

Preliminary and Incomplete

Draft: August 2007

An Estimated Open-Economy General Equilibrium Model with Housing Investment and Financial Frictions

by

Ian Christensen, Paul Corrigan, Caterina Mendicino and Shin-Ichi Nishiyama

Department of Monetary and Financial Analysis

Bank of Canada

Ottawa, Ontario, Canada K1A 0G9

Contact address: Monetary and Financial Analysis, Bank of Canada.

IChristensen@Bank-Banque-Canada.ca; cori@bankofcanada.ca; menc@bankofcanada.ca;
nish@bankofcanada.ca;

The views expressed in this paper are those of the authors.
No responsibility for them should be attributed to the Bank of Canada.

1. Introduction

The role of financial arrangements in economic fluctuations has been a central issue in a number of academic and policy debates over the last 20 years. This has led to a growing literature that aims at quantifying the importance of financial frictions for business cycle fluctuations. Most of the literature focuses on credit frictions affecting either firms or households in isolation¹. Yet, empirical evidence from micro studies suggests that both households and firms face credit constraints. The coexistence of financial frictions for both firms and households might be important to assess the quantitative importance of financial accelerator effects if the two types of borrowers seek financing at similar times during the business cycle.

The purpose of this paper is to quantify the role of credit frictions over the business cycle for the Canadian economy. To this end we develop and estimate a Dynamic Stochastic General Equilibrium (DSGE) model with borrowing constraints affecting both consumption and investment behavior. We also estimate a version of this model without the borrowing constraints and compare their performance. The objective of this comparison is to assess whether the borrowing constraints improve the model's ability to capture key features of the data.

In order to study the relation between collateralized debt and macroeconomic fluctuations, a recent strand of the business cycle literature has included into an otherwise standard business cycle framework, the borrowers-lenders setup developed by Kiyotaki and Moore (1997). Accordingly, credit flows are introduced into the model economy through the assumption of heterogeneity in terms of discount factors and agents' borrowing is tied to the value of their collateral asset. Iacoviello (2005) develops and estimates a monetary business cycle model in which both households and firms face a collateral constraint tied to the value of their real estate holdings. The main goal of his paper is to match the time series correlation between real spending and house prices. His findings suggest that borrowing constraints play a role in replicating the dynamics of private spending to both a housing price and inflation shock. Campbell and Hercowitz (2006) highlight the importance of mortgage market institutional features in affecting macroeconomic volatility. More recently, Iacoviello and Neri (2007) quantify the impact of the housing market for the US business cycle. Mendicino and Pescatori (2007) document the role of collateralized debt and heterogeneous agents for the optimal design of monetary policy rules.

¹See, for example, Carlstrom and Fuerst (1997), Bernanke et al. (1999), Aoki et al. (2004), Meier and Muller (2006) and Christensen and Dib (2007), Iacoviello and Neri (2007).

Following Iacoviello (2005) we assume borrowing constraints both at the household and firm level and ex-ante heterogeneity in households' discount factors. Our model extends Iacoviello (2005) model along 3 dimensions. First, we introduce open-economy features into this closed economy framework to make the model a better characterization of the Canadian economy. This extension allows for foreign savers to supply funds to the domestic economy, which affects the response of interest rates and asset prices to shocks.² Ambler et al. (2004), Bergin (2003) and Ortega and Rebei (2006), among others, have estimated open economy models using Canadian data, but their models do not include a role for collateralized debt in the domestic economy. Second, we relax the assumption of a fixed housing stock, allowing for investment in structures. Allowing the stock of housing to fluctuate reduces the volatility of the price of housing, potentially affecting the role of collateralized debt in business cycle fluctuations. Third, we alter the firm's borrowing constraint to allow them to borrow against the value of their capital holdings in addition to their real estate holdings. In fact, although real estate is the most important source of collateral for households, it is not clear that this is the case for firms. These features add realistic elements to the model that are likely to be important to assess the importance of financial frictions for an open economy such as Canada.

We estimate the model with Canadian data using Bayesian methods. Our findings suggest that collateralized debt plays a significant role in generating a positive response of consumption to an increase in housing prices. Standard model evaluation criteria, based on statistics calculated from the full sample, suggest that the model with a financial accelerator mechanism is not much different from a version of the model without the financial accelerator. Nevertheless, the model attributes an important role to housing demand shocks for the strength of consumption during the housing boom of the late 1980s. This finding suggests that a full evaluation of the importance of the financial accelerator for understanding features of the data should also consider episodes in history such as housing booms or severe recessions.

We present the details of the model in section 2. Section 3 outlines the estimation strategy and the data. Section 4 describes the empirical results. In Section 5 we conduct some experiments to highlight the importance of the financial accelerator mechanisms in the model. In the final section we conclude and highlight future work

²Gertler and Natalucci (2003) and Devereux et al. (2006) explore the importance of the financial accelerator in a small open economy. The papers focus on the impact of currency crises when there is foreign currency denominated debt.

2. Model

As in Iacoviello (2005), we consider a sticky-price economy populated by an entrepreneur and two types of households. Credit flows are generated by assuming ex-ante heterogeneity in agents' subjective discount factors. Entrepreneurs and impatient consumers differ from patient consumers in that they discount the future at a faster rate. Hence, in equilibrium, patient agents are net lenders while entrepreneurs and impatient agents are net borrowers. To prevent borrowing from growing without limit, we assume that borrowers face credit constraints tied to the expected future value of collateral assets. We also assume perfectly competitive intermediate-good-producing firms, retailers that operates in a monopolistic competitive market, and a monetary authority.

2.1 Households

Households consume, supply labor and derive utility from housing services, and real money holdings.

$$\max E_0 \sum_{t=0}^{\infty} (\beta_i)^t \left[\zeta_t \ln(c_{i,t} - bC_{i,t-1}) + j_t \ln h_{i,t} - \frac{(L_{i,t})^\eta}{\eta} + x \ln \frac{M_{i,t}}{P_t} \right],$$

where $i = 1, 2$, $\beta_1 > \beta_2$, and $bC_{i,t-1}$ represent habits in consumption. ζ_t is a preference shock that affects the desire of households to consume today. In contrast, $j_{i,t}$ is a shock to the weight on housing services. We refer to this as a housing demand shock.

Lenders. Patient households (denoted by 1), have a higher propensity to save, i.e. $\beta_1 > \beta_2$. So, in equilibrium, they supply loans to impatient households, $b_{1,t}$, and accumulate properties for housing purposes, $h_{1,t}$. Patient households also buy foreign bonds, b_t^* . The return on foreign debt depends on a country specific risk premium ς . Lenders also get dividends, F_t , from the final-good-producing firms. They maximize their expected utility subject to the budget constraint,

$$\begin{aligned} c_{1,t} + q_{h,t}(\tilde{h}_{1,t} - (1 - \delta_h)\tilde{h}_{1,t-1}) + s_t \left(\frac{R_{t-1}^* \varsigma_{t-1} b_{t-1}^*}{\pi_t^*} - b_t^* \right) = \dots \\ w_{1,t} L_{1,t} - \frac{R_{t-1} b_{1,t-1}}{\pi_t} + F_t + b_{1,t} + T_t - \frac{\Delta M_{1,t}}{P_t}, \end{aligned} \quad (1)$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, $q_{h,t}$ is the price of housing, $q_{k,t}$ is the price of capital, $w_{1,t}$ real wages, and s_t the real exchange rate. All the variables, except for the

gross nominal interest rates on domestic and foreign bonds, R_t , and R_t^* , are expressed in real terms. We assume that the housing stock is variable. Thus, differently from Iacoviello (2005), households accumulate properties that depreciate at a rate δ_h . Lenders' optimal choices are characterized by

$$w_{1,t} = \frac{L_{1,t}^{\eta-1}}{U_{c_{1,t}}} \quad (2)$$

$$j_{1,t} \frac{1}{h_{1,t}} = U_{c_{1,t}} q_{h,t} + \beta_1 E_t U_{c_{1,t+1}} q_{h,t+1} (1 - \delta_h) \quad (3)$$

$$U_{c_{1,t}} = \beta_1 E_t \frac{U_{c_{1,t+1}} R_t}{\pi_{t+1}} \quad (4)$$

$$U_{c_{1,t}} q_{k,t} - \beta_1 E_t U_{c_{1,t+1}} q_{k,t+1} (1 - \delta_k) = \beta_1 E_t U_{c_{1,t+1}} R_{k,t+1} \quad (5)$$

$$U_{c_{1,t}} q_{h,t} - \beta_1 E_t U_{c_{1,t+1}} q_{h,t+1} (1 - \delta_h) = \beta_1 E_t U_{c_{1,t+1}} R_{h,t+1} \quad (6)$$

$$\zeta'_t s_t = \beta' E_t \zeta'_{t+1} s_{t+1} \frac{R_t^* s_t}{\pi_{t+1}^*} . \quad (7)$$

Unlike Iacoviello (2005) we augment our model with a demand function for foreign loanable funds (7). The introduction of the risk-premium, ζ_t , is required for the model to feature a stationary distribution.³ Following Adolfson et al. (2005) we assume that the risk premium depends on the ratio of net foreign debt to domestic output and the expected exchange rate

$$\zeta_t = \exp \left[\phi \left(\frac{s_t b_t^*}{P_t^d Y_t} \right) + \phi_s \left(\frac{E_t s_{t+1}}{s_t} \frac{s_t}{s_{t-1}} - 1 \right) + \epsilon_{\zeta,t} \right] .$$

The inclusion of the expected exchange rate in the risk premium is motivated by empirical findings of a strong negative correlation between the risk premium and the expected depreciation, as reported by Fama (1984) and Duarte and Stockman (2005). The demand for foreign funds combined with the demand function for domestic loanable funds, implies an uncovered interest parity condition, which in log-linearized form obeys

$$\hat{r}_t - \hat{r}_t^* = (1 + \phi_s) E_t \Delta s_{t+1} + \phi_s \Delta s_t + \phi$$

where $r_t = R_t - E_t \pi_{t+1}$.

Borrowers. Impatient households (denoted by 2) maximize their stream of expected

³See, e.g., Schmitt-Grohe' and Uribe (2003) for further details.

future utility subject to a budget constraint

$$c_{2,t} + q_{h,t}(h_{2,t} - (1 - \delta_h)h_{2,t-1}) = w_{2,t}L_{2,t} - \frac{R_{t-1}b_{2,t-1}}{\pi_t} + b_{2,t} - \frac{\Delta M_{2,t}}{P_t}, \quad (8)$$

and a borrowing constraint

$$b_{2t} \leq mE_t \frac{q_{h,t+1}\pi_{t+1}h_{2t}}{R_t}. \quad (9)$$

Following Iacoviello (2005) we assume that borrowing is limited to a fraction of the value of borrowers housing stock; where, $(1 - m)$ represents the cost that, lenders have to pay in order to repossess the asset in case of default. Labor supply and borrowing demand are given by

$$w_{2,t} = \frac{L_{2,t}^{\eta-1}}{U_{c_{2,t}}} \quad (10)$$

$$U_{c_{2,t}} - \mu_t = \beta_2 E_t \frac{U_{c_{2,t+1}} R_t}{\pi_{t+1}}, \quad (11)$$

where μ_t is the Lagrange multiplier associated to the borrowing constraint.⁴ For the borrowers, the marginal benefit of holding one extra unit of housing takes into account also the marginal benefit of being allowed to borrow more

$$\frac{j_{2,t}}{h_{2,t}} + \mu_t \gamma E_t \frac{q_{t+1}\pi_{t+1}}{R_t} = U_{c_{2,t}} q_{h,t} + \beta_2 E_t U_{c_{2,t+1}} q_{h,t+1} (1 - \delta_h). \quad (12)$$

2.2 Entrepreneur's problem

Entrepreneurs gain utility from consumption of nondurable only

$$Max \sum_{t=0}^{\infty} \gamma^t [\zeta_t \ln(c_t - bC_{t-1})],$$

where $\gamma < \beta_1$ is the entrepreneurial discount factor, ζ_t is the preference for consumption identical for all agents, and bC_{t-1} represent habits in consumption. Entrepreneurs attempt to maximize their utility subject to a budget constraint

$$c_t + q_t^h (h_t - (1 - \delta^h)h_{t-1}) + q_{k,t} (k_t - (1 - \delta^k)k_{t-1}) = Y_t^{d,int} m c_t^d - \frac{R_{t-1}b_{t-1}}{\pi_t} + \dots + b_t - w_{1,t}L_{1,t} - w_{2,t}L_{2,t} \quad (13)$$

⁴Impatient households borrow up to the maximum in the neighborhood of the deterministic steady state. In fact, if we consider the Euler equation of the impatient household evaluated at the deterministic steady state $\mu_2 = \left(1 - \frac{\beta_2}{\beta_1 \pi}\right) U_{c_2} > 0$

and a borrowing constraint

$$b_t \leq E_t m^k q_{t+1} (1 - \delta^k) k_t \frac{\pi_{t+1}}{R_t} . \quad (14)$$

Entrepreneurs borrow from patient agents using their capital as collateral. With the funds raised, they pay the wage bill and repay past loans at a nominal interest rate of R . Entrepreneurs produce domestic intermediate input, $Y^{d,int}$, with their housing stock, h , their stock of non-housing capital, k , and labour supplied by patient agents, L_1 , and impatient agents, L_2 . Intermediate goods are produced in a perfectly competitive market by the following technology

$$Y_t^{d,int} = z_t (L_{1,t}^\alpha L_{2,t}^{1-\alpha})^{1-\gamma-\nu} k_{t-1}^\gamma h_{p,t-1}^\nu , \quad (15)$$

where z_t is an aggregate productivity shock. Intermediate domestic goods are sold at the competitive price mc_t^d , i.e. at domestic marginal cost. Entrepreneur's first order conditions are

$$U_{c_t} = \frac{1}{c_t - bC_{t-1}} \quad (16)$$

$$U_{c_t} = E_t \gamma \frac{R_t}{\pi_{t+1}} U_{c_{t+1}} + \lambda_t \quad (17)$$

$$U_{c_t} q_t^k = E_t \lambda_t m^k q_{t+1}^k (1 - \delta^k) (\pi_{t+1}/R_t) + E_t \gamma \mu U_{c_{t+1}} mc_{t+1}^d \frac{Y_{t+1}^d}{k_t} + E_t \gamma U_{c_{t+1}} q_{t+1}^k (1 - \delta^k) \quad (18)$$

$$U_{c_t} q_t^h = E_t \lambda_t m^h q_{t+1}^h (1 - \delta^h) (\pi_{t+1}/R_t) + E_t \gamma \nu U_{c_{t+1}} mc_{t+1}^d \frac{Y_{t+1}^d}{h_t} + E_t \gamma U_{c_{t+1}} q_{t+1}^h (1 - \delta^h) \quad (19)$$

$$w_{1,t} = \alpha(1 - \mu - \nu) mc_t^d \frac{Y_t^d}{L_{1,t}} \quad (20)$$

$$w_{2,t} = (1 - \alpha)(1 - \mu - \nu) mc_t^d \frac{Y_t^d}{L_{2,t}} , \quad (21)$$

where λ is the Lagrange multiplier on the credit constraint.

2.3 Firms

2.3.1 Wholesaler's Firm Problems

Domestic brands The producers of domestic brands buy the domestic intermediate input, $Y^{d,int}$, from entrepreneurs, at price mc^d , and transform it using a linear technology into $Y_t^d(z^d)$. Each firm faces a price rigidity à la Calvo, with a non-zero probability, θ_d , of being unable to adjust its nominal price in a given period. Firms maximize the expected present

value of their real dividends setting \tilde{P}_t^d such that

$$\sum_{k=0}^{\infty} \theta_d^k E_t \left\{ \zeta'_{t+k} \left(\frac{\tilde{P}_t^d}{P_{t+k}} - m c_{t+k}^d \right) Y_{t+k}^d(z^d) \right\} = 0. \quad (22)$$

The demand curve for each good obeys $Y_t^d(z) = \left(\frac{P_t^d(z^d)}{P_t^d} \right)^{-\epsilon} Y_t^d$. Each domestic brand is then aggregated into a domestic wholesale good, Y^d . Specifically, $Y_t^d = \left[\int_o^1 (Y_t^d(z^d))^{\frac{\epsilon-1}{\epsilon}} dz^d \right]^{\frac{\epsilon}{\epsilon-1}}$. This implies that the price of the domestic intermediate good, P_t^d , is given by $P_t^d = \left[\int_o^1 (P_t^d(z^d))^{1-\epsilon} dz^d \right]^{\frac{1}{1-\epsilon}}$. The Calvo adjustment process implies the following price index

$$P_t^d = (\theta_d (P_{t-1}^d)^{1-\epsilon} + (1 - \theta_d) (\tilde{P}_t^d)^{1-\epsilon})^{\frac{\epsilon}{1-\epsilon}}. \quad (23)$$

Imported brands Finally, there is a continuum of intermediate-good-importing firms $z^m \in [0, 1]$. They import a homogeneous intermediate foreign good at price P_t^* to produce a differentiated good $Y_t^m(z^m)$. Importers face a price rigidity a la' Calvo, with each firm facing a non-zero probability, θ_m , of being unable to adjust its nominal price in a given period. Firms that are able to revise the price, choose \tilde{P}_t^m such that

$$\sum_{k=0}^{\infty} \theta_m^k E_t \left\{ \zeta'_{t+k} \left(\frac{\tilde{P}_t^m}{P_{t+k}} - s_{t+k} \right) Y_{t+k}^m(z^m) \right\} = 0, \quad (24)$$

where the demand curve for each good obeys $Y_t^m(z^m) = \left(\frac{P_t^m(z^m)}{P_t^m} \right)^{-\epsilon} Y_t^m$. Imported intermediate goods are imperfect substitutes in the production of the composite imported good Y_t^m , where $Y_t^m = \left[\int_o^1 (Y_t^m(z^m))^{\frac{\epsilon-1}{\epsilon}} dz^m \right]^{\frac{\epsilon}{\epsilon-1}}$. Thus, the price of the intermediate imported good, P_t^m , is a composite of the individual prices for the inputs, $P_t^m = \left[\int_o^1 (P_t^m(z^m))^{1-\epsilon} dz^m \right]^{\frac{\epsilon}{1-\epsilon}}$. The Calvo adjustment process implies a process for the price index of $P_t^m = (\theta_m (P_{t-1}^m)^{1-\epsilon} + (1 - \theta_m) (P_t^{m,*})^{1-\epsilon})^{\frac{\epsilon}{1-\epsilon}}$.

2.3.2 Retailer's Problem

Retailers combine domestic brands of intermediate goods Y^d , and imported intermediate goods Y^m , to form a final good Y . Retailers operate in a perfectly competitive market using a CES production function

$$Y_t = \left[(1 - \omega)^{\frac{1}{\phi}} (Y_t^d)^{\frac{\phi-1}{\phi}} + \omega^{\frac{1}{\phi}} (Y_t^m)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (25)$$

where $\omega > 0$ is the share of imported goods in the final domestic goods basket and $\phi > 0$ is the elasticity of substitution between domestic and imported intermediate goods. Cost minimization entails the following demand curves for Y^d and Y^m

$$Y_t^d = (1 - \omega) \left[\frac{P_t^d}{P_t} \right]^{-\phi} Y_t \quad (26)$$

$$Y_t^m = \omega \left[\frac{P_t^m}{P_t} \right]^{-\phi} Y_t, \quad (27)$$

and a domestic aggregate price level P corresponding to the CPI, such that

$$P_t = [(1 - \omega) (P_t^d)^{1-\phi} + \omega (P_t^m)^{1-\phi}]^{\frac{1}{1-\phi}}. \quad (28)$$

2.4 Housing producer's problem

Housing producers are competitive firms that purchase raw output as a material input and combine it with rented housing to produce new properties via the production function $\xi_{h,t} h_{t-1}$. We assume a quadratic cost of adjusting the housing stock as in Aoki et al. (2004)

$$\xi_{h,t} \equiv \frac{1}{A_{h,t}} \frac{\psi_h}{2\delta_h} \left(\frac{I_{h,t}}{H_{t-1}} - \delta_h \right)^2, \quad (29)$$

where ψ_h governs the slope of the housing adjustment cost function. In addition, we include $A_{h,t}$, i.e. an AR(1) shock to the equilibrium condition of housing investment. Since this is a shock to the cost of producing an extra unit of housing, we interpret it as a housing-investment-specific shock. New housing capital goods are sold at price q_t^h

$$q_t^h = \frac{1}{A_{h,t}} \left(1 + \frac{\psi_h}{2\delta_h} \left(\frac{I_{h,t}}{H_{t-1}} - \delta_h \right) \right) \quad (30)$$

This equation is a standard Tobin's q relationship, in which the marginal cost of a unit of housing is related the marginal cost of adjusting the housing stock.

2.5 Capital producer's problem

Similarly, capital production firms take final goods, transform them into capital using the technology $\xi_{k,t}k_{t-1}$; where $\xi_{h,t} \equiv \frac{1}{A_{k,t}} \frac{\psi_k}{2\delta_k} \left(\frac{I_{k,t}}{k_{t-1}} - \delta_k \right)^2$ and ψ_k governs the slope of the capital producers adjustment cost function. Producers sell the capital at price $q_{k,t}$

$$q_t^k = \frac{1}{A_{k,t}} \left(1 + \frac{\psi_k}{\delta_k} \left(\frac{I_{k,t}}{k_{t-1}} - \delta_k \right) \right) , \quad (31)$$

where $A_{k,t}$ is an AR(1) shock. As in Greenwood et al. (1998) and Fisher (2006), we interpret $A_{k,t}$ as a capital-investment-specific shock.

2.6 Monetary policy

For simplicity we assume that the local central bank targets inflation, using a Taylor-type interest rate rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(1 + \rho_\pi) \hat{\pi}_t + (1 - \rho_R)(1 + \rho_Y) \hat{Y}_t + \epsilon_{R,t} . \quad (32)$$

where $\epsilon_{R,t}$ is an i.i.d. shock.

2.7 Market clearing conditions

Domestic output Y_t , can be consumed, invested or exported

$$Y_t = c_t + c_{1,t} + c_{2,t} + q_{k,t} I_t^k + q_{h,t} I_t^h + Y_t^x$$

Capital is accumulated according to

$$I_{h,t} = H_t - (1 - \delta_h) H_{t-1} \quad (33)$$

where the aggregate stock of housing is $H_t = h_t + h_{1,t} + h_{2,t}$; and the usual capital accumulation equation holds

$$I_{k,t} = k_t - (1 - \delta_k) k_{t-1}. \quad (34)$$

The domestic loan market condition implies that total borrowed funds are equal to funds lent out by patient households

$$b_t + b_{2,t} = -b_{1,t} \quad (35)$$

Finally, the trade balance equals economy-wide net saving, so that

$$Y_t^x - s_t Y_t^m = \left[s_t \left(\frac{R_t^f \zeta_{t-1}}{\pi_t^f} \right) b_{t-1}^f - b_t^f \right]. \quad (36)$$

2.8 Rest of the World

We assume Canada to be a small open economy. Thus, domestic developments do not affect the rest of the world economy. On the contrary, foreign economy's dynamics have an impact on the Canadian economy. Our foreign block is a four equation system that aims at capturing the main features of the model of the US economy (MUSE) used at the Bank of Canada to produce projections of the US economy. The model is based on empirical consideration, and the version used in this paper is formed by a IS curve, a Phillips curve, a monetary policy rule and an equation ruling the evolution of the foreign interest rate. A requirement of perfect symmetry would call for modelling the global economy fully as a closed-economy version of the open-economy financial accelerator model. We have taken a mid-way approach, modelling the global economy as a New Keynesian IS-LM model without capital or financial frictions, but with an export-demand function similar to the import-demand function of the local economy.⁵

The log-linearized equations of the global economy are as follows. The monetary policy rule, the IS curve and the hybrid Phillips curve are based on empirical considerations. A IS curve,

$$\hat{y}_t^f = \frac{b^f}{1+b^f} \hat{y}_{t-1}^f + \frac{1}{1+b^f} E \hat{y}_{t+1}^f - \frac{1-b^f}{1+b^f} \hat{r}_t^f + \frac{b^f}{1+b^f} \left(\hat{\zeta}_t^f - E \hat{\zeta}_{t+1}^f \right) + \frac{1-b^f}{1+b^f} \varepsilon_t^{IS}, \quad (37)$$

where $\hat{r}_t^f = \hat{R}^f - E \hat{\pi}_{t+1}^f$, b is the parameter that represents habits in consumption, and ζ_t^f is an AR(1) shock to the preference for consumption, and ε_t^{IS} is an AR(1) shock. The

⁵It is customary in many new open economy models intended for estimation (e.g. Bergin (2004)) to simply assume an AR or VAR structure for global variables of interest, when the global economy is taken to be exogenous to activity in the local economy.

empirically-based Phillips curve obeys

$$\begin{aligned} \hat{\pi}_t^f (1 + \beta_f G_1 + \beta_f^2 G_2 + \beta_f^3 G_3) = & G_1 \hat{\pi}_{t-1}^f + G_2 \hat{\pi}_{t-2}^f + G_3 \hat{\pi}_{t-3}^f + \beta_f G_1 E \hat{\pi}_{t+1}^f + \dots \\ & + \beta_f^2 G_2 E \hat{\pi}_{t+2}^f + \beta_f^3 G_3 E \hat{\pi}_{t+3}^f + \lambda^f \left(\hat{y}_t^f - b^f \hat{y}_{t-1}^f - (1 - b^f) \hat{\zeta}_t^f \right), \end{aligned} \quad (38)$$

where $\beta_f = \beta_1$ is the foreign agents discount factor, $G_3 = (1 - G_1 - G_2)$, and $\lambda^f = \frac{1}{1-b^f} \frac{(1-\theta_f)(1-\beta_f\theta_f)}{\theta_f}$. Here, θ_f is the probability that foreign price will remain sticky in a given period. The Phillips curve closely resemble a VAR representation and it allows to account for the correlation of current inflation with inflation lagged up to three periods. The Phillips curve adopted by the MUSE relies extensively on the polynomial adjustment cost approach based on decomposing the dynamic behavior of inflation into changes that are induced by expectations and those that are delayed responses to previous decisions. As in Kozicki and Tinsley (2002) the distance of actual inflation from its equilibrium path is approximated by a factor proportional to the output gap. In the current formulation we abstract from taking into account changes in the relative price of imports.

The foreign central bank is assumed to follow a simple interest-rate rule

$$\hat{R}_t^{*f} = \rho_y \hat{y}_t^f + \rho_\pi E \hat{\pi}_{t+4}^f. \quad (39)$$

According to the MUSE, the US monetary policy targets the deviations of foreign output from the deterministic steady-state and expected inflation four quarters ahead. Moreover, the interest rate evolves according to

$$\Delta R_t^f = (1 - \rho_{R^f,1}) * (\hat{R}_t^{*f} - \hat{R}_{t-1}^{*f}) + (1 - \rho_{R^f,2}) * (\hat{R}_{t-1}^{*f} - \hat{R}_{t-2}^{*f}) + \rho_{R^f,1} * \rho_{R^f,2} \Delta R_{t-1}^f + \varepsilon_t^{MP}, \quad (40)$$

where ε_t^{MP} is an I.I.D. shock, \hat{R}_t^{*f} is the interest rate given by the Taylor-type rule and R_t^f is the observed nominal federal funds rate. This formulation of the foreign monetary policy conduct, reflects some degree of inertia in the response to past interest rate changes and a gradual adjustment of the federal funds rate. The model's US policy-setting rule is based on the framework developed by Nelson and Sack (2002) for the UK.

By analogy with the import demand function of the local economy, the demand for the domestic economy's exports is captured by

$$\hat{Y}_t^x = \hat{Y}_t^g - \phi \hat{P}_t^x + \epsilon_{ex,t}$$

and $\hat{P}_t^x = \frac{P_t^x}{P_t^g}$ is the real price of local brands in the global economy and $\epsilon_{ex,t}$ is an export demand shock. It can be shown that the process for $\pi_t^x \equiv \frac{P_t^x}{P_{t-1}^x}$ obeys

$$\hat{\pi}_t^x - \beta_f E \hat{\pi}_{t+1}^x = \frac{(1 - \theta_x)(1 - \beta\theta_x)}{\theta_x} (-\hat{s}_t - \hat{P}_t^x), \quad (41)$$

where θ_x is the probability that the price of a local brand will remain sticky in the global economy in a given period.

2.8.1 Shock processes

Apart from $\epsilon_{R,t}$ and ε_t^{MP} that are zero-mean i.i.d. shock with variance σ_ϵ , the other structural shocks in the model, $\chi_t = \{\zeta_t, j_t, \epsilon_{s,t}, z_t, A_{h,t}, A_{k,t}, \varepsilon_t^{IS}, \varepsilon_t^{PH}, \epsilon_{ex,t}\}$, follow an AR(1) process

$$\ln(\chi_t) = \rho_\chi \ln(\chi_{t-1}) + \varepsilon_{\chi_t}, \quad \varepsilon_{\chi_t} \sim^{iid} N(0, \sigma_{\varepsilon_\chi}), \quad 0 < \rho_\chi < 1. \quad (42)$$

3. Data and model estimation strategy

The vector of structural parameters of the model, Λ , describing preferences, technology, the monetary policy rule and the shocks is estimated using Bayesian techniques. First, for given parameter values we solve the model by using standard first-order approximation techniques (see, for example Uhlig, (1999)). Then, we use the Kalman filter to compute the likelihood $L(\Gamma_t | \Lambda)$ for the given sample of data Γ_t , as in Hamilton (1994). We add some informative priors, $\varphi(\Lambda)$, into the estimation in order to downweight regions of the parameter space that are widely accepted to be uninteresting. Using Bayes's rule, the posterior distribution can be written as the product of the likelihood function of the data given the parameters, $L(\Gamma_t | \Lambda)$, and the prior, $\varphi(\Lambda)$:

$$P(\Lambda | \Gamma_t) \propto L(\Gamma_t | \Lambda) \varphi(\Lambda) \quad (43)$$

We start by estimating the posterior distribution's mode by maximizing the log posterior function. Second, we obtain a random draw of size 300.000 from the posterior distribution using the random-walk Metropolis-Hastings algorithm. The posterior distribution of the parameters can be used to draw statistical inference on the parameters themselves or functions of the parameters, such as second moments.

3.1 The Data

We estimate the model using Canadian data for consumption, capital investment, residential investment, the trade balance, real house prices, the short term nominal interest rate, the real bilateral exchange rate (with the United States) and the inflation rate. The availability of the house price data and a desire to have a sample over which the conduct of monetary policy and the statistical properties of inflation have been relatively stable restrict us to considering data from 1980Q1 to 2006Q3.

Consumption is measured by real personal expenditure on consumer goods and services, while residential investment is real residential structures investment. For capital investment we use real business fixed investment (including equipment and structures). These three series are transformed into per capita terms using data on the population age 15 and over. We take real exports net of real imports to measure the trade balance. Our measure of real house prices is the Multiple Listing Service existing house sales price. This index measures the average sale price of all existing residential dwellings sold in a given period.⁶ We calculate the real house price by deflating this house price series using the CPI. The overnight rate, the interest rate at which major financial institutions borrow and lend one-day (or "overnight") funds among themselves, is our measure of the short-term nominal interest rate. Finally, we use the Bank of Canada's measure of "core" CPI inflation. This measure of inflation in consumer prices excludes the effects of price changes from the eight most volatile components of the CPI (e.g. mortgage interest costs, vegetables and gasoline) and changes in indirect taxes.⁷ Finally, the real exchange rate is calculated as the nominal exchange rate (price of a U.S. dollar in Canadian dollars) times U.S. CPI divided by Canadian CPI.

The borrowing constraint in the model has implications for the dynamics of the components of GDP making these series important for identifying the share of constrained households. In addition, using separate series for consumption, capital investment and housing investment gives us the best chance of estimating the adjustment costs parameters for the two types of investment.

All of our series are taken at a quarterly frequency. The real series are logged and

⁶This series is highly correlated with an alternative index of resale housing prices from Royal LePage that measures the prices of houses with similar characteristics in different regions across the country. For this reason we do not think that composition bias is important enough to affect our results.

⁷Unlike the U.S. consumer price index, the Canadian prices data do not include the cost of imputed rents. Nonetheless, house prices do affect our measure of inflation through owned accommodation prices, which include such costs as home insurance, house depreciation and property taxes.

detrended using a linear trend.⁸ The model also implies that the value of the housing stock, $q_t h_t$, is a constant proportion of consumption. The different trends in the raw consumption and housing investment data imply that the real price of housing should also have an upward trend if it is to obey the balanced growth restriction of the model. However, the trend in our real house price series is higher than is implied by this relationship. Since this may reflect some other structural change that is not well captured by the model, for example financial innovation, we also remove a linear trend from the house price data.

The inflation rate and nominal interest rate are expressed as quarterly rates. Demers (2003) argues that the Canadian inflation is well characterized by three separate regimes each with a different mean inflation rate. The regime switches that he identifies correspond with the drop in inflation in the early 1980s and the adoption of a formal inflation target in the early 1990s. Using the break points identified in Demers (2003) we calculate subsample means of inflation (creating a step function of inflation means) and remove these from our inflation and nominal interest rate data. Finally, we remove an internal estimate of the trend in the real interest rate from the resulting interest rate series, so that the data captures the gap between the real interest rate and its long run value.

We estimate the foreign block outside the main model and, given the importance of the United States for Canadian trade (it represents about 85 per cent of Canadian exports), use U.S. data on output, inflation and nominal interest rates. These data are transformed in a similar way to the Canadian equivalents. The parameter estimates for the foreign block are kept fixed during the estimation of the full model. Those parameter values are reported in Table 7.

A detailed description of the data sources and plots of the detrended data are presented in Figure 5 and the Data Appendix.

3.2 Calibrated Parameters

We calibrate a number of parameters based on sample means or other information because they are often difficult to identify. The calibrated parameters include: the discount factors

⁸The series for the components of output all show a clear upward trend, but our model implies that these variables fluctuate around a constant mean. Furthermore, adding a deterministic trend to GDP in the model (and thus a common trend to its components) will not solve the problem since the components of GDP have different trends. For example, a regression of consumption on a constant and time returns estimates for the time trend coefficient for consumption of 0.007. In contrast, this coefficient is 0.005 for housing investment and 0.096 for business investment. Rather than complicating the model to address these differential trends, we use the linearly detrended data to estimate the model.

$\beta_1, \beta_2, \beta_e$, the weight on housing in the utility function j , factor shares γ , depreciation rates δ_h, δ_k , the steady-state gross markups for all price-setting firms, and the household loan-to-value ratio m^h .

We set the housing preference parameter j to match the ratio of personal sector residential housing (land plus structures in the National Balance Sheet Accounts) to quarterly *GDP*, which for our sample period averages about 6.9. We follow Iacoviello (2005) who draws from micro-studies of the range of discount factors of consumers, in setting the discount factor of patient agents β_1 to 0.99 and the discount factor of impatient agents β_2 to 0.95⁹. The patient agent's discount factor implies a steady-state real interest rate of 4 per cent on an annual basis. We set the elasticity of demand for individual domestic intermediate goods ϵ so as to give an average markup of five percent in steady-state. Individual imported intermediate goods have the same elasticity of demand. The share of imported goods in the final domestic goods basket, ω , is set to 0.3. and the elasticity of substitution between domestic and imported goods, ϕ , is set to 1. We set η , the parameter describing the disutility from working to 1.01¹⁰. As a typical house has a much longer lifetime than a typical piece of equipment, the housing depreciation rate δ_H should be lower than δ_k . The value of δ_H compatible with the housing investment to GDP ratio was 0.011, within the range of values in the literature Kostenbauer (2001). The depreciation rate for capital is set to $\delta_k = 0.023$, implying an annual depreciation of the capital stock of 9.5 per cent. The depreciation rate for capital along with the capital share in production of final goods, $\gamma = 0.33$, imply a ratio of business fixed investment to GDP of about 0.125, approximately that seen in the data. We treat non-residential construction as part of business fixed investment.

We also need to set a value for the loan-to-value ratio m^h . This value should reflect the typical loan-to-value ratio for a constrained household who borrows the maximum possible against their real estate holdings¹¹. By Canadian law, mortgages in excess of 75 per cent of the value of the property must be insured. In practice, first-time home buyers (perhaps the

⁹This value is in accordance with estimates of discount factors for poor or young households (see Samwick (1998) and Lawrence (1991)) and falls into the empirical distribution for discount factors estimated by Carroll and Samwick (1997).

¹⁰The rationale for assuming an almost flat labour supply curve is to ensure that hours are more strongly procyclical than real wages, as observed in the data.

¹¹Ultimately we would like to incorporate information from observed financial variables such as mortgage debt into the model. However, aggregate measures of mortgage debt in Canada include debt held by unconstrained households who have had time to accumulate other assets and are better thought of as patient consumers. It is not possible to capture this in the model since lenders own housing, but do not have mortgages. This forces the impatient households to hold all of the observed stock of debt if we are to match the household debt-to-asset ratio in the data. In future, we may be able to exploit information from microdata to determine a more appropriate target debt-to-asset ratio.

most likely to be constrained) can obtain loans at considerably higher loan-to-value ratios than 75 per cent. It is possible to obtain such insurance for loans up to 95 percent of the value of an owner-occupied residence. Constrained households are likely to have loan-to-value ratios in the 0.75 to 0.95 range. We set $m^h = 0.80$, slightly lower than the value chosen by Iacoviello and Neri (2007) based on U.S. data on new-home buyers.

3.3 Priors

Table 3 summarizes our priors. We set the prior mean on the income share of unconstrained households to 0.70, with a standard error of 0.05. This is within the range of estimates of the fraction of households who are financially constrained reported in the literature. Campbell and Mankiw (1991) estimate the fraction of constrained agents from Canadian macro data to be near 50 per cent. Estimates based on micro data for the U.S. (Jappelli (1990)) and the U.K. (Benito and Mumtaz (2006)) estimate that around 20 per cent of the population to be liquidity constrained. We set the prior mean of the habit parameter in consumption b to 0.7 with a standard error of 0.05.

We set the prior mean for m^k , the share of firms capital assets that can serve as collateral, to 0.5 with a standard error of 0.1. This is lower than the estimated value in Iacoviello (2005), but we also allow for all capital to be used as collateral, whereas he assumes only structures are collateralized. The aggregate corporate sector balance sheet data has a debt-to-asset ratio of 0.7? if we consider only fixed assets and 0.25 if we include all assets.

Our priors for the monetary policy rule are that the coefficient on inflation $\varepsilon_\pi = 1.65$ corresponding to that in the standard Taylor rule. The prior mean for the ε_r is set to 0.75 close to the estimate in Lam and Tkacz (2004). Prior mean for ε_y is 0.125. For the fixed capital and housing capital adjustment costs we chose a gamma-distribution with mean of about 1 and a standard error of 0.5 in both cases. We set a prior mean for the probability that a retailer will be unable to adjust prices, θ , to 0.67 implying that retail prices are fixed for 3 quarters on average. At our prior means the model is able to match a set of key steady-state ratios as documented in our estimation sample.

The prior mean for the elasticity of the country risk premium with respect to the ratio of foreign debt-to-output, ϕ , is based on previous work on estimation of small-open economy models by Adolfson et al. (2005) and Christoffel et al. (2007).

The prior assumptions on the distribution of the shocks are based on the estimates re-

ported by Iacoviello and Neri (2007) for the US economy. We use inverse gamma priors on all the structural parameters governing the standard deviations of the shocks. For the persistence parameters of the shock processes, we choose a beta-distribution with a prior mean of 0.8 and standard deviation of 0.1.

4. Empirical results

We estimate the model presented above and a representative agent version of the model in which there are no borrowing constraints.¹² We refer to our model as the FA model since it has an active financial accelerator and the model without the borrowing constraints as the NoFA model. Our objective is to document the role that the financial accelerator mechanism plays in the ability of the model to capture the moments of interest in the data. At present we report mainly on the fit of the model over the entire sample. However, we believe that a full evaluation of this mechanism would also consider the whether the financial accelerator helps the model to account for episodes in history such as housing booms/busts or a severe recession.

4.1 Posterior Distributions

We report the posterior mean and 0.95 probability interval for the structural parameters, along with their priors, for the model with the financial accelerator effect (FA model, in table 2) and the model without the borrowing constraint (NOFA model, in table 3). Except where noted, most of the parameters are within the ranges specified in the priors.

The habit persistence parameter, b is estimated to be 0.7969, on the high end of our prior but within acceptable levels.

In the FA model, the estimates for the two adjustment cost parameters are very similar ($\psi_h = 1.63$; $\psi_k = 1.61$). The estimates for the the capital adjustment costs imply an elasticity of investment to its price at the high end of the range of estimates reported in Chirinko (1993). In the NOFA model, while the estimates for ψ_h are about the same, that for ψ_k is somewhat higher ($\psi_k = 2.09$).

The coefficient on inflation in the monetary policy rule, ε_π , is 2.54, the weight on output,

¹²In this model the household still derives utility from housing services, but it also purchases capital and rents it to the intermediation goods producer. The other household are no longer in this model so that there are no borrowers or lenders. All other features of the model are preserved.

ε_y , is 0.025, while the interest rate smoothing term, ε_R , is 0.71. The smoothing parameter is slightly lower than other estimates on Canadian data. However the weights are consistent with other evidence on Canadian monetary policy over most of the period; in an inflation targeting regime, clearly inflation should have a high weight and output a low one.

The mean estimate of Calvo price stickiness θ^d implies an average duration of price stickiness of slightly less than five quarters. Such estimates for average price stickiness are high relative to findings on price adjustment from micro studies, however, they are in line with previous DSGE models' estimates – even with more elaborate systems of nominal stickiness. However, relatively speaking, the price stickiness for imports θ^m and for exports θ^x are much lower, the average import price being sticky for three quarters and the average export price being sticky for 2.4 quarters.

The estimated parameters most important for the financial frictions, α and m^k , merit more detailed discussion. The estimate of the wage share of unconstrained households, α , is 0.85 implying a share of labor income to credit constrained agents of about 15 percent, on the low end of the range of empirical estimates in the literature. The estimate for the loan to value ratio for firms, m^k , is 0.32.

The data seem to favour an FA model with, on the one hand, very high α 's, suggesting few constrained consumers, and low m^k 's, suggesting highly credit-constrained firms that can only invest from retained earnings.

4.2 Model Dynamics

Figures 1(a) to (h) plot the posterior distributions of the impulse response functions for a one standard deviation shock. A one percent rise in a variable is denoted as 0.01 on the y-axis and the number of quarters elapsed since the shock are indicated on the x-axis.

Figure 1(a) illustrates the effect of a neutral technology shock. As perceived wealth rises, consumption rises slowly (along with wealth), as do housing investment and prices. As consumption dominates output, output will also rise persistently with a technology shock. Capital investment rises, along with productivity. Inflation falls, as do nominal and real interest rates. With lower real interest rates, real exchange rates rise; while exports rise, real exchange rates rise further, and the trade balance falls.

Figure 1(b) illustrates the effect of a consumption preference shock. Demand for consumer

goods rises; consumption rises, and capital investment falls slightly. This shock leads to higher interest rates, reducing demand for housing and so housing investment and house prices. Import demand rises, along with consumer demand, and the trade balance falls.

Figure 1(c) illustrates the effect of an investment-specific technology shock. Naturally, capital investment rises immediately, as capital is now cheaper and the quantity demanded rises. Consumption, however, rises only very slowly after the shock; as capital accumulation rises, output rises, but the process is slow enough as to be of secondary importance for output at business cycle frequencies. Again, higher aggregate demand (from higher capital investment) increases import demand, and the trade balance falls.

Figure 1(d) illustrates the effect of a housing-specific technology shock. Housing investment rises since housing is cheaper, but the response of house prices to the supply shock is fairly small, as are the responses of the other real and nominal variables. This is not surprising, as housing investment is a relatively small portion of private output (around 7 percent). Again, higher aggregate demand (from higher housing investment) increases import demand, and the trade balance falls.

Figure 1(e) illustrates the effect of a housing demand shock, a shock for which agents favour housing more relative to consumption and leisure. Since this is a demand shock for housing, residential investment and prices rise. Rising house prices relax the credit constraint for impatient agents, leading to a rise in aggregate consumption, at least in the short run (up to eight quarters). In addition, entrepreneurs sell some of their housing stock to other households and increase their consumption and investment spending. While output and employment as a whole rise, impatient employment actually falls in the short run, as impatient agents can borrow to consume rather than work. Again, higher aggregate demand increases import demand, and the trade balance falls.

Figure 1(f) illustrates the effect of a monetary shock. Nominal and real interest rates rise and inflation falls. Consumption and investment fall in response to this shock, with the peak effect on consumption occurring in the second period reflecting the impact of habits on the dynamics of consumption. Activity in the housing market slows and house prices and housing investment fall. The real exchange rate falls as Canadian investments become more attractive to foreign investors. Slightly counter-intuitive is a rise, not a fall, in the trade balance. While exports fall with real exchange rates, the slowdown in aggregate demand is sufficient to reduce the values of imports even further.

Figure 1(g) illustrates the effect of an export demand shock. Naturally, exports rise, stimulating aggregate demand, output and inflation; real exchange rates fall, causing a rise in imports; however, as one would expect, the trade balance improves. While in a closed economy an aggregate demand shock (for example to government spending) would reduce either consumption or investment, the windfall from the trade surpluses reduces Canada's foreign debt, increasing aggregate wealth, so that consumption actually rises along with investment and the capital stock.

Figure 1(h) illustrates the effect of a shock to the risk premium on the domestic country's foreign debt. As foreign investors become more reluctant to hold Canadian assets, the real exchange rate rises and Canadian interest rates rise. As the real exchange rate accounts for a portion of marginal cost, inflation rises and output falls as the cost of production rises; however, there is some substitution away from imports to domestic inputs in production, and employment rises. In the short run, the trade balance falls, as imports become more expensive, but rises in the long run as sticky prices of imports come to reflect the changes in the real exchange rate, and import demand falls.

In an open economy DSGE model, the amount of pass-through of a change in real exchange rates depends on the relative importance of imports in the economy, the type of shock causing the change, and how long the change is expected to last. As the shock to real exchange rates has a short half-life, the pass-through to consumer prices is not large, in spite of the otherwise low price stickiness for imports. A one percent increase in real exchange rates here results in a rise in economy-wide inflation of only 0.07 percent.

4.2.1 Sensitivity Analysis.

We now compare the responses from the FA model presented above (in blue) against the NOFA model. The objective is to document the implications of the borrowing constraints for the dynamics of our model. We have also estimated each model separately so we report the results for each model using parameter values set to their posterior means. In each case we set the size of the initial shock to be the same in both models, so these plots capture the different dynamics implied by each model but the magnitudes of the responses are not affected by differences in the size of the initial shock.

Figure 2(a) shows that in both models higher interest rates reduce aggregate demand. However the impact on output of an monetary policy shock is about a third larger when constrained firms and households are present. The larger impact is particularly evident for

domestic components of output; the drop of capital investment nearly doubles, and the drop of consumption more than doubles. Firms and impatient households, being more sensitive to variations in the interest rate, adjust their consumption much more to a change in real interest rates from a monetary policy shock than do patient households.

Of particular interest is the response to a housing demand shock (Figure 2(b)). This shock can be interpreted as a shock to the price of housing.¹³ In the standard representative agent model, consumption falls in response to an increase in the housing price because households give up consumption today to purchase more housing. As shown above, in the model with collateralized debt, a rise in housing prices relaxes the borrowing constraint and allows impatient households to increase consumption closer to desired levels. Entrepreneurs also take advantage of higher house prices by selling some of their housing and increasing consumption and capital investment.

4.3 Business Cycle Properties

In the next three sections we consider how some model-implied moments compare with the data. We generate the model statistics by simulating the model and Hodrick-Prescott filtering the simulated data. We compare these statistics to the data moments calculated using the raw data series (logged where appropriate).

Standard deviations. In Table 4 we report the standard deviations of key variables in the data and from the model at business cycle frequencies. The volatility of output implied by the FA model is close to that in the data. The value in the data is GDP excluding government expenditures (since there is no government sector in the model. The NoFA model generates somewhat less output volatility. Consumption volatility is higher in the models than in the data. Borrowing and lending abroad by patient agents is relatively unrestricted in our model, given the parameter estimates that suggest little variation in the economy-wide risk premium given a change in net foreign debt. This serves to make overall consumption (dominated by patient consumption) and output (dominated by consumption) to be more persistent, and so more volatile in the long run, than would be the case in a closed-economy model.

The model does give us volatility for investment (in housing as well as capital) and for house prices not far from what we see in the data. It is also able to generate a high degree of volatility in residential investment, though still less than in the data. The model over predicts

¹³Indeed the variance decompositions show that this is the shock responsible for the largest part of house price fluctuations in the model.

the volatility of inflation, but is very close for interest rate volatility. While the volatility of the trade surplus is about what we see in the data, the volatility of the real exchange rate given by the model is less than half that from the data.

Variance decomposition. We report, in Table 5, the infinite horizon variance decompositions of output, consumption, capital investment, housing investment, house prices, inflation and interest rates. A large part of the variation of consumption and output is explained by technology shocks (25 percent for output, 22 percent for consumption) and capital supply shocks (15 percent for output, 18 percent for consumption). Consumption preference shocks also play a key role for consumption, of course (30 percent) but are less important for output (7 percent). Open-economy shocks have a role to play as well; export demand shocks are more important for output (26 percent) than for consumption (8 percent), while real exchange rate shocks are more important for consumption (12 percent) than for output (8 percent). Shocks originating in the foreign economy play a small role in the model are not reported in the tables.

It is worth comparing the importance of the shocks most affected by financial frictions in the FA and NoFA models. In the FA model, housing demand shocks explain about 16 percent of output variation (given that output includes housing investment), and about 7 percent of consumption variation. In the NoFA model, by contrast, housing demand shocks explain only 4 percent of output and 6 percent of consumption. In the FA model i.i.d. monetary policy shocks account for about 3 percent of the variance in output and consumption, in line with the VAR findings of Christiano et al. (2005) for the U.S., and about 3 percent of consumption variance; in the NOFA model, the figure is less than one percent. In the NOFA model, much of the variance attributed to financial shocks under FA is attributed to consumption shocks; consumption shocks explain 43 percent of consumption under NOFA, but only 30 percent under FA. The greater importance of the housing demand and monetary policy shocks in the FA model, and less importance of the consumption preference shock, suggests that the borrowing constraints are helping the model to capture some of the co-movement between consumption and house prices.

In both models, fluctuations in capital investment are almost entirely (95 percent for FA, 98 percent for NOFA) explained by capital supply shocks. In both models, the biggest influence on housing investment is housing supply shocks (75 percent), though housing demand shocks also account for an important share (22 percent) of housing investment fluctuations.

Housing demand shocks account for the bulk of the variation in house prices (65 percent

FA, 68 percent NOFA), with housing supply shocks making up most of the rest (26 percent FA, 25 percent NOFA). Neutral technology shocks also play a minor role (4 per cent).

The biggest influence on inflation over the cycle is the neutral technology shock (51 percent FA, 48 percent NOFA), but real exchange rate shocks (23 percent FA, 20 percent NOFA) and of course monetary shocks (24 percent FA, 25 percent NOFA) also play key roles. As nominal interest rates closely track inflation, the same shocks explain most of the variation in nominal interest rates as well.

Cross-correlations. Table 6 reports the cross-correlations for our key variables at business cycle frequencies. In the model, consumption, both types of investment and house prices are positively correlated with output, as they should be. However, the implied correlations between output and consumption and output and investment are considerably weaker than in the data, and the correlation of house prices to output is higher than in the data. Inflation is negatively related to output in the model, but positively correlated with both in the data. The nominal interest rates is negatively correlated with output and consumption in the model but positively correlated in the data. Inflation and nominal interest rates are positively correlated in the model and the data.

In the model consumption shows a positive correlation with house prices (0.16), but this is low relative to the data (0.55). If we set $\alpha = 1$ so that constrained households become unimportant, this correlation drops even lower to 0.005. Similarly, the correlations of consumption with capital and housing investment are close to zero in the model, but considerably stronger in the data. Given that both types of investment are driven by investment-efficiency shocks which have relatively little impact on consumption, this is not surprising.

Historical decompositions. The statistics reported above help us to evaluate the ability of each model to capture the full sample moments in the data. However, we are also interested in how well the FA model can explain particular episodes in the Canadian business cycle where financial frictions are most likely to have been important. Figures 3(a) and 3(b) compare detrended Canadian consumption to counterfactual consumption paths implied by the model for cases in which a single shock was operative over history. Figure 3(a) shows the decomposition for the FA model, figure 3(b) for the NOFA model.

These figures show that much of the rise in consumption during the housing boom of the late 1980s is attributed by the FA model to the housing demand shock and, to a lesser extent, the monetary policy shock. This is also apparent during the recent housing boom, though

in this period monetary policy shocks appear to play a somewhat larger role. In the NOFA model, however, the housing demand shock acts as a drag on consumption in this period and more recently (this is from the substitution effect away from consumption and toward housing). The shock that best explains consumption's rise and fall in the NoFA model is the consumption preference shock. This suggests that the borrowing constraints in the FA model are helping it to explain the co-movement of house prices and consumption observed during housing booms.

5. Conclusion

The aim of this paper is to investigate the role of credit frictions over the business cycle for the Canadian economy. To this end, we estimate a model with borrowing constraints both at the household and firm level for Canada. Differently from much of the previous literature, in our model the housing stock is variable, which allows us to consider the dynamics of residential investment. In order to capture richer housing price and residential investment dynamics we also introduce a housing investment specific shock. We introduce open economy features because this has the potential to affect the importance of the financial accelerator and to make the analysis more relevant to Canada.

The estimated model generates an increase in aggregate consumption after a shock that primarily acts to raise house prices. A positive housing preference shock, increases the price of housing and the value of collateral, improving the borrowing capacity of credit constrained agents and leading to higher aggregate consumption. In contrast, a version of the model without the collateral constraints generates a decline in consumption. Thus, along this dimension, the borrowing constraints change the prediction of the models not simply in terms of the magnitude of the response, but also its direction.

We estimate two versions of the model, one with the financial accelerator effects and one where these effects are removed. Standard model evaluation criteria, which compare moments from the model against those calculated over the full sample, suggest that the differences between these two estimated models are not large. Despite this finding, the model with borrowing constraints attributes an important contribution of housing demand shocks to the strong consumption growth seen in the late 1980s. This was a period often described as a housing boom in Canada. The model without borrowing constraints relies more heavily on a shock that directly affects consumption to explain this period. We think this is sign that a

full evaluation of the importance of the financial accelerator mechanism should also consider the performance of the two models over specific episodes such as housing booms or severe recessions.

Further refinements to the structure of the model should focus on the firms. In particular, adding firms who are not borrowing constrained could improve the ability of our model to capture the dynamics of investment. Finally, in future work we would like explore how we might use more information from financial data in the estimation of the model.

References

- Adolfson, M., S. Laseen, J. Linde, and M. Villani. 2005. "Evaluating an Estimated New Keynesian Small Open Economy Model." Photocopy, Sveriges Riksbank.
- Ambler, S., A. Dib, and N. Rebei. 2004. "Optimal Taylor rules in an estimated model of small open economy." Working paper 2004-36, Bank of Canada.
- Aoki, K., J. Proudman, and G. Vlieghe. 2004. "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach." *Journal of Financial Intermediation* (13): 414–435.
- Benito, A. and H. Mumtaz. 2006. "Consumption excess Sensitivity, Liquidity Constraints and the Collateral Role of Housing." Bank of England, Working Paper no. 306.
- Bergin, P. 2004. "How well can the new open economy macroeconomics explain the exchange rate and current account?" National Bureau of Economic Research Working Paper 10356.
- Bernanke, B., M. Gertler, and S. Gilchrist. 1999. "The Financial Accelerator In a Quantitative Business Cycle Framework." *Handbook of Macroeconomics* 1: 1342–1385.
- Campbell, J. and G. Mankiw. 1991. "The Response of Consumption to Income: A Cross-Country Investigation." *European Economic Review* 34(4): 723–56.
- Carroll, C. and A. Samwick. 1997. "The Nature of Precautionary Wealth." *Journal of Monetary Economics* 40(1): 41–72.
- Chirinko, R.S. 1993. "Business Fixed Investment Spending: Modeling Strategies, Empirical Results, and Policy Implications." *Journal of Economic Literature* 31: 1875–1911.
- Christiano, L., M. Eichenbaum, and C. Evans. 2005. "Nominal rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113: 1–45.
- Christoffel, K., G. Coenen, and A. Warne. 2007. "Conditional Versus Unconditional Forecasting With the New Area Wide Model of the Euro Area." Preliminary Draft European Central Bank.
- Demers, F. 2003. "The Canadian Phillips Curve and Regime Shifting." Bank of Canada, Working Paper 2003-32.
- Devereux, M.B., P.R. Lane, and J. Xu. 2006. "Exchange Rates and Monetary Policy in Emerging Market Economies." *Economic Journal* 116(511): 478–506.

- Fisher, J.D.M. 2006. “The Dynamic Effects of Neutral and Investment-Specific Technology.” *Journal of Political Economy* 114(3): 413–451.
- Gertler, M. and F. Natalucci. 2003. “External Constraints on Monetary Policy and the Financial Accelerator.” BIS Working Paper No. 139 (Journal of Money Credit and Banking, forthcoming).
- Greenwood, J., Z. Hercowitz, and P. Krusell. 1998. “The Role of Investment-Specific Technological Change in the Business Cycle.” *European Economic Review* (44): 91–115.
- Hamilton, J. 1994. *Time Series Analysis*. Princeton: Princeton University Press.
- Iacoviello, M. 2005. “House prices, borrowing constraints and monetary policy in the business cycle.” *American Economic Review* 95(3): 739–764.
- Iacoviello, M. and S. Neri. 2007. “The Role of Housing Collateral in an Estimated Two-Sector Model of the US Economy.” Photocopy, Boston College.
- Jappelli, T. 1990. “Who is Credit Constrained in the US Economy?” *Quarterly Journal of Economics* 94: 219–234.
- Kiyotaki, N. and J. Moore. 1997. “Credit Cycles.” *The Journal of Political Economy* 105: 211–248.
- Kostenbauer, K. 2001. “Housing Depreciation in the Canadian CPI.” *Statistics Canada Publication* 62F0014MIE(15).
- Kozicki, S. and P. Tinsley. 2002. “Alternative Sources of the Lag Dynamics of Inflation.” Federal Reserve Bank of Kansas Working Paper No. RWP02-12.
- Lam, J.P. and G. Tkacz. 2004. “Estimating Policy-Neutral Interest Rates for Canada Using a Dynamic Stochastic General Equilibrium Framework.” *Journal of Economics and Statistics* 140(1): 89–126.
- Lawrence, E. 1991. “Poverty and the rate of Time Preference: Evidence from Panel Data.” *Journal of Political Economy* 99(1): 54–77.
- Meier, A. and G. Muller. 2006. “Fleshing Out the Monetary Transmission Mechanism: Output Composition and the Role of Financial Frictions.” *Journal of Money, Credit, and Banking* 38: 195–214.

Ortega, E. and N. Rebei. 2006. “the welfare implications of inflation versus price level targeting in a small two-sector, small open economy.” Working paper 2003-12 Bank of Canada.

Samwick, A. 1998. “Discount Rate Heterogeneity and Social Security Reform.” *Journal of Development Economics* 57(1): 117–146.

Appendix A: Tables

Table 1:FA Parameters

Parameter		Prior Distribution			Posterior Distribution		
		type	mean	std	mean	5%	95%
Habit formation	γ	beta	0.7	0.05	0.7969	0.7499	0.8454
Adj. Cost for Housing	ψ_h	gamma	1	0.5	1.6301	1.2002	2.0633
Adj. Cost for capital	ψ_k	gamma	1	0.5	1.6199	1.1615	2.0450
Price Stickiness Domest. ℓ	θ_d	beta	0.67	0.05	0.7855	0.7380	0.8362
Price Stickiness Imports	θ_m	beta	0.67	0.05	0.6573	0.5831	0.7275
Price Stickiness Exports	θ_x	beta	0.67	0.05	0.5871	0.4941	0.6826
Elasticity of Country Risk Premium	Φ	inv. gamma	0.001	Inf	0.0005	0.0002	0.0008
UIP Lag	Φ_s	beta	0.25	0.15	0.1251	0.0199	0.2259
Housing factor share	ν	beta	0.02	0.01	0.0459	0.0269	0.0647
Share of Patient HHs	α	beta	0.65	0.075	0.8458	0.7926	0.9013
Firm LTV rate	mk	beta	0.5	0.1	0.3155	0.1793	0.4462
<i>Monetary Policy Rule</i>							
Int. Rate Smoothing	ρ_r	beta	0.75	0.1	0.7068	0.6545	0.7573
Response to Inflation	ρ_π	gamma	1.65	0.25	2.5398	2.0902	2.9792
Response to Output	ρ_y	gamma	0.125	0.0625	0.0254	0.0077	0.0420
<i>Auto Regressive Coefficients of shocks</i>							
Housing Demand	ρ_j	beta	0.85	0.1	0.9505	0.9234	0.9783
Consumption	ρ_c	beta	0.85	0.1	0.5671	0.3886	0.7423
Housing Supply	ρ_{Ah}	beta	0.85	0.1	0.9442	0.9072	0.9847
Capital Supply	ρ_{Ak}	beta	0.85	0.1	0.9166	0.8768	0.9560
Technology	ρ_z	beta	0.85	0.1	0.8321	0.7848	0.8780
Export Demand	ρ_x	beta	0.85	0.1	0.7943	0.7304	0.8584
Exchange Rate	ρ_s	beta	0.85	0.1	0.8532	0.7335	0.9502

Continued on Next Page

Table 1 Continued							
Parameter		Prior Distribution			Posterior Distribution		
		type	mean	std	mean	5%	95%
<i>Standard Deviations of shocks</i>							
Housing Supply	σ_{Ah}	inv. gamma	Inf	0.06	0.0622	0.0452	0.0783
Capital Supply	σ_{Ak}	inv. gamma	Inf	0.15	0.0745	0.0524	0.0951
Housing Demand	σ_j	inv. gamma	Inf	0.1	0.1201	0.0753	0.1615
Technology	σ_z	inv. gamma	Inf	0.025	0.0280	0.0199	0.0357
Monetary Policy	σ_{MP}	inv. gamma	Inf	0.003	0.0036	0.0031	0.0041
Consumption	σ_c	inv. gamma	Inf	0.04	0.0361	0.0280	0.0440
Export Demand	σ_X	inv. gamma	Inf	0.04	0.0374	0.0329	0.0416
Exchnng Rate	σ_S	inv. gamma	Inf	0.005	0.0039	0.0022	0.0056

Table 2: NOFA Parameters

Parameter		Prior Distribution			Posterior Distribution		
		type	mean	std	mean	5%	95%
Habit formation	γ	beta	0.7	0.05	0.7863	0.7318	0.8429
Adj. Cost for Housing	ψ_h	gamma	1	0.5	1.6247	1.1842	2.0303
Adj. Cost for capital	ψ_k	gamma	1	0.5	2.0859	1.3753	2.8003
Domest. Price Stickiness	θ_d	beta	0.67	0.05	0.7263	0.6709	0.7846
Imports Price Stickiness	θ_m	beta	0.67	0.05	0.6823	0.6162	0.7453
Exports Price Stickiness	θ_x	beta	0.67	0.05	0.5787	0.4833	0.6728
Elasticity of Country Risk Premium	Φ	inv. gamma	0.001	Inf	0.0004	0.0002	0.0007
UIP Lag	Φ_s	beta	0.25	0.15	0.1618	0.0478	0.2757
Housing Factor Share	ν	beta	0.02	0.01	0.015	0.004	0.0251
Monetary Policy Rule							
Int. Rate Smoothing	ρ_r	beta	0.75	0.1	0.7041	0.6522	0.7540
Response to Inflation	ρ_π	gamma	1.65	0.1	2.3651	1.9283	2.7855
Response to Output	ρ_y	gamma	0.125	0.1	0.0173	0.0051	0.0297
<i>Auto Regressive Coefficients of Shocks</i>							
Housing Demand	ρ_j	beta	0.85	0.1	0.9416	0.9088	0.9763
Consumption	ρ_c	beta	0.85	0.1	0.6377	0.4819	0.7916
Housing Supply	ρ_{Ah}	beta	0.85	0.1	0.9406	0.9019	0.9812
Capital Supply	ρ_{Ak}	beta	0.85	0.1	0.9411	0.9063	0.9764

Continued on next page

Table 2 Continued							
Parameter		Prior Distribution			Posterior Distribution		
		type	mean	std	mean	5%	95%
<i>Auto Regressive Coefficients of Shocks (ctd...)</i>							
Technology	ρ_z	beta	0.85	0.1	0.8452	0.7913	0.8995
Export Demand	ρ_x	beta	0.85	0.1	0.8011	0.7374	0.8672
Exchange Rate	ρ_s	beta	0.85	0.1	0.8408	0.7333	0.9517
<i>Standard Deviations of shocks</i>							
Housing Supply	σ_{Ah}	inv. gamma	Inf	0.06	0.0621	0.0460	0.0777
Capital Supply	σ_{Ak}	inv. gamma	Inf	0.15	0.0684	0.0460	0.0893
Housing Demand	σ_J	inv. gamma	Inf	0.1	0.1467	0.0815	0.2066
Technology	σ_Z	inv. gamma	Inf	0.025	0.0225	0.0171	0.0278
Monetary Policy	σ_{MP}	inv. gamma	Inf	0.003	0.0035	0.003	0.0040
Consumption	σ_c	inv. gamma	Inf	0.04	0.0356	0.0268	0.0442
Export Demand	σ_X	inv. gamma	Inf	0.04	0.0364	0.0321	0.0407
Exchnng Rate	σ_S	inv. gamma	Inf	0.005	0.0037	0.0020	0.0054

Table 3: Standard Deviations

	Standard Deviations		
	Data	FA Model	NOFA Model
GDP	0.041	0.0403	0.0373
C	0.027	0.0382	0.0382
I^k	0.105	0.0994	0.1256
I^h	0.141	0.1380	0.1346
qh	0.105	0.1098	0.1087
π	0.0025	0.0040	0.0039
R	0.0069	0.0067	0.0060
s	0.138	0.0579	0.0596
TS	0.055	0.0569	0.0557

Table 4: FA Variance Decomposition

	A^h	A^k	j	$u_r(MP)$	$\zeta(Cons)$	z	s	X
<i>GDP</i>	1.57	14.71	15.6	2.88	6.62	24.99	7.85	25.8
<i>C</i>	0.63	16.51	7.35	3.27	29.65	22.06	12.44	8.1
I^k	0.02	94.44	2.18	0.81	0.01	1.29	0.82	0.43
I^h	75.22	0.69	21.86	0.11	0.07	1.19	0.56	0.3
<i>qh</i>	26.31	0.73	65.27	0.44	0.15	4.35	2.27	0.46
π	0.02	0.91	0.66	23.7	0.03	50.7	23.4	0.58
<i>R</i>	0.04	1.09	0.96	16.36	0.3	53.12	26.93	1.2
<i>s</i>	0.71	9.35	1.91	4.2	0.73	32.54	38.92	11.63
<i>TS</i>	0.73	0.42	9.37	4.46	3.98	3.46	16.61	60.98

Table 5: NOFA Variance Decomposition

	A^h	A^k	j	u_r	ζ	z	s	X
<i>GDP</i>	2.53	23.57	3.77	0.19	11.42	27.53	6.32	22.73
<i>C</i>	0.51	12.98	5.87	0.55	42.89	20.59	8.65	5.92
I^k	0	98.47	0.05	0.06	0.01	0.95	0.3	0.07
I^h	74.76	0.68	22.56	0.07	0.08	1.05	0.43	0.25
<i>q</i>	24.46	0.58	68.33	0.3	0.16	3.72	1.69	0.37
π	0	0.52	0.3	24.73	0.06	47.98	19.75	0.29
<i>R</i>	0.03	0.68	0.77	18.51	0.37	48.33	23.95	0.45
<i>s</i>	0.4	11.56	0.84	2.93	0.93	24.54	37.36	11.14
<i>TS</i>	1.11	1.81	2.84	2.87	5.72	3.01	14.84	62.52

Table 6: Correlations

w/ GDP	Data	FA	NOFA
C	0.77	0.70	0.66
I_h	0.53	0.36	0.30
I_k	0.74	0.44	0.51
q_h	0.00	0.39	0.27
π	0.30	-0.31	-0.34
R	-0.07	-0.41	-0.40
s	0.08	0.10	0.11
TS	0.18	0.11	0.20
w/ C			
I_h	0.83	0.03	-0.06
I_k	0.71	0.24	0.28
q_h	0.55	0.14	0.00

Table 7: Parameters For the Foreign Block

Parameter		Value
<i>Estimated Parameters</i>		
Habit Formation	B_f	0.6349
Coefficient on 1st lag of inflation	$G1$	0.06
Coefficient on 2nd lag of inflation	$G2$	0.6332
<i>Monetary Policy Rule</i>		
Response to Output	ρ_{yf}	0.2795
Response to Inflation	$\rho_{\pi f}$	1.7608
Smoothing Parameter 1	$\rho_{r,f1}$	0.4107
Smoothing Parameter 2	$\rho_{r,f2}$	0.7605
<i>Shocks</i>		
<i>Auto Regressive Coefficient</i>		
Persistence of a Phillips Curve shock	ρ_{isf}	0.791
Persistence of a Consumption Preference shock	ρ_{pcf}	0.5332
<i>Standard Deviations</i>		
Monetary Policy	σ_{rf}	0.0019
IS Curve AR(1)	σ_{isf}	0.0031
Consumption Preference AR(1)	σ_{pcf}	0.0156
<i>Calibrated Parameters</i>		
Foreign Agent Discount Factor	β_f	0.99

Appendix B: IRFS

Figure 1(a): Technology Shock

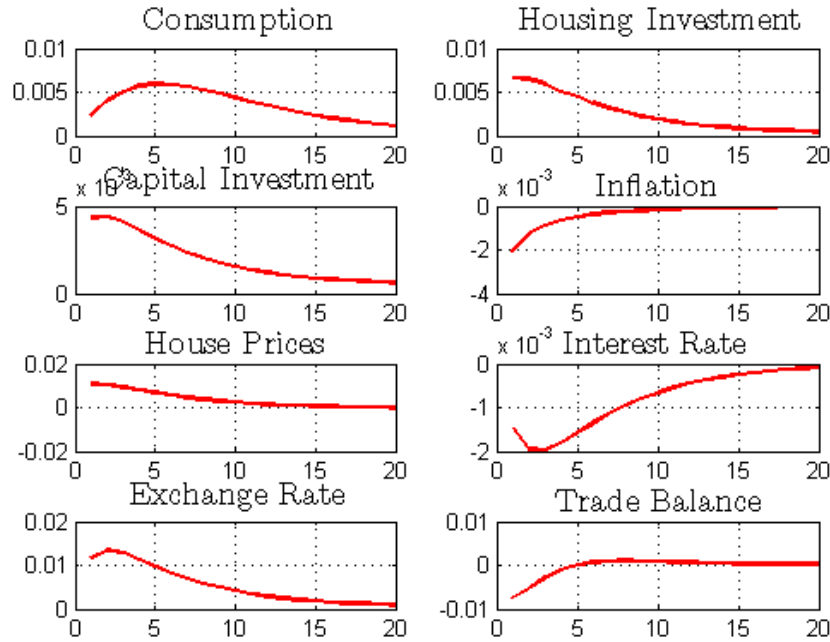


Figure 1(b): Consumption Preference Shock

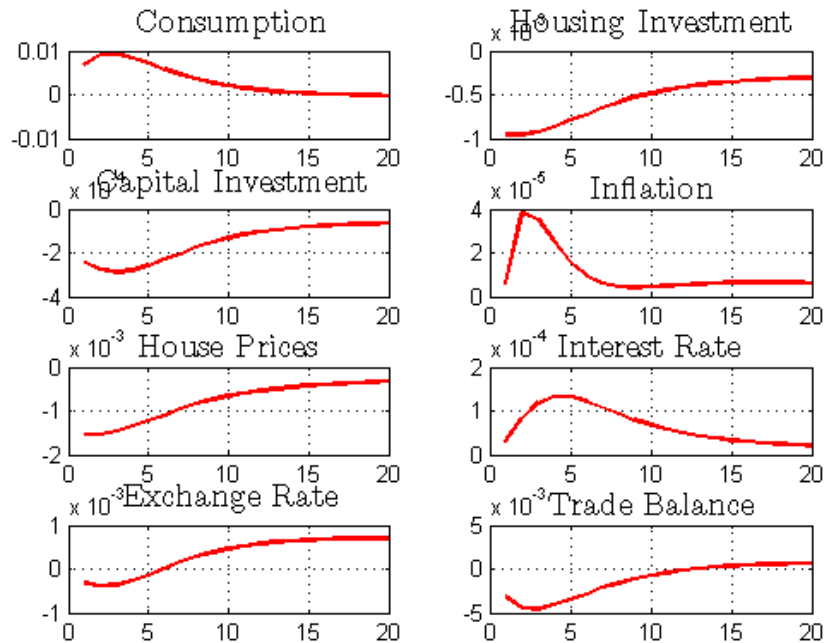


Figure 1(c): Technology Specific Investment Shock

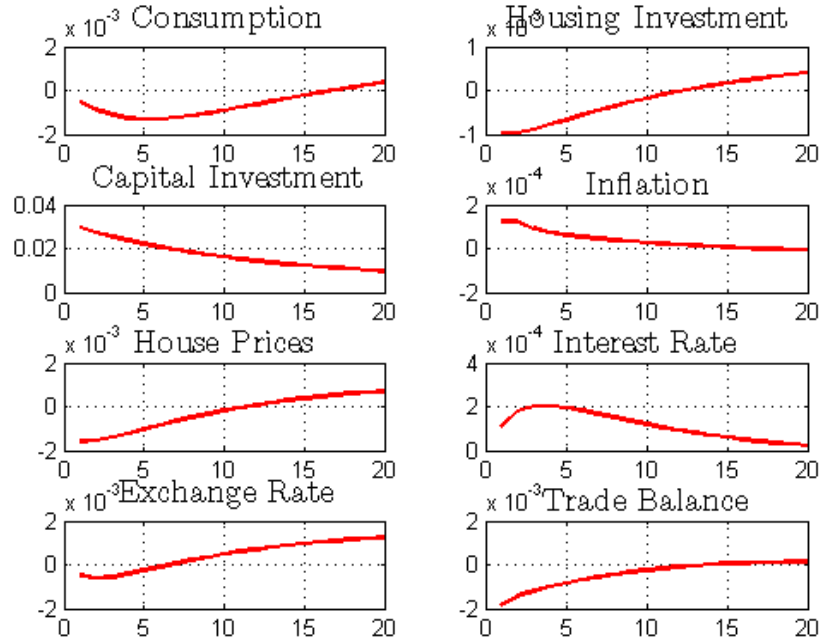


Figure 1(d): Housing Specific Investment Shock

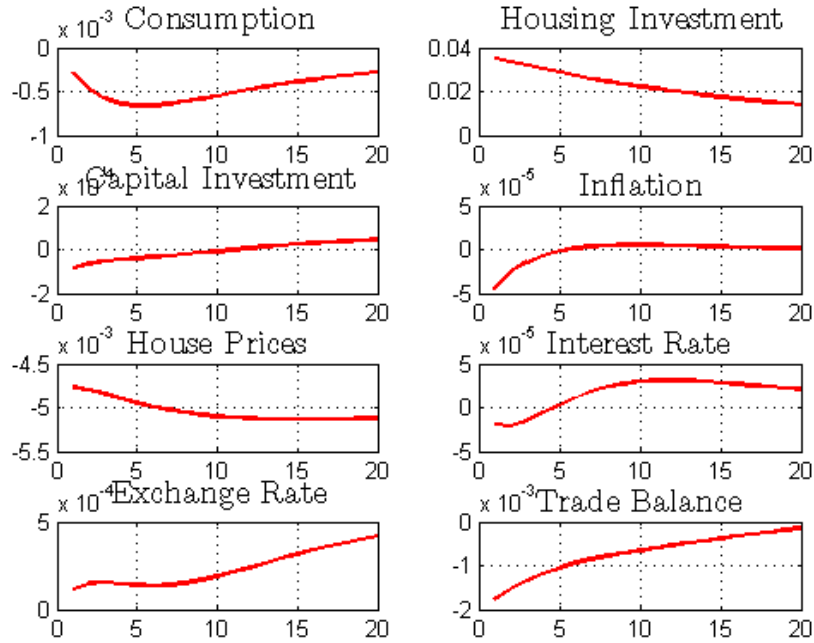


Figure 1(e): Housing Demand Shock

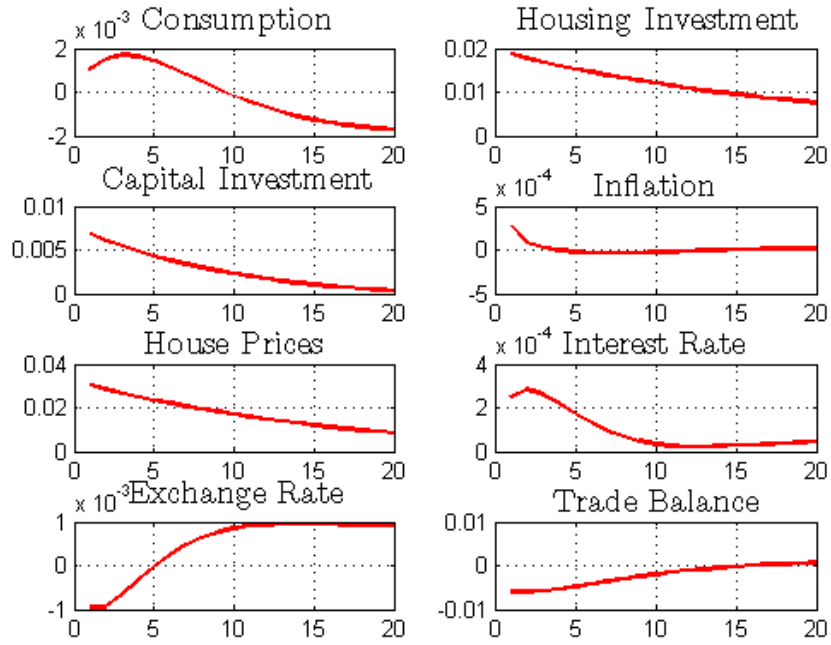


Figure 1(f): Monetary Policy Shock

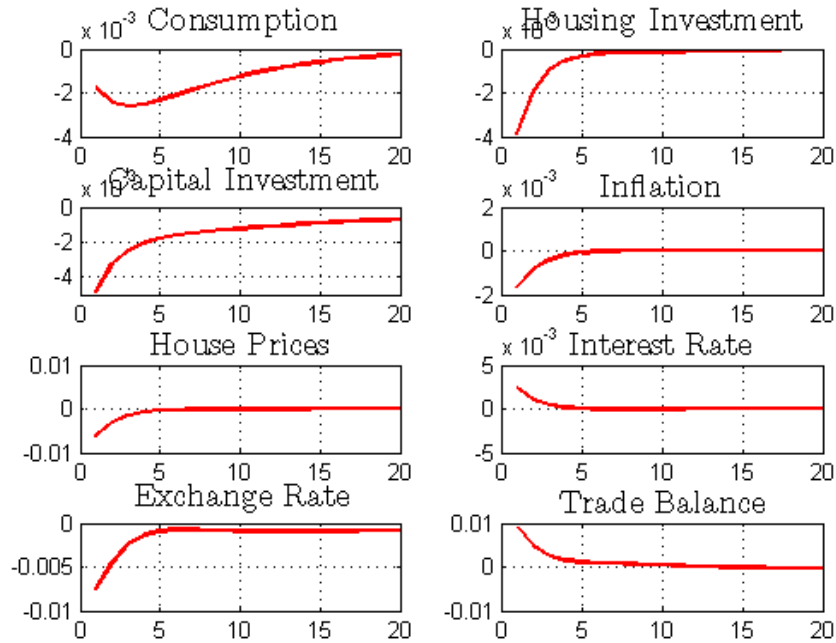


Figure 1(h): Export Demand Shock

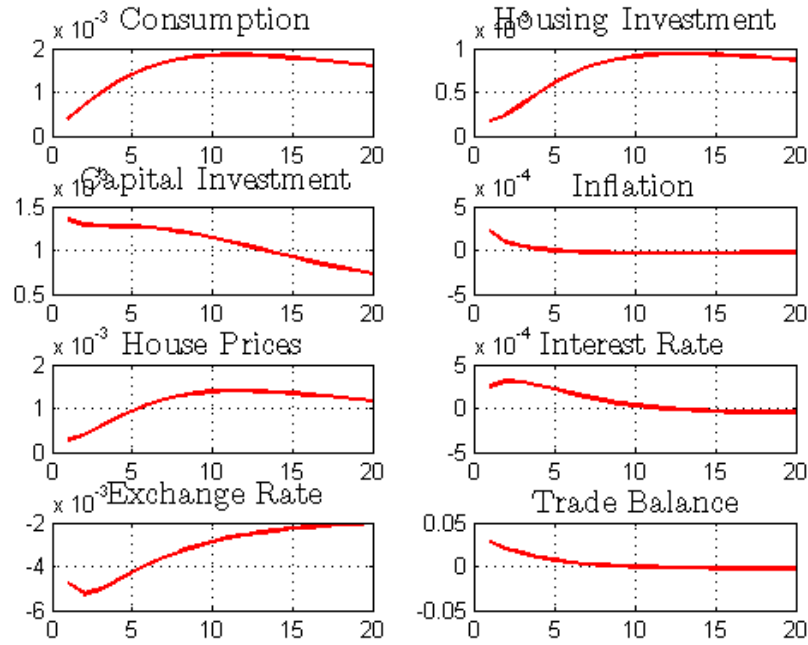


Figure 1(i): Risk Premium Shock

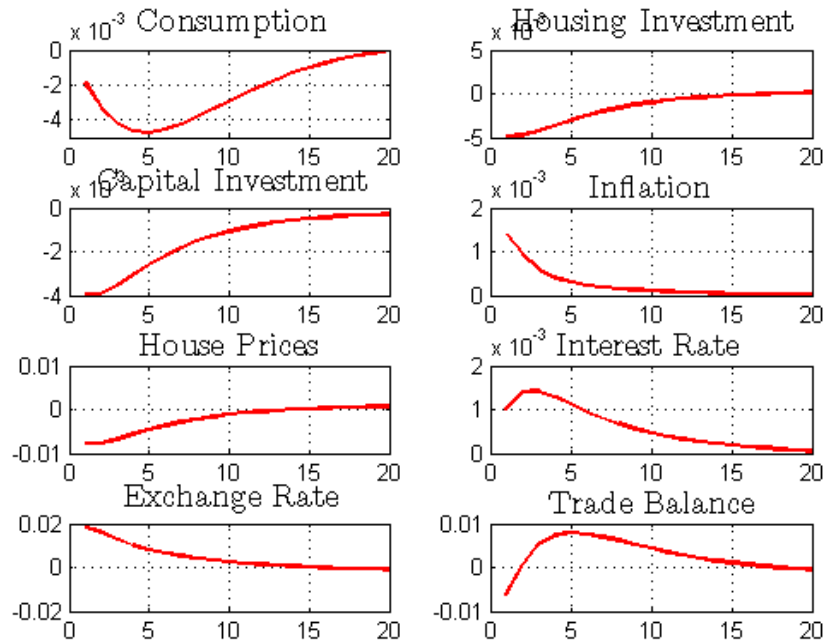


Figure 2(a): Monetary Policy Shock

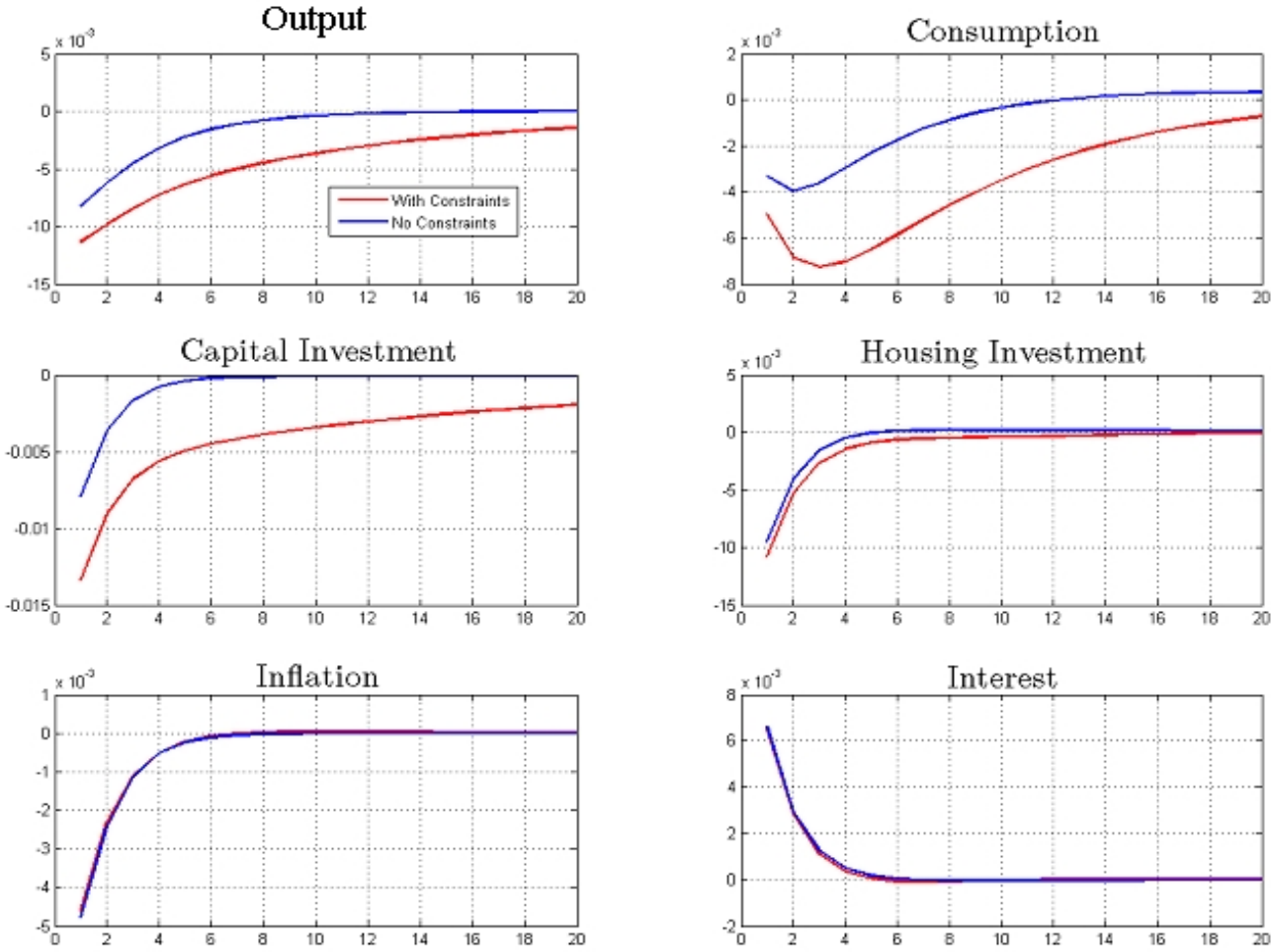


Figure 2(b): Housing Demand Shock

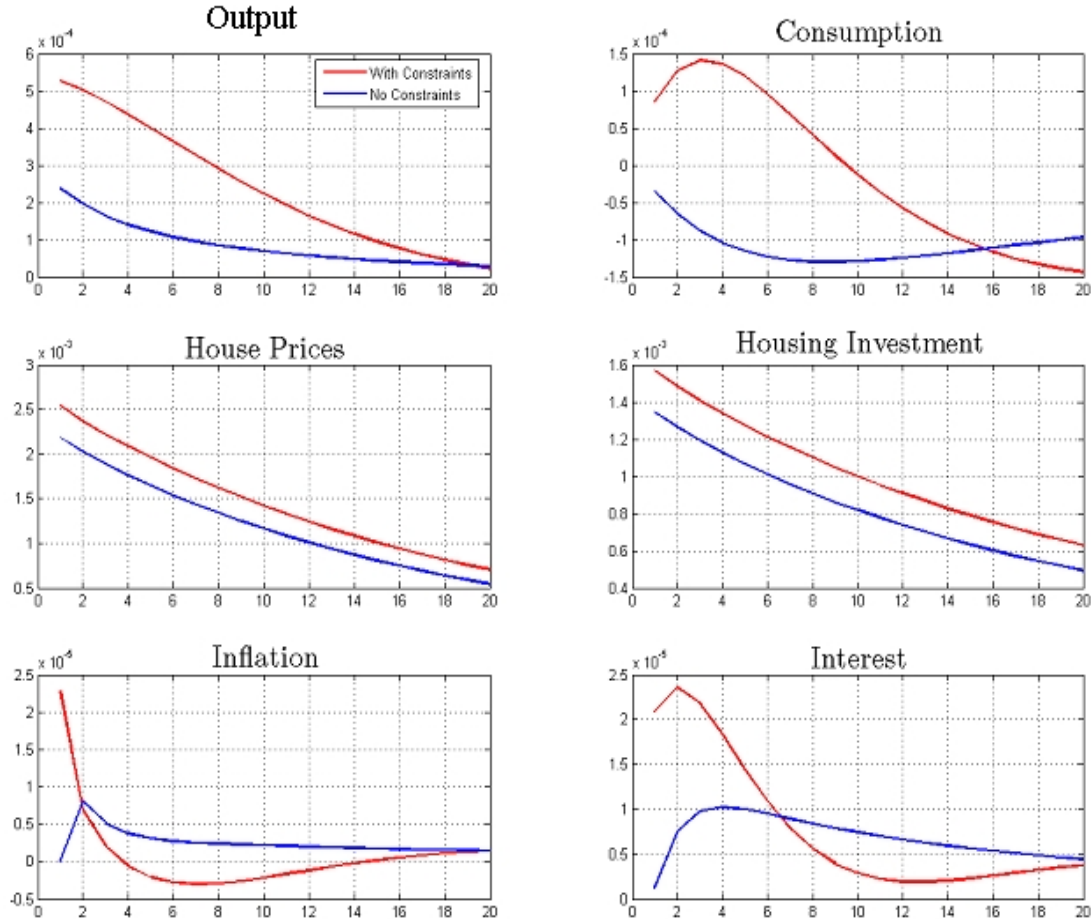


Figure 3(a): Consumption Historical Decomposition FA Model

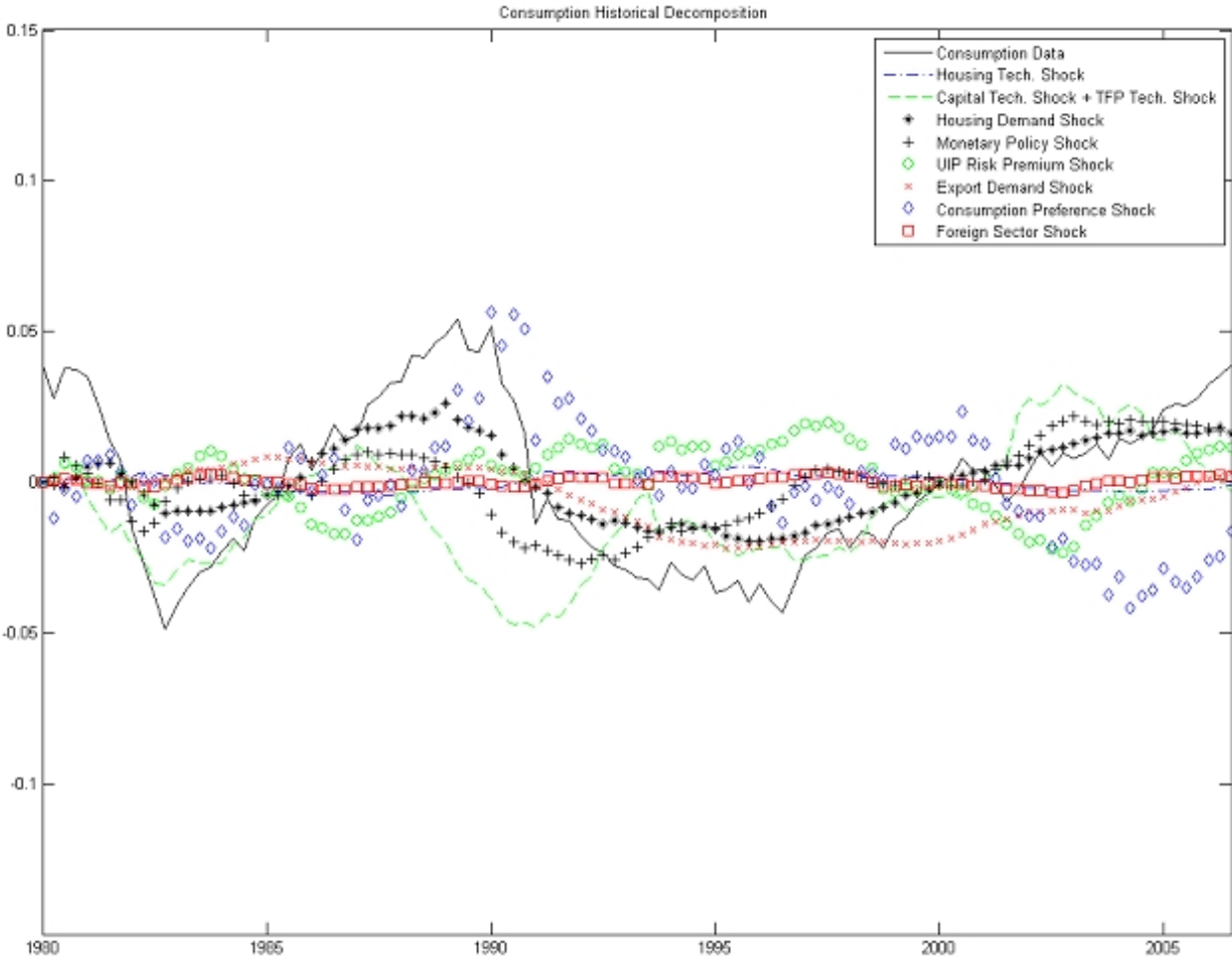


Figure 3(b): Consumption Historical Decomposition NOFA Model

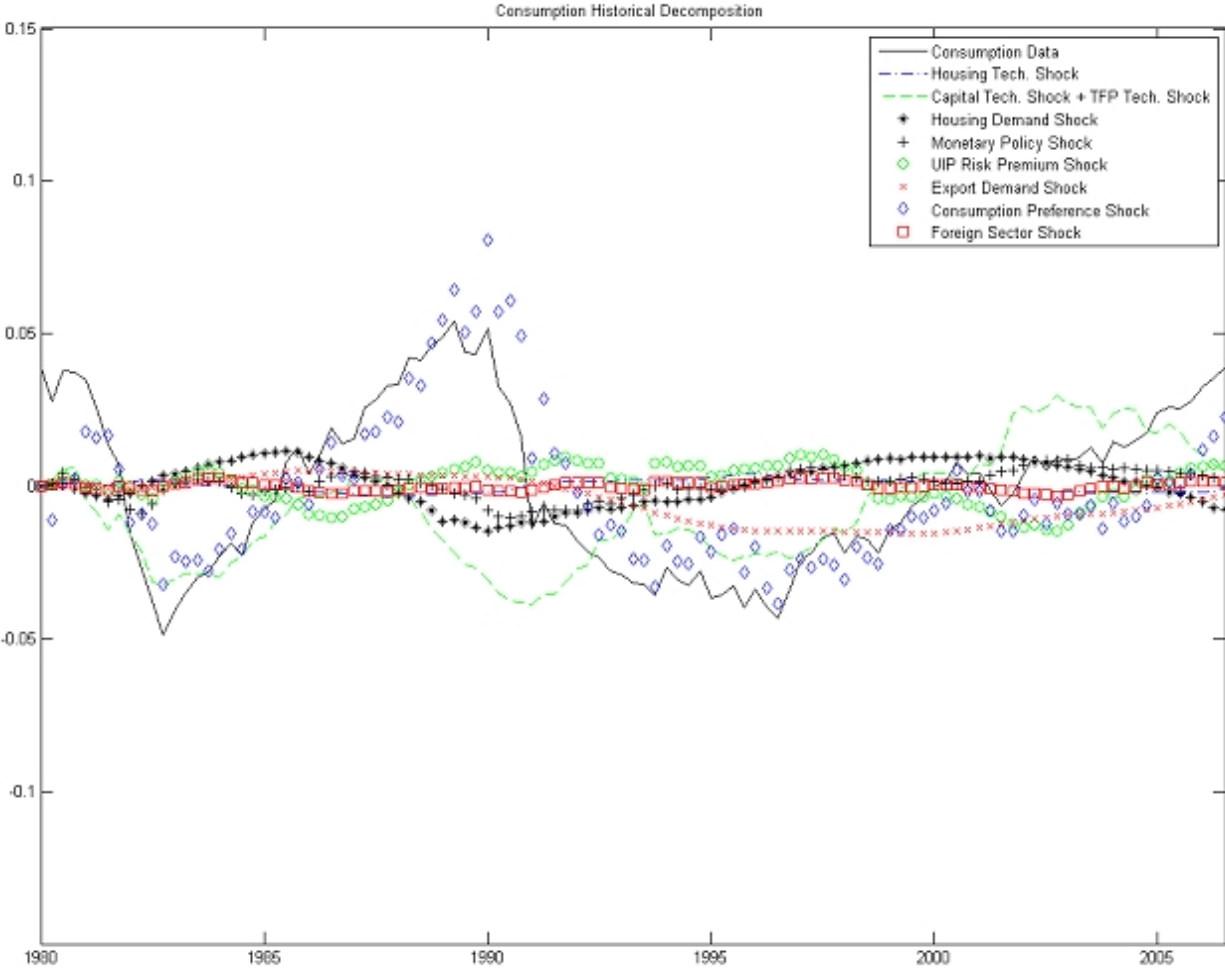


Figure 4: Datasets

