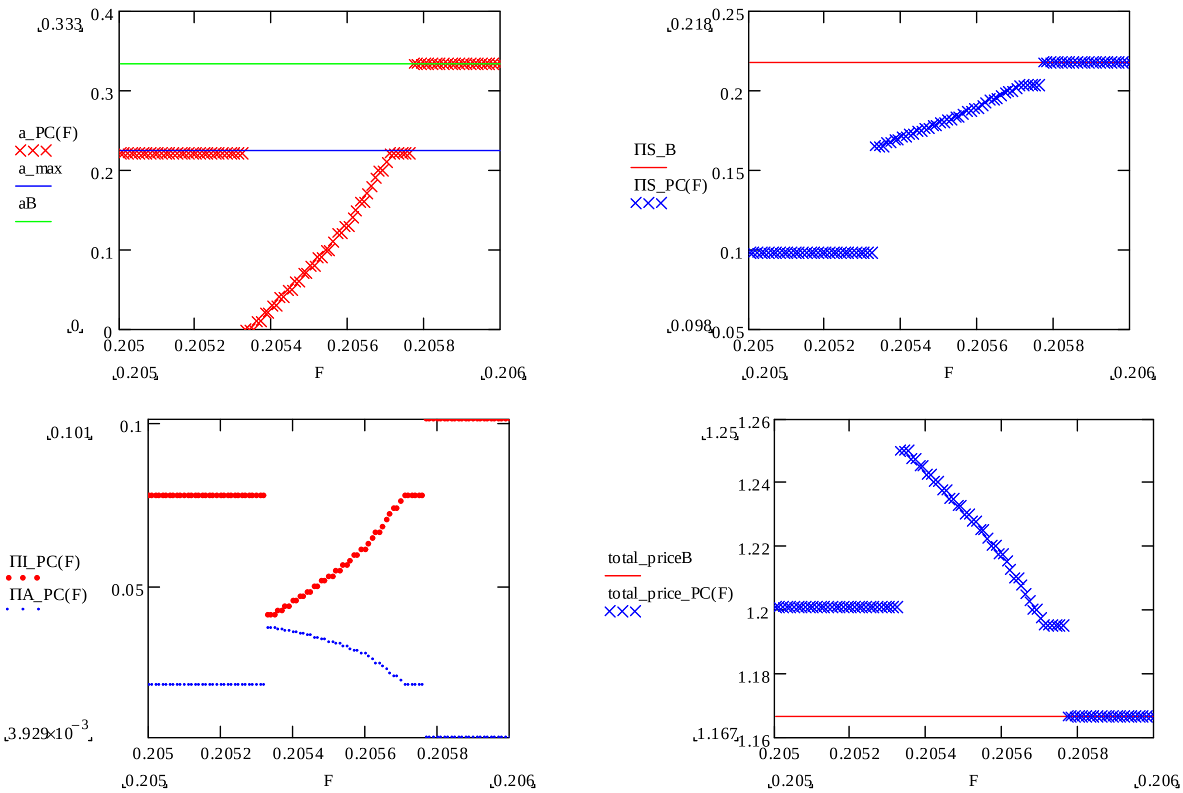
(iii) entry deterrence: the payment system sets $\left(a^P\right)^* = \widehat{a} < \left(a^P\right)^B$ *and deters entry; this can only occur if there exists* $a^P \in [0, \min\{a^{\max}, \left(a^P\right)^B\}]$ *such that* $(f^C)^* \left(a^P\right) > 1/2$.

Proof. If $EC(\left(a^P\right)^B) < 0$, blockaded entry is a possible equilibrium outcome. Otherwise, there is either entry accommodation or entry deterrence. Entry deterrence can only occur if there exists $a^P \in [0, \min\{a^{\max}, \left(a^P\right)^B\}]$ such that $(f^C)^* \left(a^P\right) > 1/2$. Indeed, if EC is decreasing with a^P for all values of a^P, the payment system must increase the interchange fee to deter entry. Since in the benchmark case the interchange fee is set at the maximum level compatible with positive profit for the Acquirer, a^P cannot be set above $\left(a^P\right)^B$. But we have $EC(\left(a^P\right)^B) \geq 0$, hence, entry cannot be deterred. Finally, from Proposition 5 we know that EC is increasing with a^P only for $(f^C)^* \left(a^P\right) > 1/2$. ■

Proposition 6 shows that the threat of the competition with the private card may lead the payment system to *decrease* its interchange fee, in comparison to the benchmark case. Hence, in no situation, the payment system will be tempted to increase its interchange to prevent the merchant from issuing private cards. If merchant 1's incentives to issue private cards decrease with the interchange fee, the payment system cannot deter merchant 1 from entering the market, because of the constraint that the Acquirer's profit remains positive. The only way for the payment platform to deter entry would be to set a higher interchange fee, which could only occur if the Issuer could compensate the Acquirer so that he made positive profit.

Suppose that entry is not blockaded, that is, $EC(\left(a^P\right)^B) \geq 0$. Since $(f^C)^* \left(a^P\right)$ is increasing with the cost of the Issuer, c_I, entry deterrence is more likely to occur for high values of c_I. In other words, the payment system will tend to deter entry only when the Issuer is inefficient by setting a low interchange fee. Finally, the payment system prefers to accommodate rather than to deter entry if $(\Pi_I + \Pi_A)(\left(a^P\right)^{PC}) \geq (\Pi_I + \Pi_A)(\widehat{a})$.

A numerical example As an illustration, we use the following parameter values: $t = 5$, $c = 0$, $c_I = 0.4$, $c_A = 0.3$, $c_M = 0.3$, and $b_S = 0.4$. The following figures show the optimal interchange fee, the equilibrium banks' profits as well as the total price set by the payment platform as a function of the entry cost, F.



The equilibrium as a function of the entry cost, F

The figure in the upper left shows the optimal interchange fee, which takes into account the optimal reaction of the platform. For low values of F, the payment system cannot deter entry. Therefore, it has to accommodate entry, and in this example it sets an interchange equal to $a^{\max}$, which is lower to $\left(a^P\right)^B$. For intermediate values of F, entry deterrence is a possibility and we find that it is preferred to entry accommodation. Therefore, the payment system lowers the interchange fee (down to zero, in this example). Then, the entry deterring interchange fee increases as F increases, since the entry threat becomes milder. When F is sufficiently high, there is no entry threat and the payment system sets the reference interchange fee; entry is blockaded.

The figure in the upper right shows, as expected, that the profit of the payment platform increases as the entry threat becomes milder (F increases). The figure in the lower left shows that the distribution of profits between the Issuer and the Acquirer depends on the outcome of the entry game. In particular, when the platform starts to deter entry, the Issuer's profit is reduced, whereas the Acquirer's profit is increased. This is due to the fact that the platform decreases the interchange fee to deter entry, which harms the Issuer while it benefits the Acquirer. Finally, the figure in the lower right shows that the total price varies non-monotically with the entry cost. When the equilibrium moves from the entry accommodation region to the entry deterrence region, the total price jumps up. Then, it decreases as F increases.

4 Extensions and discussions

In this section, we start by discussing the impact of the market structure on banking retail markets on merchant 1's incentives to issue private cards. Then, we give the condition under which merchant 2 does not react to merchant 1's entry.

4.1 The impact of the banking market structure on the incentives to issue private cards.

The market structure on the Issuing and Acquiring sides could impact the incentives of merchant 1 to issue private cards. So far, we assumed that the payment platform organised the interactions between a monopolistic Issuer and a monopolistic Acquirer. In what follows, we begin by assuming perfect competition on the acquisition side, and then we discuss the case in which there is perfect competition on both sides of the market.

Perfect competition leads the acquirers to choose a merchant fee that is equal to the marginal cost of the acquisition activity, that is, $m^* = a + c_A$.[21] We denote by $\overline{a}$ the maximum interchange fee such that both merchants accept payment cards.

Now, we study the condition under which merchant 1 enters the market for payment card transactions at stage 2. With perfect competition on the acquisition side, simulations suggest than a higher interchange fee *increases* merchant 1's incentives to issue private cards. In Section 3.3, we proved that a higher interchange fee has two opposite effects on merchant 1's incentives to issue private cards. If the acquirers are perfectly competitive, the second effect is dominant and a higher interchange fee raises merchant 1's incentives to enter the market for payment card transactions.

Let $\underline{a}$ be the minimum level of interchange fee such that merchant 1 issues private cards. If the payment platform wants to deter entry, it has to set $\left(a^P\right)^* = \underline{a}$. If entry is accommodated, since the Acquirers make zero profit, banks' joint profits are equal to the profit of the Issuer, and increase with the level of interchange fee. Hence, if the payment platform accommodates entry, it chooses the maximum interchange fee compatible with merchant acceptance, that is $\left(a^P\right)^* = \overline{a}$. The results of Proposition is modified as follows. If $a^B \leq \underline{a}$, entry is blockaded. If $a^B > \underline{a}$ and if $\left(\Pi_I\right)^B(\underline{a}) < \left(\Pi_I\right)^{PC}(\overline{a})$, the payment platform accommodates entry, whereas if $\left(\Pi_I\right)^B(\underline{a}) \geq \left(\Pi_I\right)^{PC}(\overline{a})$, the payment platform deters merchant 1 from issuing private cards.

[21] The decisions of the consumers and the merchants at stage 4 and 5 remain unchanged. At stage 3, the best responses of the Issuer and of merchant 1 are the same as in section 3.3.2.

If there is perfect competition on both sides of the market, banks make zero profit. Hence, the interchange fee has no impact on banks' joint profits. It can be shown that the payment platform minimises the probability of entry when it chooses an interchange fee that is equal to Baxter's interchange fee, that is $\left(a^P\right)^* = b_S - c_A$.[22]

To sum up, this analysis shows that the market structure influences the level of the interchange fee that deters entry.

4.2 Analysis of the reaction of the second merchant

In our setting, we assumed that only merchant 1 can issue private cards. Consider now the following modification to our setting. Once the platform has chosen the interchange fee, merchant 1 decides whether or not to issue private cards. Then, merchant 2 observes merchant 1's decision; if merchant 1 has issued a private card, merchant 2 can decide to follow and issue his own private cards. We denote by f_i^{PC} the private card fee of merchant i, and we focus on the subgame in which merchant 1 has issued private cards, to determine the conditions under which merchant 2 does not react to this decision.

If both merchants issue private cards, we show in Appendix L that they choose a private card fee that is equal to zero. There are now five possible equilibrium outcomes:

- blockaded entry: the payment platform set $\left(a^P\right)^* = \left(a^P\right)^B$ and both merchants do not issue private cards.

- entry accommodation of both merchants: both merchants issue private cards and the payment platform makes zero profit.

- entry accommodation of merchant 1: the payment platform accomodates the entry of merchant 1 by setting $\left(a^P\right)^* = \left(a^P\right)^{PC}$, and for this value of the interchange fee, the entry of merchant 2 is blockaded.

- entry deterrence of merchant 1: the payment platform sets $\left(a^P\right)^* = \widehat{a}$, which prevents merchant 1 from entering the market.

- entry deterrence of merchant 2: the payment platform chooses a level of interchange fee that prevents merchant 2 from entering the market. For this value of the interchange fee, the entry of merchant 1 can be either accomodated or blockaded.

In Appendix L, we give the condition under which the fixed costs of issuing private cards for merchant 2 are sufficiently high, such that he never reacts to merchant 1's decision to issue

[22] See Appendix K.

private card. Our analysis also shows that, if this condition is not verified, the payment platform may decide to deter merchant 2 from issuing private cards, otherwise, banks would have to exit from the market since payment cards are never used when both merchants issue private cards. An interesting insight is that the interchange fee that deters merchant 2 from entry may also deter merchant 1 from issuing private cards. Hence, the probability of entry accomodation of merchant 1 may become lower, because of the threat of merchant 2's reaction.

5 Conclusion

Our paper shows that, with monopolies both on the issuing and on the acquisition side, a payment platform may increase its level of interchange fee to deter a merchant from entering the market for payment card transactions. The effect of the competition with the private card is to reduce the card fee and to increase the cost of card acceptance for the merchant that does not issue private cards.

Further research is needed to understand better other forms of entry accommodation that can be designed by the payment platform. For instance, in many countries (e.g. France, Spain), several merchants have started issuing cards with the support of financial institutions that are members of payment card associations. The payment platform could also think of other types of contracts that would enable merchants to "opt-in" the payment system, such as cobranding agreements. Or a large retailer, as the merchant Target in the United-States, could decide to become an issuing member of the payment association. Research is also needed to understand the other opt-out strategies of the merchants. For instance, several merchants could decide, as for the Aurore Card in France, to build private payment associations that compete with payment card associations.

References

[1] BAXTER W. (1983): "Bank Interchange of Transactional Paper: Legal and Economic Perspectives", Journal of Law & Economics, vol. 26, no. 3, October, pp. 541-588.

[2] CHAKRAVORTI S. (2003): "Theory of Credit Card Networks: A survey of the literature", Review of Network Economics, vol. 2, no. 2, June, pp. 50-68.

[3] ROCHET J-C. (2003): "the Theory of Interchange Fees: A synthesis of recent contributions", Review of Network Economics, vol. 2, no. 2, June, pp. 97-124.

[4] ROCHET J-C. & TIROLE J. (2002): "Cooperation Among Competitors: The Economics of Payment Card Associations", The RAND Journal of Economics, vol. 33, no. 4, winter, pp. 549-570.

[5] ROCHET J-C. & TIROLE J. (2003a): "An Economic Analysis of the Determination of Interchange Fees in Payment Card Systems", Review of Network Economics, vol. 2, no. 2, June, pp. 69-79.

[6] ROCHET J-C. & TIROLE J. (2006a): "Externalities and Regulation in Card Payment Systems", Review of Network Economics, vol. 5, no. 1, March, pp. 1-14.

[7] ROCHET J-C. & TIROLE J. (2006b): "Two-sided markets: a progress report", RAND Journal of Economics, vol. 37, no. 3, March, pp. 645-667.

[8] ROCHET J-C. & TIROLE J. (2007): "Must-Take Cards and the Tourist Test", Mimeo.

[9] SCHMALENSEE R. (2002): "Payment systems and Interchange Fees", Journal of Industrial Economics, vol. 50, no. 2 (June), pp. 103-122.

[10] WRIGHT J. (2002): "Optimal Payment Card Systems," European Economic Review, vol. 47, no. 4, August, pp. 587-612.

[11] WRIGHT J. (2004): "Determinants of Optimal Interchange Fees in Payment Systems", Journal of Industrial Economics, vol. 52, no. 1, March, pp. 1-26.

6 Appendix

6.1 Appendix A: A basic model of bypass by a local monopolist

6.1.1 Appendix A-1: Proof of Lemma 1

We study the merchant's decision to accept payment cards, if he does not issue private cards. If he accepts payment cards, a monopolistic merchant makes profit

$$\pi_m = (p-c)\left(D^C + D^{Cash}\right) + D^C(b_S - m),$$

where D^C denotes the demand of card users, and D^{Cash} denotes the demand of cash users. Whereas, if he refuses payment cards, he makes profit

$$\pi_m = (p-c)\, D^{Cash}.$$

If the merchant refuses payment cards, the consumers only buy the good if $p \leq v$, therefore the merchant maximises its profit by choosing $p^* = v$. Hence, the market is covered, and the merchant makes profit $\pi_m = v - c$.

We now determine the price that is chosen by a monopolistic merchant if he accepts payment cards. There are three cases. Either the monopolistic merchant sets $p \in [0; v]$, and the market is covered. In this case, the demand of card users is $D^C = 1 - f^C$, while the demand of cash users is $D^{Cash} = f^C$. If the monopolistic merchant sets $p \in \left[v; 1 + v - f^C\right]$, the good is not bought by cash users. Hence, in this case, the demand of card users is $D^C = 1 + v - p - f^C$, and the demand of cash users is $D^{Cash} = 0$. If the monopolistic merchant sets $p \in \left[1 + v - f^C; +\infty\right)$, the good is not bought by the consumers, and the merchant makes zero profit. The profit of a merchant who accepts payment cards is therefore

$$\pi_m = \begin{cases} p - c + (b_S - m)(1 - f^C) & \text{if} \quad p \in [0; v] \\ (p - c + b_S - m)\left(1 + v - p - f^C\right) & \text{if} \quad p \in \left[v; 1 + v - f^C\right] \\ 0 & \text{if} \quad p \in \left[1 + v - f^C; +\infty\right) \end{cases}$$

The merchant chooses the price that maximises its profit. We determine the maximum profit that can be made by the merchant on the intervals $[0; v]$ and $\left[v; 1 + v - f^C\right]$. Since π_m is increasing in p over $[0; v]$, π_m is maximal for $p^* = v$ over $[0; v]$, and this maximum is equal to $\pi_m = v - c + (b_S - m)(1 - f^C)$.

We now determine the maximum of π_m over $\left[v; 1 + v - f^C\right]$. If $v - c - \left(1 - f^C\right) \leq m - b_S \leq v - c + \left(1 - f^C\right)$,[23] the profit is maximised for $p^* = (1 - f^C + m + v + c - b_S)/2$, and this maximum is equal to

$$\pi_m = \frac{1}{4}\left(1 - f^C + v + b_S - m - c\right)^2.$$

If $v - c - \left(1 - f^C\right) \geq m - b_S$, the profit is maximal for $p^* = v$, and $\pi_m = v - c + (b_S - m)\left(1 - f^C\right)$. If $m - b_S \geq v - c + \left(1 - f^C\right)$, the profit is maximal for $p^* = 1 + v - f^C$. Since $D^C = 0$, the merchant makes zero profit.

We now determine the optimal price for the merchant. The merchant makes more profit by choosing $p^* = (1 - f^C + m + v + c - b_S)/2$ than $p^* = v$ if and only if

$$\frac{1}{4}\left(1 - f^C + v + b_S - m - c\right)^2 \geq v - c + (b_S - m)\left(1 - f^C\right).$$

[23] This corresponds to the condition $p^* \in \left[v; 1 + v - f^C\right]$.

Since $1 - f^C + v + b_S - m - c \geq 0$, this condition is equivalent to

$$1 - f^C + v - c - 2\sqrt{v - c + (b_S - m)(1 - f^C)} \geq m - b_S. \tag{A1}$$

Hence, $p^* = (1 - f^C + m + v + c - b_S)/2$ maximises the merchant's profit if and only if

$$v - c - \left(1 - f^C\right) \leq m - b_S \leq 1 - f^C + v - c - 2\sqrt{v - c + (b_S - m)(1 - f^C)}. \tag{A2}$$

If this condition does not hold, the merchant's profit is maximised for $p^* = v$.

We now study the decision of a merchant to accept cards. If (A2) does not hold, the merchant makes more profit by accepting payment cards if and only if

$$v - c + (b_S - m)\left(1 - f^C\right) \geq v - c.$$

Since $f^C \in [0; 1]$, this condition is equivalent to $m \leq b_S$.

Otherwise, if (A2) holds, the merchant makes more profit by accepting payment cards if

$$\frac{1}{4}\left(1 - f^C + v + b_S - m - c\right)^2 \geq v - c.$$

Since $1 - f^C + v + b_S - m - c \geq 0$, this condition can be restated as

$$m - b_S \leq 1 - f^C + v - c - 2\sqrt{v - c}.$$

This condition cannot be verified since $m - b_S \geq v - c - \left(1 - f^C\right)$. Indeed, otherwise, we would have $1 - f^C > \sqrt{v - c}$, which is impossible since $1 - f^C \in [0; 1]$ and $v - c > 2$ by assumption. Hence, if (A2) holds, the payment card is refused by the merchants.

6.1.2 Appendix A-2: Proof of Lemma 2

If merchant M_0 issues a private card, and if $f^{PC} > f^C$, the private card is never used by the consumers. If $f^{PC} \leq f^C$, consumers always prefer using the private card to the payment card. A consumer obtains a utility of $v - p + b_B - f^{PC}$ if he pays with the private card, whereas he obtains $v - p$ if he pays cash. There are three cases. Either merchant M_0 sets $p \in [0; v]$, and both private card users and cash users buy the good. In this case, the demand of private card users is $D^{PC} = 1 - f^{PC}$, while the demand of cash users is $D^{Cash} = f^{PC}$. If merchant M_0 sets $p \in \left[v; v + 1 - f^{PC}\right]$, the good is not bought by cash users. Hence, in this case, the demand of private card users is $D^{PC} = 1 + v - f^{PC} - p$, and the demand of cash users is $D^{Cash} = 0$.

If merchant M_0 sets $p \in \left[v+1-f^{PC};+\infty\right)$, the good is not bought by the consumers. The profit of the merchant is therefore

$$\pi_{M_0}^{PC} = \begin{cases} p-c+(b_S-c_M+f^{PC})(1-f^{PC})-F & \text{if} \quad p \in [0;v] \\ \left(p-c+b_S-c_M+f^{PC}\right)\left(1+v-f^{PC}-p\right)-F & \text{if} \quad p \in \left[v;v+1-f^{PC}\right] \\ -F & \text{if} \quad p \in \left[v+1-f^{PC};+\infty\right) \end{cases}$$

Merchant M_0 chooses the price that maximises its profit. We determine the maximum profit on the intervals $[0;v]$ and $\left[v;v+1-f^{PC}\right]$.

Since π_m is increasing in p over $[0;v]$, π_m is maximal for $p^* = v$ over $[0;v]$. If $p \in \left[v;1+v-f^{PC}\right]$, we also find that the profit is maximal for $p^* = v$. Indeed, the first order condition yields $p^* = -f^{PC} + (1+v+c_M+c-b_S)/2$, but this expression is lower than v since $b_S - c_M \in [0,1]$ and $v - c \geq 2$. Therefore, merchant M_0 sets $p^* = v$ and makes profit

$$\pi_{M_0}^{PC} = v - c + (b_S - c_M + f^{PC})(1-f^{PC}) - F.$$

6.1.3 Appendix A-3: Proof of Lemma 3

If $f^{PC} > f^C$, the choice of f^{PC} is irrelevant as no consumer pays with the private card at merchant M_0's. If $f^{PC} \leq f^C$, merchant M_0 chooses the private card fee that maximises its profit, $\pi_m = v-c+(b_S-c_M+f^{PC})(1-f^{PC})-F$. Solving for the first order condition[24] gives an optimal private card fee of $(f^{PC})^* = \min\left\{(1-b_S+c_M)/2, \left(f^C\right)^*\right\}$. Merchant M_0 makes profit $\pi_{M_0}^{PC} = v-c+(1+b_S-c_M)^2/4-F$ if $(f^{PC})^* = (1-b_S+c_M)/2$; otherwise, he makes profit $\pi_{M_0}^{PC} = v-c+\left(b_S-c_M+(f^C)^*\right)\left(1-(f^C)^*\right)-F$.

6.1.4 Appendix A-4: Bypass conditions with other market structures.

If there is perfect competition on the acquisition side, the merchant fee is equal to the acquirers' perceived marginal cost, that is $m^* = a^P + c_A$, and the card fee chosen by the monopolistic issuer is equal to $\left(f^C\right)^* = (1-a^P+c_I)/2$. Substituting for m^* in the card acceptance condition, we obtain that merchants accept cards if $a^P \leq b_S - c_A$. At stage 1, the payment platform chooses the interchange fee that maximises banks' joint profits, subject to $a^P \leq b_S - c_A$, $\Pi_A \geq 0$ and $\Pi_I \geq 0$. Substituting for $\left(f^C\right)^*$ and m^* in Π_I and Π_A respectively, we obtain that $\Pi_A + \Pi_I = (1+a^P-c_I)^2/4$. Therefore, the payment platform chooses the maximum interchange fee that is compatible with merchants' acceptance of payment cards, that is $\left(a^P\right)^* = b_S - c_A$.

[24]The second order condition is verified.

Hence, with the same card fee and interchange fee, the bypass conditions remain the same as in Proposition 1.

If there is perfect competition on the issuing side, the card fee is $\left(f^C\right)^* = c_I - a^P$, and a monopolistic acquirer chooses the maximum merchant fee that is compatible with merchants' acceptance of payment cards, that is $m^* = b_S$. Merchant M_0 sets $(f^{PC})^* = \min\left\{(1 - b_S + c_M)/2; \left(f^C\right)^*\right\}$. At stage 1, the payment platform chooses the interchange fee that maximises banks' joint profit, subject to $\Pi_A \geq 0$ and $\Pi_I \geq 0$. Substituting for $\left(f^C\right)^*$ and m^* in Π_I and Π_A respectively, we obtain that $\Pi_A + \Pi_I = (b_S - a^P - c_A)(1 + a^P - c_I)$. Solving for the first order condition yields to an unconstrained optimum of $\left(a^P\right)^* = (-1 + b_S - c_A + c_I)/2$. For this value of the interchange fee, the acquirers' profit is positive, as $m^* - c_A - a^P = (1 + (b_S - (c_I + c_A)))/2 \geq 0$ since $|b_S - (c_I + c_A)| \in [0, 1]$ by assumption. Hence, the payment platform chooses $\left(a^P\right)^* = (-1 + b_S - c_A + c_I)/2$. We have therefore $\left(f^C\right)^* = (1 - b_S + c_I + c_A)/2$. Therefore, merchant M_0 sets $(f^{PC})^* = \left(f^C\right)^*$ if $c_M > c_I + c_A$ and $(f^{PC})^* = (1 + b_S + c_M)/2$, otherwise. Replacing for $\left(f^{PC}\right)^*$ in merchant M_0 's profit, we obtain the bypass condition $(1 + b_S - c_M)^2/4 \geq F$ if $c_M \leq c_I + c_A$ and

$$(1 + b_S + c_I + c_A - 2c_M)(1 + b_S - c_I - c_A)/4 \geq F,$$

otherwise. This is the same bypass condition as in Proposition 1.

6.2 Appendix B: Proof of Proposition 2

Assume that both merchants accept payment cards at stage 5. Consumers such that $b_B \in \left[0; f^C\right]$ always pay cash, while consumers such that $b_B \in \left[f^C; 1\right]$ always pay by card. Each consumer trades off between shopping at merchant 1's and shopping at merchant 2's. A consumer with benefit b_B, and located at x, buys from merchant 1 if and only if $-p_1 - tx \geq -p_2 - t(1 - x)$. For $(i; j) \in \{1; 2\}^2$ and $i \neq j$, we define $w_i = [t + (p_j - p_i)]/(2t)$. Consumers such that $b_B \geq f^C$ purchase by card, therefore the demand of card payments for merchant i is $D_i^C = (1 - f^C)w_i$. The total demand for card payments is $D_T^C = D_1^C + D_2^C = 1 - f^C$. Similarly, consumers such that $b_B \leq f^C$ pay cash, hence the demand for cash payments of merchant i is $D_i^{Cash} = f^C w_i$. Each merchant chooses the price that maximises its profit,

$$\pi_i^{C,C} = (1 - f^C)w_i(p_i - c - m + b_S) + f^C w_i(p_i - c).$$

Writing the first order condition, we obtain the prices chosen at the equilibrium of the subgame[25], $p_i = c + t + (m - b_S)(1 - f^C)$, and the equilibrium profits,

$$\pi_i = \frac{t}{2}, \tag{7}$$

for $(i; j) \in \{1; 2\}^2$ and $i \neq j$. Note that assumption 1 ensures that no merchant corners the market in equilibrium.

Now, suppose that merchant 1 deviates from this presumed equilibrium, and decides to refuse payment cards. A consumer located at x with benefit $b_B \geq f^C$ wants to use his payment card, therefore he buys from merchant 1 if and only if $-p_1 - tx \geq -p_2 - t(1 - x) + b_B - f^C$.

Aggregating over all customers such that $b_B \geq f^C$, we obtain the demand of the consumers who wish to use their payment cards, and still choose to shop at merchant 1, even if the latter refuses cards:

$$(1 - f_C)w_1 - \frac{1}{4t}(1 - f^C)^2.$$

The demand of the consumers who wish to use cash and choose merchant 1 is equal to $f^C w_1$. Merchant 1 and merchant 2 choose respectively the prices p_1 and p_2 that maximise their profits:

$$\pi_1^{NC,C} = \left(w_1 - \frac{1}{4t}(1 - f^C)^2\right)(p_1 - c),$$

and

$$\pi_2^{NC,C} = \left((1 - f^C)w_2 + \frac{1}{4t}(1 - f^C)^2\right)(p_2 - c + b_S - m) + f_C w_2(p_2 - c).$$

Solving for the first order conditions[26] yields equilibrium prices

$$\begin{aligned} p_1 &= t + c + \frac{1}{3}((m - b_S)(1 - f^C) - \frac{(1 - f^C)^2}{2}), \\ p_2 &= t + c + \frac{1}{3}(2(m - b_S)(1 - f^C) + \frac{(1 - f^C)^2}{2}), \end{aligned}$$

and equilibrium profits

$$\begin{aligned} \pi_1^{NC,C} &= \frac{1}{2t}\left[t + \frac{1}{3}((m - b_S)(1 - f^C) - \frac{(1 - f^C)^2}{2})\right]^2, \\ \pi_2^{NC,C} &= \frac{1}{2t}\left[t + \frac{1}{3}((m - b_S)(1 - f^C) + \frac{(1 - f^C)^2}{2})\right]^2 + \frac{(b_S - m)f^C(1 - f^C)^2}{4t}. \end{aligned}$$

Merchant 1 has no incentive to deviate from the equilibrium in which both merchants accept

[25] The second order condition is always satisfied.

[26] The second order conditions are always satisfied.

cards if and only if $\pi_1^{C,C} \geq \pi_1^{NC,C}$, that is, if and only if

$$\frac{t}{2} \geq \frac{1}{2t}(t + \frac{1}{3}((m - b_S)(1 - f^C) - \frac{(1 - f^C)^2}{2}))^2,$$

which can be written, if $f^C \neq 1$, as

$$m \leq b_S + \frac{(1 - f^C)}{2}. \tag{8}$$

This condition is the same for merchant 2.

At stage 3, the issuer and the acquirer maximise their profits, $\Pi_I = (1 - f^C)(f^C + a^P - c_I)$, and $\Pi_A = (1 - f^C)(m - a^P - c_A)$, with respect to f^C and m, respectively, subject to (8). The constraint is binding for the acquirer since $d\Pi_A/dm = 1 - f^C \geq 0$. Therefore, the best response of the acquirer is to choose $m = b_S + (1 - f^C)/2$. Solving for the first-order condition of profit maximisation for the issuer yields the best response[27]

$$f^C = \frac{1 + c_I - a^P}{2}. \tag{9}$$

In this case, the optimal merchant fee is

$$m = b_S + \frac{1 - c_I + a^P}{4}. \tag{10}$$

At stage 1, the payment card system chooses the interchange fee that maximises banks' joint profits,

$$\Pi_I + \Pi_A = \frac{1}{2}\left(b_S + \frac{3 + c_I - a^P}{4} - (c_I + c_A)\right)\left(1 + a^P - c_I\right),$$

subject to $\Pi_i \geq 0$, where $i = I, A$. Substituting for f^C and m given by (9) and (10) into Π_I, we have $\Pi_I = \left(1 - c_I + a^P\right)^2/4$, which shows that $\Pi_I \geq 0$ for all $a^P \geq 0$. Therefore, the problem of the payment system can be restated as maximizing $\Pi_I + \Pi_A$ subject to $\Pi_A \geq 0$.

We form the Lagrangian $L = \Pi_I + \Pi_A + \lambda\Pi_A$. The first order conditions are $\partial L/\partial a^P = 0$, $\lambda\Pi_A = 0$, $\Pi_A \geq 0$ and $\lambda \geq 0$. If $\Pi_A > 0$, then we have necessarily $\lambda = 0$. It can be shown that the optimal a^P is then $a^P = 2(b_S - c_A) + 1 - c_I$. Substituting for this expression into the Acquirer's margin, $m - c_A - a^P$, yields $m - c_A - a^P = -b_S + (c_I + c_A - 1)$, which is strictly negative as $b_S \geq 0$ and $c_I + c_A < 1$. Since this contradicts $\Pi_A > 0$, it follows that the constraint is binding, that is, we have $\Pi_A = 0$ at the optimum. Substituting for f^C and m given by (9)

[27] The second order condition is verified.

and (10) into Π_A, we have

$$\Pi_A = \frac{1-c_I+a^P}{2} \times \left(b_S - c_A + \frac{1-c_I-3a^P}{4}\right).$$

We have $1-c_I+a^P > 0$ as $c_I < 1$, therefore $\Pi_A = 0$ implies that $\left(a^P\right)^B = (4b_S - 4c_A + 1 - c_I)/3$. The optimal transaction fees are then $\left(f^C\right)^B = (1 + 2(c_I + c_A - b_S))/3 > 0$, and $m^B = (4b_S + 1 - (c_I + c_A))/3 > 0$.

6.3 Appendix C: Equilibrium profits

Both merchants accept cards If both merchants accept cards, and $f^{PC} > f^C$, the equilibrium profits are given by (7). Otherwise, if $f^{PC} \leq f^C$, the equilibrium profits are

$$\begin{aligned}
\pi_1^{C,C} &= \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(\Delta f)^2}{2} + \left(f^{PC} - c_M\right)(1 - f^{PC}) + (\Delta f + m)(1 - f^C) + b_S \Delta f\right)\right)^2 \\
&\quad + \frac{(\Delta f)(f^{PC} + b_S - c_M) f^{PC}}{2t} \times \left(\frac{\Delta f}{2} + 1 - f^C\right),
\end{aligned}$$

and

$$\begin{aligned}
\pi_2^{C,C} &= \frac{1}{2t}\left(t + \frac{1}{3}\left(-\frac{(\Delta f)^2}{2} - \left(f^{PC} - c_M\right)(1 - f^{PC}) - (\Delta f + m)(1 - f^C) - b_S \Delta f\right)\right)^2 \\
&\quad + \frac{(m - b_S)(1 - f^C)(f^C - f^{PC})\left(f^C + f^{PC}\right)}{4t}.
\end{aligned}$$

Merchant 1 does not accept payment cards, while merchant 2 accepts them If $f^{PC} \leq f^C$, profits are identical to the previous case. Otherwise, if $f^{PC} > f^C$, the equilibrium profits are

$$\begin{aligned}
\pi_1^{NC,C} &= \frac{1}{2t}\left(t + \frac{1}{3}\left((m - b_S)(1 - f^C) - \frac{(\Delta f)^2}{2} + (f^{PC} + b_S - c_M)(1 - f^{PC}) + \left(1 - f^{PC}\right)\Delta f\right)\right)^2 \\
&\quad + \frac{(f^{PC} - c_M + b_S)}{4t}(\Delta f)(1 - f^{PC})(f^C + f^{PC}),
\end{aligned}$$

and

$$\begin{aligned}
\pi_2^{NC,C} &= \frac{1}{2t}\left(t + \frac{1}{3}\left(-(m - b_S)(1 - f^C) + \frac{(\Delta f)^2}{2} - (f^{PC} + b_S - c_M)(1 - f^{PC}) - \left(1 - f^{PC}\right)\Delta f\right)\right)^2 \\
&\quad + \frac{f^C(\Delta f)(m - b_S)}{2t}\left(1 - \frac{f^{PC} + f^C}{2}\right).
\end{aligned}$$

Merchant 1 accepts all cards, while merchant 2 refuses them, or both merchants refuse cards If merchant 1 accepts all cards and merchant 2 refuses payment cards and

$f^{PC} \leq f^C$, or if both merchants refuse cards, the equilibrium profits are

$$\pi_1^{C,NC} = \pi_1^{NC,NC} = \frac{1}{2t}\left(t+\frac{1}{3}\left(\frac{(1-f^{PC})^2}{2}+(f^{PC}+b_S-c_M)(1-f^{PC})\right)\right)^2 + \frac{f^{PC}(f^{PC}+b_S-c_M)(1-f^{PC})^2}{4t},$$

and

$$\pi_2^{C,NC} = \pi_2^{NC,NC} = \frac{1}{2t}\left(t+\frac{1}{3}\left(-\frac{(1-f^{PC})^2}{2}-(f^{PC}+b_S-c_M)(1-f^{PC})\right)\right)^2.$$

6.4 Appendix D: Proof of Proposition 4

6.4.1 Appendix D1: π_1 decreases with f^{PC} if $f^{PC} < f^C$.

Both merchants accept cards If $f^{PC} < f^C$, consumers who shop at merchant's 1 pay with the private card, hence, whether merchant 1 accepts cards or not is irrelevant. Merchant 1's profit is

$$\pi_1^{C,C} = \frac{1}{2t}\left(t+\frac{1}{3}\left(\frac{(\Delta f)^2}{2}+\left(f^{PC}-c_M\right)\left(1-f^{PC}\right)+(m+\Delta f)(1-f^C)+b_S(\Delta f)\right)\right)^2 + \frac{f^{PC}(f^{PC}+b_S-c_M)(\Delta f)}{2t}\left[(1-f^C)+\frac{\Delta f}{2}\right].$$

Derivating with respect to f^{PC}, we obtain

$$\frac{\partial \pi_1^{C,C}}{\partial f^{PC}} = \frac{-H_1}{36t},$$

where $H_1 = 4\Delta f\,(b_S)^2 + Xb_S + Y$, $X_1 = X_1\left(t, f^C, f^{PC}, m, c_M\right)$, and $Y_1 = Y_1\left(t, f^C, f^{PC}, m, c_M\right)$.

We want to prove that $H_1 \geq 0$, which would lead that $\partial\pi_1^{C,C}/\partial f^{PC} \leq 0$. We do it in a few steps. First, we prove that

$$\left.\frac{\partial \pi_1^{C,C}}{\partial f^{PC}}\right|_{f^{PC}=0} \leq 0.$$

Indeed, we have

$$\left.\frac{\partial \pi_1^{C,C}}{\partial f^{PC}}\right|_{f^{PC}=0} = -\frac{(b_S-c_M)}{36t}K_1,$$

where $K_1 = 12t + 4m\left(1-f^C\right) + 4\left(b_S-c_M\right) - \left[4b_S\left(1-f^C\right)+14f^C-7\left(f^C\right)^2\right]$. Given that $b_S \leq 1$ and $f^C \in [0,1]$, it can be shown that the term into brackets is always strictly lower than

8. Hence, $3t \geq 2$ implies that $K_1 > 0$. Since $b_S > c_M$ by assumption, we have

$$\left. \frac{\partial \pi_1^{C,C}}{\partial f^{PC}} \right|_{f^{PC}=0} \leq 0.$$

Second, we prove that

$$\left. \frac{\partial^2 \pi_1^{C,C}}{\partial \left(f^{PC}\right)^2} \right|_{f^{PC}=0} \leq 0. \tag{11}$$

Indeed, we have

$$\left. \frac{\partial^2 \pi_1^{C,C}}{\partial \left(f^{PC}\right)^2} \right|_{f^{PC}=0} = \frac{-M_1}{9t},$$

where

$$M_1 = 3t + 9\left(b_S - c_M\right) + \left(b_S - c_M\right)\left(1 - \left(b_S - c_M\right)\right) + m\left(1 - f^C\right) - \left[\left(b_S + 4f^C\right)\left(1 - f^C\right) + 4f^C\right].$$

The last term into brackets is lower than 5. Hence, if $3t \geq 5$, and given that $b_S \geq c_M$, we have $M_1 \geq 0$, which implies that (11) holds.

Third, we find that the third-order derivative of $\pi_1^{C,C}$, denoted by $\pi_1^{(3)}$, has the sign of $114f^{PC} - 54 + 33\left(b_S - c_M\right)$. When $f^{PC} = 0$, we have $\pi_1^{(3)} < 0$ as $b_S - c_M \leq 1$. When $f^{PC} = 1$, we have $\pi_1^{(3)} > 0$ as $b_S - c_M > 0$. Therefore, we have $\pi_1^{(3)} < 0$ for low values of f^{PC} and $\pi_1^{(3)} > 0$ for high values of f^{PC}, which implies that $\pi_1^{(2)}$ is first decreasing then increasing.

Given these properties, we know that either $\pi_1^{(1)}$ is always negative, or it is first negative then positive (as a function of f^{PC}). The second case occurs when $\pi_1^{(2)}$ becomes positive for high values of f^{PC} and $\pi_1^{(1)}$ increases sufficiently to become positive. Therefore, the global optimum of $\pi_1(f^{PC})$ when $f^{PC} \in \left[0, f^C\right)$ is either 0 or f^{C-}. We have

$$\pi_1^{C,C}(0) = \frac{1}{2t}\left[t + \frac{1}{3}\left(m\left(1 - f^C\right) + \left(f^C - c_M\right)\left(1 - f^C\right) + \left(b_S - c_M\right)f^C + \frac{\left(f^C\right)^2}{2}\right)\right]^2,$$

and

$$\pi_1^{C,C}\left(f^{C-}\right) = \frac{1}{2t}\left[t + \frac{1}{3}\left(m\left(1 - f^C\right) + \left(f^C - c_M\right)\left(1 - f^C\right)\right)\right]^2,$$

hence $\pi_1^{C,C}(0) > \pi_1^{C,C}\left(f^{C-}\right)$ if and only if $f^C + 2\left(b^S - c_M\right) > 0$, which is true (for all $f^C \geq 0$) since $b^S > c_M$.

Merchant 2 refuses all payment cards If $f^{PC} \leq f^C$, merchant 1's profit is

$$\pi_1^{C,NC} = \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(1 - f^{PC})^2}{2} + (f^{PC} + b_S - c_M)(1 - f^{PC})\right)\right)^2 + \frac{f^{PC}(f^{PC} + b_S - c_M)(1 - f^{PC})^2}{4t}.$$

Derivating with respect to f^{PC}, we obtain

$$\frac{\partial \pi_1^{C,NC}}{\partial f^{PC}} = \frac{-H_2}{36t},$$

where $H_2 = 4(1-f^{PC})(b_S)^2 + X_2 b_S + Y_2$, $X_2 = X_2\left(t, f^{PC}, c_M\right)$, and $Y_2 = Y_2\left(t, f^{PC}, c_M\right)$.

We want to prove that for any $b_S \geq 0$, $f^{PC} \in [0,1]$, we have $\partial \pi_1^{C,NC}/\partial f^{PC} \leq 0$. We use the same steps as above. First, we prove that

$$\left.\frac{\partial \pi_1^{C,NC}}{\partial f^{PC}}\right|_{f^{PC}=0} \leq 0.$$

Indeed, we have

$$\left.\frac{\partial \pi_1^{C,NC}}{\partial f^{PC}}\right|_{f^{PC}=0} = -\frac{(b_S - c_M)}{36t} K_2,$$

where $K_2 = 12t - 7 - 8c_M + 4(b_S + c_M)$. Since $4(b_S + c_M) > 0$, and since $c_M < 1$, $t \geq 5/4$ implies that $K_2 > 0$. Since $b_S > c_M$ by assumption, we have

$$\left.\frac{\partial \pi_1^{C,NC}}{\partial f^{PC}}\right|_{f^{PC}=0} \leq 0.$$

Second, we prove that

$$\left.\frac{\partial^2 \pi_1^{C,NC}}{\partial \left(f^{PC}\right)^2}\right|_{f^{PC}=0} \leq 0. \tag{D1}$$

Indeed, we have

$$\left.\frac{\partial^2 \pi_1^{C,NC}}{\partial \left(f^{PC}\right)^2}\right|_{f^{PC}=0} = \frac{-M_2}{36t},$$

where

$$M_2 = 12t - 16 + (b_S - c_M)(40 - 4b_S + 4c_M).$$

Since $b_S < 1$, then $40 - 4b_S + 4c_M > 0$. Hence, given that $b_S > c_M$, by Assumption 1, we have $M_2 \geq 0$, which implies that (D1) holds. Third, we find that the third-order derivative of π_1, denoted by $\pi_1^{(3)}$, has the sign of $114 f^{PC} + 33\left(b_S - c_M\right) - 54$.

When $f^{PC} = 0$, we have $\pi_1^{(3)} < 0$ as $b_S - c_M \leq 1$. When $f^{PC} = 1$, we have $\pi_1^{(3)} > 0$ as $b_S - c_M \geq 0$. Therefore, $\pi_1^{(3)} < 0$ for low values of f^{PC} and $\pi_1^{(3)} > 0$ for high f^{PC}. It implies that $\pi_1^{(2)}$ is first decreasing then increasing. Given these properties, we know that either $\pi_1^{(1)}$ is always negative, or it is first negative then positive (as a function of f^{PC}). The second case occurs when $\pi_1^{(2)}$ becomes positive for high values of f^{PC} and $\pi_1^{(1)}$ increases sufficiently to become positive. Therefore, the global optimum of $\pi_1^{C,NC}\left(f^{PC}\right)$ when $f^{PC} \in \left[0, f^{C}\right)$ is either

0 or f^{C-}. We have

$$\pi_1^{C,NC}(0) = \frac{1}{2t}\left[t + \frac{1}{3}\left(b_S - c_M + \frac{1}{2}\right)\right]^2,$$

and

$$\pi_1^{C,NC}\left(f^{C-}\right) = \frac{1}{2t}\left[t + \frac{1}{3}\left(b_S - c_M + \frac{1}{2} - f^C(b_S - c_M) - \frac{(f^C)^2}{2}\right)\right]^2.$$

Since $b^S - c_M > 0$, we have $\pi_1(0) > \pi_1\left(f^{C-}\right)$.

To sum up, in cases 1-4, the global maximum of $\pi_1^{C,NC}\left(f^{PC}\right)$ over $\left[0, f^C\right)$ is obtained at $f^{PC} = 0$.

6.4.2 Appendix D2: Merchant 1 undercuts f^C by setting $f^{PC} < f^C$

We show that, in all cases, merchant 1 always makes more profit if he undercuts the Issuer by setting $f^{PC} < f^C$.

Case 1: Both merchants accept payment cards. If $f^{PC} > f^C$, merchant 1 makes profit $\pi_1^{C,C} = t/2$. If $f^{PC} < f^C$, we know from Appendix B1 that merchant 1's profit is maximum for $f^{PC} = 0$, in which case he makes

$$\pi_1^{C,C} = \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(f^C)^2}{2} + (b_S - c_M) + (f^C + m - b_S)(1 - f^C)\right)\right)^2.$$

Let $\gamma = m - a - c_A$ and $\delta = f^C + a - c_I$ be the Acquirer's and Issuer's margins, respectively. Since the margins are positive, we have $\gamma \geq 0$ and $\delta \geq 0$. We also have $f^C + m - b_S = \delta + \gamma + c_I + c_A - b_S$. Since $b_S \leq c_I + c_A$ by assumption, it follows that $f^C + m - b_S \geq 0$. As we have $b_S > c_M$ too, then merchant 1 makes more profit if he undercuts the Issuer by setting $f^{PC} < f^C$.

Case 2: Merchant 2 is the only one who accepts cards. If $f^{PC} < f^C$, merchant 1 makes profit

$$\begin{aligned}\pi_1^{NC,C} &= \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(\Delta f)^2}{2} + (f^{PC} - c_M)(1 - f^{PC}) + (\Delta f + m)(1 - f^C) + b_S\Delta f\right)\right)^2 \\ &\quad + \frac{(\Delta f)(f^{PC} + b_S - c_M)f^{PC}}{2t} \times \left(\frac{\Delta f}{2} + 1 - f^C\right),\end{aligned}$$

whereas if $f^{PC} > f^C$, merchant 1 makes profit

$$\pi_1^{NC,C} = \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(\Delta f)^2}{2} + (f^{PC} - c_M)(1 - f^{PC}) + (\Delta f + m)(1 - f^C) + b_S\Delta f\right)\right)^2 + \frac{(f^{PC} + b_S - c_M)}{4t}(\Delta f)(1 - f^{PC})(f^C + f^{PC}).$$

We show that merchant 1 makes more profit if $f^{PC} < f^C$. Since $b_S \geq c_M$, we have $f^{PC} + b_S - c_M \geq 0$. If $f^{PC} < f^C$, then $(\Delta f) \geq 0$. So,

$$\frac{(\Delta f)(f^{PC} + b_S - c_M)f^{PC}}{2t} \times \left(\frac{\Delta f}{2} + 1 - f^C\right) \geq 0.$$

If $f^{PC} > f^C$, then $(\Delta f) \leq 0$. So, we have

$$\frac{(f^{PC} + b_S - c_M)}{4t}(\Delta f)(1 - f^{PC})(f^C + f^{PC}) \leq 0.$$

Therefore, merchant 1 makes more profit if he chooses $f^{PC} < f^C$.

Case 3: Merchant 1 is the only one who accepts cards. If $f^{PC} < f^C$, merchant 1 makes profit

$$\pi_1^{C,NC} = \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(1 - f^{PC})^2}{2} + (f^{PC} + b_S - c_M)(1 - f^{PC})\right)\right)^2 + \frac{f^{PC}(f^{PC} + b_S - c_M)(1 - f^{PC})^2}{4t},$$

whereas if $f^{PC} > f^C$, he makes profit

$$\pi_1^{C,NC} = \frac{1}{2t}\left[t + \frac{1}{3}((b_S - m)(1 - f^C) + \frac{(1 - f^C)^2}{2})\right]^2 + \frac{(b_S - m)f^C(1 - f^C)^2}{4t}.$$

Notice that this situation is possible if and only if the non deviation condition in the Benchmark Case is not verified, that is, if we have $m \geq b_S + (1 - f^C)/2$. Therefore, in case 3, if $f^{PC} > f^C$, we have $(b_S - m) \leq 0$. So, $(b_S - m)f^C(1 - f^C)^2/(4t) \leq 0$. Consequently, to prove that merchant 1 makes more profit if he undercuts f^C, it suffices to prove that $C \geq D$, where

$$C = (1 - f^{PC})^2/2 + (f^{PC} + b_S - c_M)(1 - f^{PC}),$$

and $D = (b_S - m)(1 - f^C) + (1 - f^C)^2/2$. Rearranging C, and using that $1 - f^{PC} = 1 - f^C + \Delta f$, we obtain

$$C = \frac{(1 - f^C)^2}{2} + (f^{PC} + b_S - c_M)(1 - f^C) + \frac{(\Delta f)^2}{2} + (f^{PC} + b_S - c_M + 1 - f^C)(\Delta f).$$

Since $(f^{PC}+b_S-c_M)(1-f^C) \geq (b_S-m)(1-f^C)$, and since $\frac{(\Delta f)^2}{2}+(f^{PC}+b_S-c_M+1-f^C)(\Delta f) \geq 0$, we have $C \geq D$. Therefore, merchant 1 makes more profit if he chooses $f^{PC} < f^C$.

Case 4: Both merchants refuse payment cards. This case is not relevant, as both merchants refuse cards (and hence, merchant 1's profit does not depend on whether $f^{PC} < f^C$ or $f^{PC} > f^C$).

To sum up, in all cases, merchant 1 makes more profit if he undercuts f^C by setting $f^{PC} < f^C$.

6.5 Appendix E: Proof of Lemma 5

Assume that merchant 1 sets $f^{PC}=0$. Merchant 2 does not change his decision to accept cards if and only if his profit is higher if it accepts cards than if it does not, that is,

$$\begin{aligned}
&\frac{1}{2t}\left(t+\frac{1}{3}\left(-\frac{(f^C)^2}{2}+c_M-(f^C+m)(1-f^C)-b_Sf^C\right)\right)^2 \\
&+\frac{(m-b_S)(1-f^C)(f^C)^2}{4t} \geq \frac{1}{2t}\left(t+\frac{1}{3}\left(-\frac{1}{2}-(b_S-c_M)\right)\right)^2.
\end{aligned}$$

This condition is equivalent to $g(m,f^C) \geq 0$, where

$$\begin{aligned}
g(m,f^C) \quad = \quad & \left(t+\frac{1}{3}\left(-\frac{(f^C)^2}{2}+c_M-(f^C+m)(1-f^C)-b_Sf^C\right)\right)^2 \\
&+\frac{(m-b_S)(1-f^C)(f^C)^2}{2} \\
&-\left(t+\frac{1}{3}\left(-\frac{1}{2}-(b_S-c_M)\right)\right)^2.
\end{aligned}$$

Condition under which the demand is positive We know that, if both merchants accept cards and $f^{PC}=0$, the demand for card payments at merchant 2's is

$$\begin{aligned}
D_2^C \quad &= \quad (1-f^C)w_2-\frac{1}{2t}(1-f^C)f^C \qquad \text{(E1)}\\
&= \quad \frac{1}{2t}(1-f^C)\left(t+\frac{1}{3}(-(b_S+1+f^C)f^C+c_M-m(1-f^C))\right).
\end{aligned}$$

Therefore, we have $D_2^C \geq 0$ if and only if $m \leq \overline{m}$, where

$$\overline{m}=\frac{3t+c_M-(b_S+1+f^C)f^C}{(1-f^C)}.$$

Existence and characterization of $\widetilde{m}(f^C)$ First, we show that there exists an $\widetilde{m}(f^C)$ such that merchant 2 does not deviate from the equilibrium in which he accepts cards for $m \leq \widetilde{m}(f^C)$. Indeed, note that g is a convex polynomial function of m of degree 2, as $\partial^2 g/\partial m^2 = 2\left(1-f^C\right)^2/9 > 0$. Besides, we have

$$g(\overline{m}, f^C) = \frac{\left(f^C\right)^4}{4} + \frac{(f^C)^2}{2}\left[3t + c_M - b_S - f^C(1+f^C)\right] - \left(\frac{3t - b_S + c_M - 1/2}{3}\right)^2.$$

We are going to show that $g(\overline{m}, f^C) \leq 0$. To prove that, we show that $g(\overline{m}, f^C)$ is increasing with f^C and that $g(\overline{m}, 1) \leq 0$. We have

$$\frac{\partial g(\overline{m}, f^C)}{\partial f^C} = f^C\left[3t - b_S + c_M - \left(f^C\right)^2 - \frac{3}{2}f^C\right].$$

Since $c_M - b_S \geq -1$, and $\left(f^C\right)^2 + \frac{3}{2}f^C \in [0, 5/2]$, to have $\partial g(\overline{m}, f^C)/\partial f^C \geq 0$, it suffices that $t \geq 7/6$, which is true by Assumption 1. Now, replacing for $f^C = 1$ in $g(\overline{m}, f^C)$, we obtain

$$g(\overline{m}, 1) = \frac{-1}{18}\left(6t - 7 + 2c_M - 2b_S\right)\left(3t - 2 - b_S + c_M\right).$$

Assumption 1 implies that both parenthesis are positive, therefore, we have $g(\overline{m}, 1) \leq 0$. Hence, $g(\overline{m}, f^C) \leq 0$.

Now, notice that $b_S + 1 - f^C \leq \overline{m}$. Indeed, this condition is equivalent to

$$3t - b_S + c_M - f^C(1+f^C) - (1-f^C)^2 \geq 0,$$

which is true if $t \geq 4/3$, by Assumption 1. We have

$$g(b_S + \frac{3(1-f^C)}{4}, f^C) = \frac{-(1-f^C)^2}{144}\left(24t - 55\left(f^C\right)^2 - 8\left(b_S - c_M\right) + 5 - 2f^C\right). \tag{E3}$$

We have $\left(f^C\right)^2 \leq 1$, $(b_S - c_M) \in [0,1]$ and $2f^C \leq 2$, therefore, (E3) is negative if $t > 5/2$, which is true by Assumption 1. Finally, we obtain that

$$g(b_S + \frac{1-f^C}{2}, f^C) = \frac{(1-f^C)^2(f^C)^2}{4} \geq 0.$$

This shows that $g\left(m, f^C\right)$ is first positive then negative over $[0, \overline{m}]$, and that it crosses $y = 0$ only once, at $\widetilde{m}(f^C)$. Besides, since $g(b_S + \frac{3(1-f^C)}{4}, f^C) < 0$ and $g(b_S + \frac{1-f^C}{2}, f^C) \geq 0$, we have

$$\widetilde{m}(f^C) \in \left(b_S + \frac{(1-f^C)}{2}; b_S + \frac{3(1-f^C)}{4}\right).$$

6.6 Appendix F: Proof of Proposition 3

The first order conditions of profit maximisation for the Acquirer and the Issuer are

$$\frac{dD_2^C}{dm}(m - a^P - c_A) + D_2^C = 0, \tag{F1}$$

and

$$\frac{dD_2^C}{df^C}(f^C + a^P - c_I) + D_2^C = 0, \tag{F2}$$

respectively. Proposition 4 shows that $f^{PC} = 0$ is a dominant strategy for merchant 1. Therefore, we replace for $f^{PC} = 0$ in (F1) and (F2). We have

$$D_2^C = \frac{1}{2t}(1 - f^C)\left(t + \frac{1}{3}(-(b_S + 1 + f^C)f^C + c_M - m(1 - f^C))\right).$$

We define

$$R = \frac{2t}{(1 - f^C)}D_2^C. \tag{F3}$$

Since $\frac{dD_2^C}{dm} = \frac{-(1-f^C)^2}{6t}$ and $\frac{dD_2^C}{df^C} = \frac{-R}{2t} + \frac{(1-f^C)}{6t} \times (m - 1 - b_S - 2f^C)$, by simplifying (F1) and (F2), we obtain

$$\begin{aligned} \frac{-(1-f^C)}{3}(m - a^P - c_A) + R &= 0, \\ (f^C + a^P - c_I)(-R + \frac{(1-f^C)}{3} \times (m - b_S - 1 - 2f^C)) + (1 - f^C)R &= 0. \end{aligned}$$

Before solving for the equilibrium, we start by showing that the Issuer's and the Acquirer's profit functions are concave.

6.6.1 Appendix F1: Concavity of profit functions

Writing the second derivative of Π_A with respect to m, we obtain

$$\frac{\partial^2 \Pi_A}{\partial^2 m} = \frac{-(1-f^C)^2}{3t} < 0,$$

so the second order condition for the Acquirer is verified.

Writing the second derivative of Π_I with respect to f^C, we obtain

$$\begin{aligned} \frac{\partial^2 \Pi_I}{\partial^2 f^C} = & -1 - \frac{1}{3t}\left[(f^C + a^P - c_I)(m - b_S - 3f^C) - (b_S + 1 + f^C)f^C\right] \\ & + \frac{1}{3t}\left[-c_M + (1 - f^C)(2m - b_S - 1 - 2f^C)\right]. \end{aligned}$$

The third derivative of Π_I with respect to f^C is given by

$$\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, f^C) = \frac{1}{t}\left(4f^C + a^P - c_I - m + b_S\right).$$

Replacing for $f^C = 0$ yields

$$\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, 0) = \frac{1}{t}\left(a^P - c_I - m + b_S\right).$$

Since the Acquirer's profit must be positive, we have $m - a^P - c_A \geq 0$. So

$$a^P - c_I - m + b_S \leq b_S - c_I - c_A.$$

Since $b_S - c_I - c_A < 0$ by assumption, we have

$$\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, 0) < 0.$$

Replacing for $f^C = 1$ yields

$$\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, 1) = \frac{1}{t}\left(4 + a^P - c_I - m + b_S\right) = \frac{1}{t}(3 + a^P - c_I + 1 + b_S - m).$$

In Appendix C, we proved that $\widetilde{m}(f^C) \leq b_S + 3\left(1 - f^C\right)/4$, so $m \leq \widetilde{m}(f^C) \leq 1 + b_S$. Since the margin of the Issuer, $f^C + a^P - c_I$, must be positive, and $f^C \in [0, 1]$, we have $3 + a^P - c_I \geq 0$. So

$$\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, 1) > 0.$$

Therefore, as $\frac{\partial^3 \Pi_I}{\partial^3 f^C}$ is increasing with f^C, there exists a unique $\widetilde{f}^C \in (0; 1)$ such that $\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, f^C) > 0$ if $f^C > \widetilde{f}^C$ and $\frac{\partial^3 \Pi_I}{\partial^3 f^C}(m, f^C) \leq 0$ otherwise. To show that $\frac{\partial^2 \Pi_I}{\partial^2 f^C}(m, f^C) \leq 0$, it suffices to prove that $\frac{\partial^2 \Pi_I}{\partial^2 f^C}(m, 0) \leq 0$, and that $\frac{\partial^2 \Pi_I}{\partial^2 f^C}(m, 1) \leq 0$. Replacing for $f^C = 0$ yields

$$\frac{\partial^2 \Pi_I}{\partial^2 f^C}(m, 0) = \frac{-1}{3t}(3t + (m - b_S)(a^P - c_I) + 1 + b_S - m + c_M - m).$$

We know from Appendix C that $1 + b_S - m > 0$. We now show that $\left|a^P - c_I\right| \leq 1$. Since the Issuer's margin is positive, $1 + a^P - c_I \geq f^C + a^P - c_I \geq 0$. So $-1 \leq a^P - c_I$. Since the Acquirer's margin is positive, we have $a^P - c_I \leq m - c_A - c_I$. Hence, $a^P - c_I \leq 1 + b_S - c_A - c_I$. By assumption, $b_S - c_A - c_I \leq 0$. Therefore, $-1 \leq a^P - c_I \leq 1$. Since $\left|a^P - c_I\right| \leq 1$ and $|m - b_S| \leq 1$ then $(m - b_S)(a^P - c_I) \geq -1$. We also know that $-m \geq -b_S - 1 \geq -2$.

Therefore, to prove that $\partial^2\Pi_I/\partial^2 f^C(m, 0, f^{PC}) \leq 0$, it suffices that $3t - 3 \geq 0$, which is equivalent to $t \geq 1$. This is true by Assumption 1. Replacing for $f^C = 1$ yields

$$\frac{\partial^2\Pi_I}{\partial^2 f^C}(m, 1) = \frac{-1}{3t}(3t + m(1 + a^P - c_I) - (3 + b_S)(1 + a^P - c_I) - (b_S - c_M) - 2)).$$

Since $3 + b_S \leq 4$, and $1 + a^P - c_I \leq 2$, we have $-(3 + b_S)(1 + a^P - c_I) \geq -8$. We also have $-(b_S - c_M) \geq -1$. Therefore, to show that $\dfrac{\partial^2\Pi_I}{\partial^2 f^C}(m, 1) \leq 0$, it suffices that $3t - 11 \geq 0$, which is equivalent to $t \geq \dfrac{11}{3}$. This is true by Assumption 1.

To sum up, by Assumption 1, Π_I and Π_A are concave with respect to f^C and m, respectively.

6.6.2 Appendix F2: The best response of the Issuer is strictly positive.

We have that

$$\left.\frac{\partial\Pi_I}{\partial f^C}\right|_{f^C=0} = \frac{1}{2t}\left[\left(1 + a^P - c_I\right)\left(t + \frac{1}{3}(c_M - b_S)\right) + \frac{(b_S + 1 - m)}{3}\right].$$

Since $\widetilde{m}(f^C) \leq b_S + \dfrac{3}{4}$, then $b_S + 1 - m > 0$. Since the margin of the Issuer must be positive, we also know that $1 + a^P - c_I \geq 0$. Since, by Assumption 1, t is sufficiently high such that $t + (c_M - b_S)/3 \geq 0$, we can conclude that

$$\left.\frac{\partial\Pi_I}{\partial f^C}\right|_{f^C=0} > 0.$$

Therefore, the best response of the Issuer is strictly positive.

6.6.3 Appendix F3: The Acquirer chooses the maximum merchant fee compatible with merchant acceptance.

Assume that the constraint $m \leq \widetilde{m}(f^C)$ is not binding. The best response of the Acquirer is to play m^{BR} which satisfies to the first order condition, that is

$$\frac{-(1 - f^C)}{3}(m^{BR} - a^P - c_A) + R = 0,$$

where R is defined in (F3). Rearranging the first order condition, we get

$$\frac{-2(1 - f^C)m^{BR}}{3} + t + \frac{1}{3}(-(b_S + 1 + f^C)f^C + c_M + (1 - f^C)(a^P + c_A)) = 0.$$

So, we have $m^{BR}(f^C) = \left(a^P + c_A\right)/2 + y(f^C)$, where

$$y(f^C) = \frac{3}{2(1-f^C)}\left(t + \frac{1}{3}(-(b_S + 1 + f^C)f^C + c_M\right).$$

To show that the constraint is binding if the Acquirer plays its best response, it is sufficient to prove that for $m = y(f^C)$, the non deviation condition is not verified, that is, $y(f^C) > \widetilde{m}(f^C)$ (since $m^{BR}(f^C) > y(f^C)$). A simple way of showing that the non deviation condition is violated for $m = y(f^C)$ is to prove that $y(f^C) > b_S + 1 - f^C$, as we know that $b_S + 1 - f^C \geq \widetilde{m}$. We have

$$y - b_S = \frac{3}{2(1-f^C)}\left(t + \frac{1}{3}(-(b_S - c_M) - (1 + f^C)f^C - b_S(1 - f_C)\right).$$

To show that $y(f^C) - b_S > 1 - f^C$, it is equivalent to prove that $T \equiv (y - b_S)(1-f^C) - (1-f^C)^2 > 0$. We have

$$T = \frac{3}{2}t - \frac{1}{2}\left(b_S(1-f^C) + (b_S - c_M) + 2(f^C)^2 + (1-f^C)(2-f^C)\right).$$

Since $b_S(1-f^C) < 1$, $(b_S - c_M) < 1$, $2(f^C)^2 < 2$, and $(1-f^C)(2-f^C) < 2$, we have $T > 3t/2 - 3$. To have $T > 0$, it suffices that $t > 2$. So if Assumption 1 holds, the Acquirer chooses the maximum merchant fee compatible with merchant acceptance.

6.6.4 Appendix F4: The equilibrium

We can now solve for the equilibrium. We start by showing that two lemmas.

Lemma 7 *$\widetilde{m}(f^C)$ is decreasing with f^C.*

Proof. The function $\widetilde{m}(f^C)$ is defined implicitly by the non deviation condition. Using the implicit function theorem, we obtain

$$\frac{\partial \widetilde{m}(f^C)}{\partial f^C} = -\left(\left.\frac{\partial g}{\partial m}\right|_{m=\widetilde{m}}\right)^{-1} \times \left.\frac{\partial g}{\partial f^C}\right|_{m=\widetilde{m}}.$$

Since g is decreasing with m over $[0, \widetilde{m}]$, the sign of $\frac{\partial \widetilde{m}(f^C)}{\partial f^C}$ has the same as $\left.\frac{\partial g}{\partial f^C}\right|_{m=\widetilde{m}}$. Taking the derivative of g with respect to f^C, we obtain

$$\left.\frac{\partial g}{\partial f^C}\right|_{m=\widetilde{m}} = \frac{2Y}{3}(\widetilde{m}(f^C) - (b_S + 1 - f^C)) + \frac{\widetilde{m}(f^C) - b_S}{2}\left(-3(f^C)^2 + 2f^C\right),$$

where

$$Y = t - \frac{1}{3}\left[\frac{1}{2} + (b_S - c_M) - (1-f^C)(\frac{1-f^C}{2} + b_S - \widetilde{m}(f^C))\right].$$

We now show that $\left.\frac{\partial g}{\partial f^C}\right|_{m=\widetilde{m}} < 0$. First, we have $Y \geq 0$ by Assumption 1.

Indeed, since $\widetilde{m}(f^C) - \frac{3}{4}(1-f^C) < 0$, we have $\frac{(1-f^C)}{2} - \widetilde{m}(f^C) > \frac{-(1-f^C)}{4}$. Hence, $\frac{(1-f^C)}{3}\left(\frac{(1-f^C)}{2} - \widetilde{m}(f^C)\right) > \frac{-(1-f^C)^2}{12} \geq \frac{-1}{12}$. Since $\frac{1}{2} + (b_S - c_M) < \frac{3}{2}$, if $t - \frac{7}{12} \geq 0$, then $Y \geq 0$. Therefore, it suffices that $t \geq \frac{7}{12}$, which is true by Assumption 1.

Besides, we have $\widetilde{m}(f^C) - (b_S + 1 - f^C) \leq 0$, so $\left.\frac{\partial g}{\partial f^C}\right|_{m=\widetilde{m}} < 0$ if and only if

$$Y \geq \frac{3}{4}\frac{\widetilde{m}(f^C) - b_S}{1 - f^C + b_S - \widetilde{m}(f^C)}\left(-3(f^C)^2 + 2f^C\right). \tag{F4-1}$$

We have $1 - f^C + b_S - \widetilde{m}(f^C) \leq b_S - \widetilde{m}(f^C)$ as $\widetilde{m}(f^C) - b_S \geq \left(1 - f^C\right)/2$. Therefore, a sufficient condition for (F4-1) to hold is

$$Y \geq \frac{3}{4}\left(-3f^C + 2\right)f^C. \tag{F 4-2}$$

We have $\left(-3f^C + 2\right)f^C \leq 1/3$, so (F 4-2) is equivalent to $Y \geq 1/4$, that is, $t \geq 5/6$, which is true by Assumption 1. If this condition holds, then $\widetilde{m}(f^C)$ is decreasing with f^C. ■

Lemma 8 *$(f^C)^{BR}$ is increasing with m.*

Proof. The function $(f^C)^{BR}$ is defined implicitly by the first order condition of the maximisation of the Issuer's profit. Using the implicit function theorem, we obtain

$$\frac{\partial (f^C)^{BR}}{\partial m} = -\left(\left.\frac{\partial^2 \Pi_I}{\partial^2 f^C}\right|_{f^C=(f^C)^{BR}}\right)^{-1} \times \left.\frac{\partial^2 \Pi_I}{\partial f^C \partial m}\right|_{f^C=(f^C)^{BR}}.$$

Since we have shown that the second order condition is verified, the sign of $\frac{\partial (f^C)^{BR}}{\partial m}$ is the same as the sign of $\left.\frac{\partial^2 \Pi_I}{\partial f^C \partial m}\right|_{f^C=(f^C)^{BR}}$. Taking the derivative of the first order condition with respect to m, we obtain

$$\left.\frac{\partial^2 \Pi_I}{\partial f^C \partial m}\right|_{f^C=(f^C)^{BR}} = \frac{(1-(f^C)^{BR})}{6t}\left[3(f^C)^{BR} + 2(a^P - c_I) - 1\right].$$

So, if $(f^C)^{BR} \leq \frac{1 + 2c_I - 2a^P}{3}$, then $\frac{\partial (f^C)^{BR}}{\partial m} \leq 0$, and $\frac{\partial (f^C)^{BR}}{\partial m} > 0$ otherwise. We are going to show that $(f^C)^{BR} > \frac{1 + 2c_I - 2a^P}{3}$, which will prove that $\frac{\partial (f^C)^{BR}}{\partial m} > 0$. To do so, we replace for $f^C = \dfrac{1+2c_I-2a^P}{3}$ in the first order condition. Since π_I is concave, if

$$\frac{\partial \Pi_I}{\partial f^C}(m, \frac{1+2c_I-2a^P}{3}) > 0$$

then we know that $(f^C)^{BR} > \left(1+2c_I-2a^P\right)/3$.

We have

$$\frac{\partial \Pi_I}{\partial f^C}(m, \frac{1+2c_I-2a^P}{3}) = \frac{\left(1-c_I+a^P\right)H}{162t}, \tag{F 4-3}$$

where

$$H = 27t - 9\left(b_S - c_M\right) - 14 + 8a\left(1-c_I\right) + 4\left(a^2 + (c_I)^2\right) - 8c_I.$$

We have $1 - c_I + a^P \geq f^C - c_I + a^P \geq 0$, therefore, (F 4-3) is positive if and only if $H \geq 0$. Since $0 \leq 9\left(b_S - c_M\right) \leq 9$, and $8c_I \leq 8$, then a sufficient condition for $H \geq 0$ is $t \geq 31/27$, which is true by Assumption 1. ■

Define $\widetilde{f}$ such that $\widetilde{m}(\widetilde{f}) = 0$. Then we want to prove that $\widetilde{f} > f^*(m=0)$. The card fee $\widetilde{f}$ is defined by the non deviation condition, in which $\widetilde{m}(\widetilde{f}) = 0$, that is

$$\begin{aligned}\left(t + \frac{1}{3}\left(-\frac{1}{2} - b_S + c_M\right)\right)^2 \quad = \quad & \left(t + \frac{1}{3}\left(-\frac{1}{2} - b_S + c_M + b_S(1-\widetilde{f}) - \widetilde{f}(1-\widetilde{f}) - \frac{(\widetilde{f})^2}{2}\right)\right)^2 \\ & - \left[b_S(1-\widetilde{f})(\widetilde{f})^2\right]/2.\end{aligned}$$

This equation can be rewritten as

$$(1-\widetilde{f})\left\{\left[\frac{2}{3}Z + \frac{1}{9} + \left(\frac{1-\widetilde{f}}{9}\right)\left(\frac{1-\widetilde{f}}{2} + b_S\right)\right]\left(\frac{1-\widetilde{f}}{2} + b_S\right) - b_S\frac{\left(\widetilde{f}\right)^2}{4}\right\} = 0,$$

where $Z = t + \frac{1}{3}\left(-\frac{1}{2} - b_S + c_M\right)$. It can be shown that the terms in the second parenthesis are strictly positive. Hence, the only solution of this equation is obtained for $\widetilde{f} = 1$. Therefore, $\widetilde{f} \geq f^*(m=0)$. Therefore, we have shown that there is a unique equilibrium such that: $\left(f^{PC}\right)^* = 0$; $\left(f^C\right)^* \in (0,1)$; $m^* = \widetilde{m}$.

6.7 Appendix G: Proof of Proposition 4

We already proved that $\widetilde{m}(f^C) > b_S + \left(1 - f^C\right)/2$, which shows that for a given f^C, the Acquirer's best response is to choose a higher merchant fee than in the benchmark case. We now compare $(f^C)^{BR}$ with the best response of the Issuer in the benchmark case, that is

$f^C = \left(1 + c_I - a^P\right)/2$. We have

$$\frac{\partial \Pi_I}{\partial f^C}(m, \frac{1 + c_I - a^P}{2}) = \frac{\left(1 - c_I + a^P\right)^2}{24t}\left(m - b_S - 1 - (1 + c_I - a^P)\right).$$

Since $m - b_S - 1 < 0$, and $1 + c_I - a^P > 0$, we have

$$\frac{\partial \Pi_I}{\partial f^C}(m, \frac{1 + c_I - a^P}{2}) < 0.$$

Since π_I is concave, this proves that $(f^C)^{BR} < \dfrac{1 + c_I - a^P}{2}$. So, for a given m, the Issuer chooses a lower transaction fee than in the benchmark case.

To sum up, for a given f^C, the Acquirer's best response is to choose a higher m than in the benchmark case. Besides, for a given m, the Issuer's best response is to set a lower f^C than in the benchmark case, so the equilibrium merchant fee is higher than in the benchmark case, while the card fee is lower.

6.8 Appendix H: Proof of Lemma 6

We start by showing that $(f^C)^{BR}$ is increasing with a^P. The function $(f^C)^{BR}(m, a^P)$ is defined implicitly by the first order condition of the maximisation of the Issuer's profit. Using the implicit function theorem, we obtain

$$\frac{\partial (f^C)^{BR}(m, a^P)}{\partial a^P} = -\left(\frac{\partial^2 \Pi_I}{\partial^2 f^C}(m, f^C, a^P)\right)^{-1} \frac{\partial^2 \Pi_I}{\partial f^C \partial a^P}(m, (f^C)^{BR}, a^P).$$

Since we have shown in Appendix D1 that π_I is concave, the sign of $\dfrac{\partial (f^C)^{BR}(m, a^P)}{\partial a^P}$ is the same as the sign of $\dfrac{\partial^2 \Pi_I}{\partial f^C \partial a^P}(m, (f^C)^{BR}, a^P)$. Taking the derivative of the first order condition with respect to a^P, we obtain

$$\frac{\partial^2 \Pi_I}{\partial f^C \partial a^P}(m, (f^C)^{BR}, a^P) = \frac{1}{2t}\left[-R + \frac{(1 - (f^C)^{BR})}{3}(m - (b_S + 1 + 2(f^C)^{BR}))\right],$$

where R is given by (F3). Since $m < 1 - f^C + b_S$, we know that $m - (b_S + 1 + 2(f^C)^{BR}) < 0$. Since $R \geq 0$, then it follows that $-R + \dfrac{(1 - (f^C)^{BR})}{3}(m - (b_S + 1 + 2(f^C)^{BR})) < 0$. This shows that

$$\frac{\partial^2 \Pi_I}{\partial f^C \partial a^P}(m, (f^C)^{BR}, a^P) < 0.$$

This proves that $(f^C)^{BR}$ is decreasing with a^P. We also know that $(f^{PC})^{BR}(f^C, m)$ does not depend on a^P as it is equal to 0. Besides, $\widetilde{m}(f^C)$ does not depend on a^P either, as the non deviation condition does not depend on a^P (see the expression of g in Appendix C). So, if a^P increases, the best response of the Acquirer remains unchanged, while the best response of the Issuer decreases. As shown in Lemma 7, $\widetilde{m}(f^C)$ is decreasing with f^C, which proves that $(f^C)^*$ is lower and that m^* is higher if the interchange fee is higher.

6.9 Appendix I: Entry condition

6.9.1 Entry condition

Merchant 1 enters the market if and only if he makes higher profit with the private card at the equilibrium of stage 3 than in the benchmark case. This condition is obtained by replacing for $f^{PC} = 0$, $(f^C)^*$ and m^* in π_1 (case L-1), that is $\pi_1^{C,C}(m^*, (f^C)^*, 0) - F \geq t/2$, which is equivalent to

$$\left(t + \frac{1}{3}\left(-c_M + \left((f^C)^* + m^*\right)\left(1 - (f^C)^*\right) + \frac{((f^C)^*)^2}{2} + b_S(f^C)^*\right)\right)^2 - 2tF \geq t^2.$$

6.10 Appendix J: Proof of Lemma 5

We start by rearranging $(EC)'(a^P)$. Solving for $\widetilde{m}(f^C)$ (see Appendix E), we find that $\widetilde{m}(f^C) = b_S + \left(1 - f^C\right)/2 - U(f^C)$, where

$$U(f^C) = \frac{(-2Q + \frac{3}{2}(f^C)^2) + \sqrt{D}}{2(1 - f^C)/3},$$

$$D = (2Q - \frac{3}{2}(f^C)^2)^2 - (1 - f^C)^2(f^C)^2,$$

and $Q = t + \left(-1/2 - b_S + c_M\right)/3$. Since $\widetilde{m}(f^C) \geq b_S + \left(1 - f^C\right)/2$, we have $U(f^C) \leq 0$.

We have $m^* = \widetilde{m}(\left(f^C\right)^*)$ and $\partial\widetilde{m}/\partial a^P = 0$, therefore,

$$\frac{dm^*}{da^P} = \left.\frac{d\widetilde{m}(f^C)}{df^C}\right|_{f^C=(f^C)^*} \times \frac{d\left(f^C\right)^*}{da^P}.$$

Replacing for this expression in $(EC)'(a^P)$ and replacing for $\widetilde{m}(\left(f^C\right)^*)$, we find that

$$(EC)'(a^P) = \frac{2}{3}\Psi \times \left[(b_S + 1 - (f^C)^* - \widetilde{m}(\left(f^C\right)^*)) + (1 - (f^C)^*)\left.\frac{d\widetilde{m}(f^C)}{df^C}\right|_{f^C=(f^C)^*}\right]\frac{d(f^C)^*(a^P)}{da^P}.$$

We know that $\Psi \geq 0$ and $d(f^C)^*/da^P \leq 0$, therefore, we have $(EC)'(a^P) \leq 0$ if and only if the term into brackets is positive. Replacing for $\widetilde{m}(\left(f^C\right)^* = b_S + \left(1 - (f^C)^*\right)/2 - U((f^C)^*)$, we find that $(EC)'(a^P) \leq 0$ if and only if

$$(1-f^C)\left(\frac{1}{2}+\left.\frac{d\widetilde{m}(f^C)}{df^C}\right|_{f^C=(f^C)^*}\right)+U((f^C)^*)\geq 0.$$

Since

$$\left.\frac{d\widetilde{m}(f^C)}{df^C}\right|_{f^C=(f^C)^*}=-\frac{1}{2}-\left.\frac{dU(f^C)}{df^C}\right|_{f^C=(f^C)^*},$$

we have that $(EC)'(a^P) \leq 0$ if and only if

$$(1-(f^C)^*)\left.\frac{dU(f^C)}{df^C}\right|_{f^C=(f^C)^*}-U((f^C)^*)\leq 0. \tag{J1}$$

We have

$$(1-f^C)\frac{dU(f^C)}{df^C}-U(f^C)=\frac{9}{2}f^C+\frac{3}{4}D^{-1/2}\frac{dD}{df^C},$$

where

$$\frac{dD}{df^C}=-f^C\left(12Q-5\left(f^C\right)^2-6f^C+2\right).$$

Replacing for Q, it can be shown that $2Q-5\left(f^C\right)^2-6f^C+2\geq 0$ for $t\geq 5/4$, which is always true by assumption 1. Hence, $dD/df^C \leq 0$. It follows that (J1) holds, hence $(EC)'(a^P)\leq 0$, if and only if

$$12\left(1-2\left(f^C\right)^*\right)Q+\left(5\left(\left(f^C\right)^*\right)^3+8\left(\left(f^C\right)^*\right)^2-5\left(f^C\right)^*+1\right)\geq 0. \tag{J2}$$

We have $5\left(f^C\right)^3+8\left(f^C\right)^2-5f^C+1\geq 0$ for any $f^C\in[0,1]$. If $\left(f^C\right)^*\leq 1/2$, the first term in (J2) is also positive, hence (J2) holds always. Therefore, if $\left(f^C\right)^*\leq 1/2$, EC is decreasing in a^P. If $\left(f^C\right)^*>1/2$, then (J2) is the sum of a negative term and a positive term. Condition (J2) holds if and only if $Q\leq\widetilde{Q}\left(\left(f^C\right)^*\right)$, where

$$\widetilde{Q}\left(f^C\right)=\frac{\left(5\left(f^C\right)^3+8\left(f^C\right)^2-5f^C+1\right)}{12\left(2f^C-1\right)}.$$

Therefore, $(EC)'(a^P)\geq 0$ if $Q\geq\widetilde{Q}\left(\left(f^C\right)^*\right)$. Since $t>11/3$ by Assumption 1 and $b_S-c_M\in[0,1]$, then we have $Q\geq 19/6$. We find that $\widetilde{Q}\left(f^C\right)>19/6$ for $f^C\in(0.5,0.516)$ and $\widetilde{Q}\left(f^C\right)\leq 19/6$ for $f^C\in(0.516,1]$. Hence, if $\left(f^C\right)^*\in(0.516,1]$, we have $Q\geq\widetilde{Q}\left(\left(f^C\right)^*\right)$, therefore, EC increasing with a^P. If $\left(f^C\right)^*\in(0.5,0.516)$, we have EC increasing with a^P if t is sufficiently high, and EC decreasing with a^P otherwise.

6.11 Appendix K: the impact of the market structure on entry

If there is perfect competition on both sides of the market, both banks charge fees that are equal to their perceived marginal cost, that is $(f^C)^* = a^P - c_I$, and $m^* = a^P + c_A$. If merchant 1 decides to issue private cards, he makes a profit (gross of the entry cost) of

$$\pi_1^{C,C}(m^*, (f^C)^*, 0) = \frac{1}{2t}\left(t + \frac{1}{3}\left(\frac{(c_I - a^P)^2}{2} - c_M + c(1 + a^P - c_I) + b_S(c_I - a^P)\right)\right)^2.$$

We now determine the level of the interchange fee that minimises the profit of merchant 1. Differentiating $\pi_1^{C,C}$ with respect to a^P yields

$$\frac{\partial \pi_1^{C,C}}{\partial a^P} = \left(t + \frac{1}{3}\left(\frac{(c_I - a^P)^2}{2} - c_M + c(1 + a^P - c_I) + b_S(c_I - a^P)\right)\right)\frac{(a^P + c_A - b_S)}{3t}.$$

Our assumption on t ensures that the term in the first parenthesis is positive, therefore $\pi_1^{C,C}$ is minimal for $a^P = b_S - c_A$. This means that, if the payment platform chooses Baxter's interchange fee, the probability that merchant 1 enters the market is minimized.

6.12 Appendix L: the reaction of merchant 2.

If merchant 2 issues private cards, he never chooses a price for the private card that is higher than the card fee, otherwise, the private card is never used by the consumers. After elimination of the dominated strategies, we find that there are two cases in which both merchants issue private cards that are used by consumers

- Case a: $f_1^{PC} \leq f_2^{PC} \leq f^C$
- Case b: $f_2^{PC} \leq f_1^{PC} \leq f^C$.

We start by studying Case a. The second case can be solved in a similar way by symmetry. We assume that, if a consumer is indifferent between the payment card and the private card, he chooses to use the private card. Using the same method as in Section 3.3.1, we determine the demand for merchant 1's and the demand for merchant 2's. Consumers trade off between shopping at merchant 1's and paying cash or with its private card, and shopping at merchant

2's and paying cash or with merchant 2's private card. We find that

$$\begin{aligned}
D_1^{PC} &= w_1(1-f_1^{PC}) + \frac{(f_2^{PC}-f_1^{PC})^2}{4t} + \frac{(f_2^{PC}-f_1^{PC})(1-f_2^{PC})}{2t}, \\
D_1^{Cash} &= f_1^{PC} w_1, \\
D_2^{PC} &= w_2(1-f_2^{PC}) - \frac{(f_2^{PC}-f_1^{PC})(1-f_2^{PC})}{2t}, \\
D_2^{Cash} &= f_2^{PC} w_2 - \frac{(f_2^{PC}-f_1^{PC})^2}{4t}.
\end{aligned}$$

Since the price of the private card is lower at merchant 1's, the latter attracts cash users from merchant 2's (the second term of D_1^{PC}) and private card users from merchant 2's (the third term in D_1^{PC}). At stage 4, each merchant chooses the price that maximises its profit, that is

$$\pi_i^{PC} = (f_i^{PC} + b_S - c_M) D_i^{PC} + (p_i - c)(D_i^{PC} + D_i^{Cash}).$$

We find that at the equilibrium of stage 4, the prices are

$$\begin{aligned}
p_1 &= t + c + \frac{1}{3}\left[-2(f_1^{PC} + b_S - c_M)(1-f_1^{PC}) - (f_2^{PC} + b_S - c_M)(1-f_2^{PC})\right] \\
&\quad + \frac{1}{3}\left[\frac{(f_2^{PC}-f_1^{PC})^2}{2} + (f_2^{PC}-f_1^{PC})(1-f_2^{PC})\right],
\end{aligned}$$

and

$$\begin{aligned}
p_2 &= t + c + \frac{1}{3}\left[-(f_1^{PC} + b_S - c_M)(1-f_1^{PC}) - 2(f_2^{PC} + b_S - c_M)(1-f_2^{PC})\right] \\
&\quad + \frac{1}{3}\left[\frac{-(f_1^{PC}-f_2^{PC})^2}{2} + (f_1^{PC}-f_2^{PC})(1-f_2^{PC})\right].
\end{aligned}$$

Merchant 1 makes profit

$$\begin{aligned}
\pi_1^{PC} &= 2t\left[\frac{1}{2} + \frac{1}{6t}((b_S - c_M)(f_2^{PC} - f_1^{PC}) + \frac{(f_2^{PC})^2 - (f_1^{PC})^2}{2}\right]^2 \\
&\quad + f_1^{PC}(f_1^{PC} + b_S - c_M)\left[\frac{(f_2^{PC}-f_1^{PC})^2}{4t} + \frac{(f_2^{PC}-f_1^{PC})(1-f_2^{PC})}{2t}\right] - F,
\end{aligned}$$

whereas merchant 2 makes profit

$$\begin{aligned}
\pi_2^{PC} &= 2t\left[\frac{1}{2} + \frac{1}{6t}((b_S - c_M)(f_1^{PC} - f_2^{PC}) + \frac{(f_1^{PC})^2 - (f_2^{PC})^2}{2}\right]^2 \\
&\quad - (f_2^{PC} + b_S - c_M)(1-f_2^{PC})(\left(f_2^{PC}\right)^2 - \left(f_1^{PC}\right)^2)/(4t) - F_2.
\end{aligned}$$

If the merchants choose the same price for the private card, merchant 2 makes profit $\pi_2^{PC} = t/2 - F_2$. Since $f_1^{PC} - f_2^{PC} \leq 0$, we have $\pi_2^{PC} \leq t/2 - F_2$. Hence, if merchant 1 chooses $f_1^{PC} \in [0;1]$, the strategy to choose $f_2^{PC} > f_1^{PC}$ is dominated for merchant 2. We now show that merchant 2 has an incentive to undercut the price that is chosen by merchant 1. If merchant 2 chooses a price that is equal to $\left(f_1^{PC}\right) - \varepsilon$, for $\varepsilon > 0$ small, he makes profit

$$\begin{aligned}\pi_2^{PC} &= 2t\left[\frac{1}{2} + \frac{\varepsilon}{6t}((b_S - c_M) + f_1^{PC} - \varepsilon/2)\right]^2 \\ &\quad + (f_1^{PC} - \varepsilon)(f_1^{PC} - \varepsilon + b_S - c_M)\left[\frac{\varepsilon^2}{4t} + \frac{\varepsilon(1 - f_1^{PC})}{2t}\right] - F_2.\end{aligned}$$

For $\varepsilon > 0$ sufficiently small and $f_1^{PC} > 0$, this profit is strictly higher than $t/2 - F_2$, as $b_S - c_M \geq 0$. Hence, merchant 2 has an incentive to undercut the price chosen by merchant 1. By symmetry of case a and b, it can also be shown that merchant 1 has an incentive to undercut the price that is chosen by merchant 2. Hence, at the equilibrium of Stage 4, the best response of each merchant i is to set $f_i^{PC} = 0$. Merchant 1 then makes profit $\pi_1^{PC} = t/2 - F$, while merchant 2 makes profit $\pi_2^{PC} = t/2 - F_2$. To determine if merchant 2 issues private cards at stage 3, we compare its profit in the two cases. We find that merchant 2 does not issue private cards if $\pi_2^{C,C} \geq \pi_2^{PC}$. Hence, merchant 2 does not react to merchant 1's decision if

$$F_2 \geq k((f^C)^*((a^P)^{PC}),$$

where,

$$\begin{aligned}k(f^C) &= \frac{t}{2} - \frac{1}{4t}(\frac{1 - f^C}{2} - U(f^C))(1 - f^C)(f^C)^2 \\ &\quad - \frac{1}{2t}\left[t + \frac{1}{3}(-\frac{(f^C)^2}{2} + c_M - (b_S + \frac{1 + f^C}{2} - U(f^C))(1 - f^C) - b_S f^C\right]^2.\end{aligned}$$