

The Determinants of Stock and Bond Return Comovements

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Research questions

- Establish **stylized facts** with respect to stock-bond return correlations.
- Explain **level** and **time variation** in stock-bond return correlations using **dynamic factor model**.

Only fundamental factors are considered (but we consider wide range)

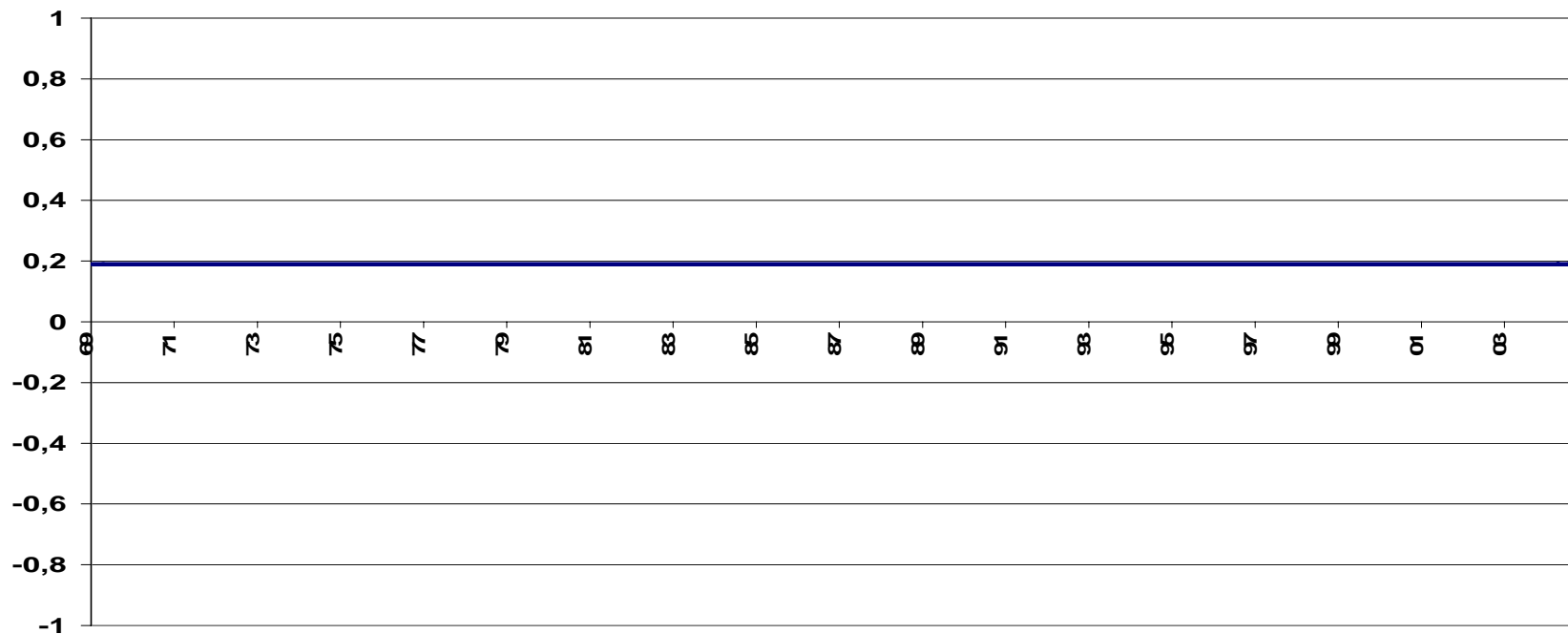
- Consider **non-fundamental** instruments to explain any **residual correlation**.

Stylized Facts

■ **Data:**

- ◆ NYSE-AMEX-NASDAQ value-weighted total excess returns from CRSP.
- ◆ 10-Year excess Bond Returns from CRSP US Treasury and Inflation Module.

■ **Unconditional Correlation:**



- Existing studies only modestly successful in generating realistic unconditional correlations using economic state variables:
 - ◆ **Shiller and Beltratti (JME,92)** : underestimate correlations using present value model with constant discount rates.
 - ◆ **Bekaert, Engstrom, and Grenadier (05)** : overestimate correlations using a consumption-based asset pricing model with stochastic risk aversion.

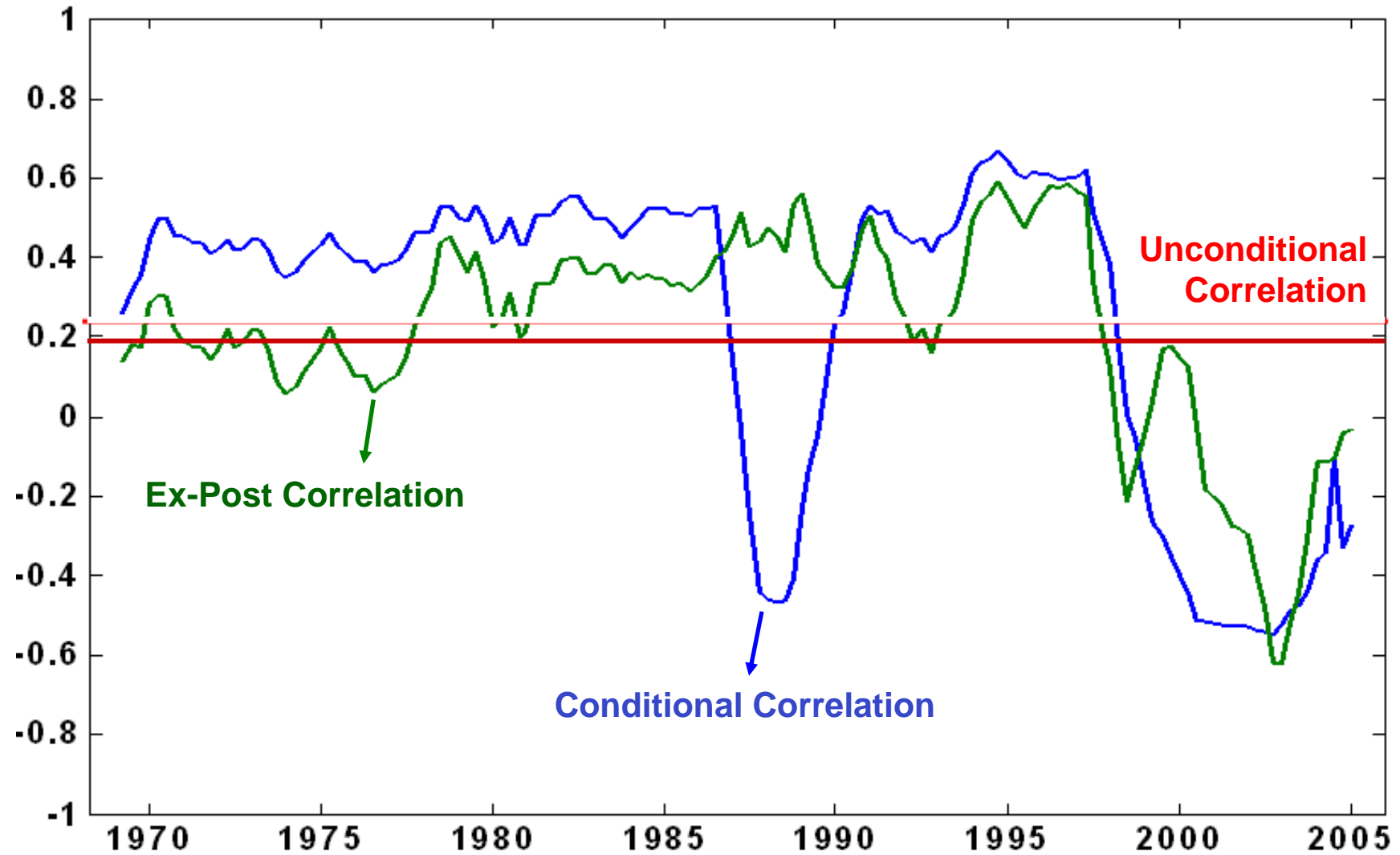
- Even more challenging: explaining conditional correlations.
- Two measures:
 - ◆ Ex-post quarterly correlations (based on daily within-quarter returns)
 - ◆ Conditional quarterly correlations estimated from:

$$r_t = E_{t-1}[r_t] + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega_t), \quad \Omega_t = \begin{bmatrix} \sigma_{s,t}^2 & \rho_{s,b,t} \sigma_{s,t}^2 \sigma_{b,t}^2 \\ \rho_{s,b,t} \sigma_{s,t}^2 \sigma_{b,t}^2 & \sigma_{b,t}^2 \end{bmatrix}$$

$$\begin{aligned} \sigma_{s,t}^2 &= \sigma_s^2(S_t) + \theta_s(S_t) \hat{\sigma}_{s,t-1}^2 \\ \sigma_{b,t}^2 &= \sigma_b^2(S_t) + \theta_b(S_t) \hat{\sigma}_{b,t-1}^2 \\ \rho_{s,b,t} &= \rho_{s,b}(S_t) + \theta_\rho(S_t) \hat{\rho}_{s,b,t-1} \end{aligned}$$

ex-post
measures

- Two state RS Model
- Best performing model out of large set of alternatives.



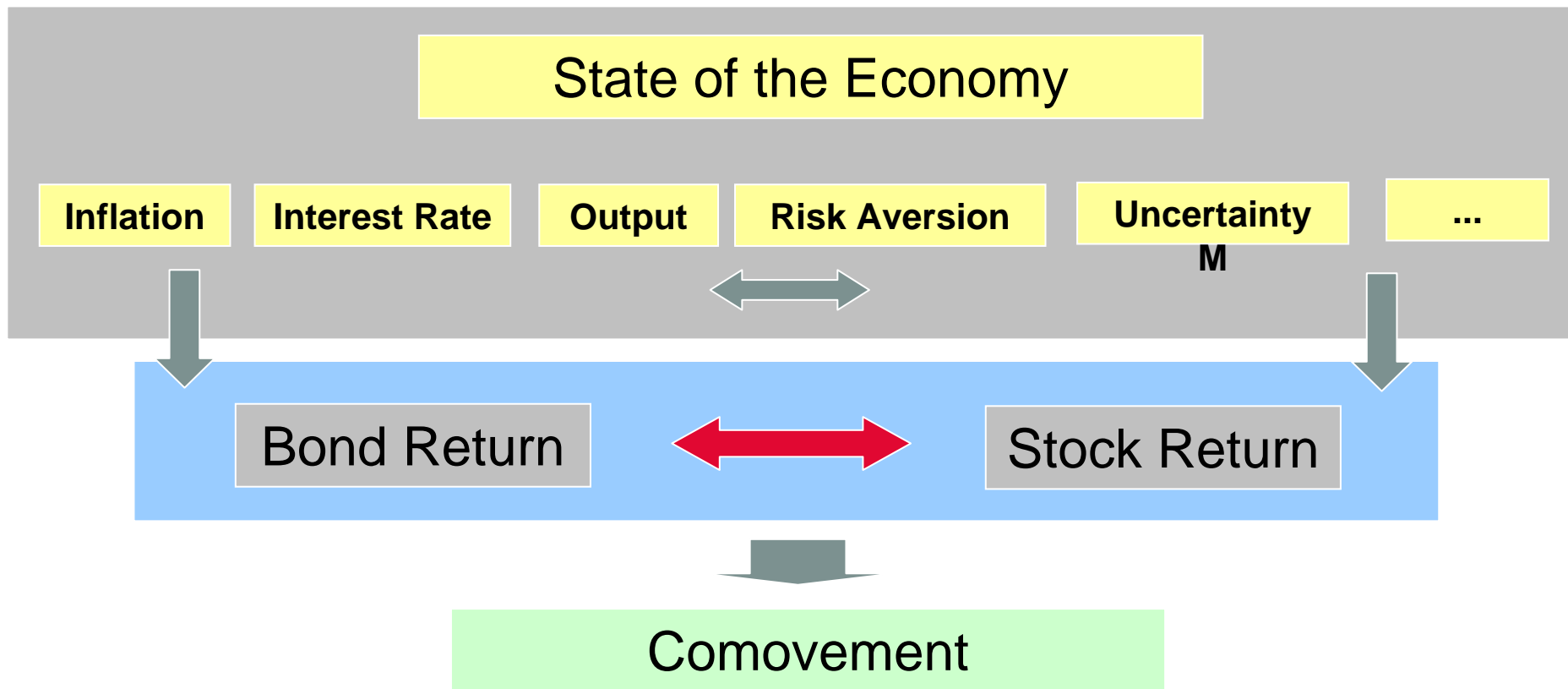
Conditional correlations

- Existing literature has mainly focused on developing *statistical* models to characterize the time variation in stock-bond correlations (Guidolin and Timmermann (JAE,06), Capiello, Engle, and Sheppard (ECB,03)).
- ... or on finding explanations for the short periods of extreme negative correlations
 - ◆ Connolly, Stivers, and Sun (JFQA,05) relate the episodes of negative correlations to a 'Flight to Safety' phenomenon.
 - ◆ Investors rebalance portfolios from stocks to bonds in times of increased market uncertainty.
 - ◆ Unfortunately, they do not correct for 'economic fundamentals'

In search for fundamentals

- Explain average stock-bond correlation and its time variation through common exposures to economic state variables.

- We consider wide range of economic state variables:
 - ◆ **Level variables:** inflation, output gap (GDP growth), short rate, risk aversion, cash flow growth.
 - ◆ **Expectation variables:** expected inflation and output gap (GDP growth)
 - ◆ **Uncertainty variables:** uncertainty in inflation and GDP growth.



- Consider following model:

$$\begin{bmatrix} r_{s,t} \\ r_{b,t} \end{bmatrix} = \begin{bmatrix} E_{t-1}(r_{s,t}) \\ E_{t-1}(r_{b,t}) \end{bmatrix} + \begin{bmatrix} \beta_{s,t-1} \\ \beta_{b,t-1} \end{bmatrix} \times F_t + \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{bmatrix}$$

Stock/Bond
returns

Expected
Stock/Bond
returns

(Time-Varying)
Factor Exposures

Model
Residuals

$$F_t = X_t - E_{t-1}(X_t) \sim N(0, \Sigma_{t-1})$$

Shock to economic
State variable

Diagonal
factor VCV

- The fundamental-implied correlation is given by:

$$\begin{aligned} \rho_{t-1}^F(r_{s,t}, r_{b,t}) &= \frac{\text{COV}_{t-1}(r_{s,t}, r_{b,t})}{\sqrt{\text{var}_{t-1}(r_{s,t})} \sqrt{\text{var}_{t-1}(r_{b,t})}} \\ &= \frac{\beta_{s,t-1}^1 \beta_{b,t-1}^1 \text{var}_{t-1}(F_t^1) + \dots + \beta_{s,t-1}^n \beta_{b,t-1}^n \text{var}_{t-1}(F_t^n)}{\sqrt{\text{var}_{t-1}(r_{s,t})} \sqrt{\text{var}_{t-1}(r_{b,t})}} \end{aligned}$$

- Implications:

- ◆ Correlations driven by Betas and Factor Variances.
- ◆ Contribution of Factor to correlation increases with the Factor's variance.
- ◆ Positive (negative) correlation if stock/bond betas have same (different) signs.

3 state variable model

- Vector of State Variables X : inflation, short rate, output gap
- Basic building blocks of New-Keynesian macro-economic models.
- Stocks:
 - ◆ *Claim on real* (stochastic) cash flows (output gap)
 - ◆ 'Real' discount rate (real rate, output gap as driver of risk premium)
 - ◆ ... but considerable evidence stock prices react to inflation
(Mundell – Tobin effect \leftrightarrow stocks poor inflation hedges)
- Bonds:
 - ◆ Discounted value of *nominal* fixed cash flows (inflation, real rate)
 - ◆ ... but risk premium may also react to output shocks

3 state variable model

- Vector of State Variables X : inflation, short rate, output gap
- Consider following VAR:

$$X_t = \mu + AX_{t-1} + \Gamma_{t-1}F_t$$

- Γ_{t-1} determines contemporaneous correlation between state variable shocks, i.e. it identifies the factors.
- We impose structure on Γ_{t-1} in two ways:
 - ◆ *Choleski Decomposition* (Non-structural VAR)
 - ◆ *Structural New-Keynesian Model* (Structural VAR)

- Factors are potentially heteroskedastic:

$$F_t \sim N(\mathbf{0}, \Sigma_{t-1})$$

- Models for diagonal Σ_{t-1} :

- | | | |
|--|---|------------------------|
| 1. Homoskedastic Model (constant) | → | Σ |
| 2. State-Dependent Model (time-varying) | → | $\Sigma(X_{t-1})$ |
| 3. Regime-Switching Model (time-varying) | → | $\Sigma(S_t)$ |
| 4. RS State-Dependent Model (time-varying) | → | $\Sigma(X_{t-1}, S_t)$ |

- VAR in output gap, inflation, interest rate (see Bikbov (05))

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} \mu^y \\ \mu^\pi \\ \mu^i \end{bmatrix} + A \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ \gamma_{21} & 1 & 0 \\ \gamma_{31}(S_t^m) & \gamma_{32}(S_t^m) & 1 \end{bmatrix} \begin{bmatrix} F_t^y \\ F_t^\pi \\ F_t^i \end{bmatrix}$$

- ◆ S_t^s shifts structural parameters in Monetary Policy equation.
- ◆ Diagonal VCV Matrix, with following specifications:

$$\text{var}_{t-1}(F_t^y) = \exp(\alpha_y(S_t^e) + \theta_{1,y}y_{t-1} + \theta_{2,y}y_{t-1}d_{t-1})$$

$$\text{var}_{t-1}(F_t^\pi) = \exp(\alpha_\pi(S_t^e) + \theta_{1,\pi}\pi_{t-1} + \theta_{2,\pi}\pi_{t-1}d_{t-1})$$

$$\text{var}_{t-1}(F_t^i) = \exp(\alpha_i(S_t^{ir}) + \theta_i(S_t^{ir})i_{t-1})$$

S_t^e shifts variance of exogenous shocks



S_t^{ir} shifts variance of monetary policy (interest rate) shocks

- Structural New-Keynesian Model (see e.g. Bekaert, Cho, Moreno (05))

$$y_t = \alpha_{IS} + \mu E_t [y_{t+1}] + (1 - \mu) y_{t-1} - \phi (i_t - E_t [\pi_{t+1}]) + F_t^y$$

$$\pi_t = \alpha_{AS} + \delta E_t [\pi_{t+1}] + (1 - \delta) \pi_{t-1} + \lambda y_t + F_t^\pi$$

$$i_t = \alpha_{MP} + \rho i_{t-1} + (1 - \rho) \left[\beta (S_t^m) E_t [\pi_{t+1}] + \gamma (S_t^m) y_t \right] + F_t^i$$

- Interpretable structural parameters $\mu, \phi, \delta, \lambda, \rho, \beta, \gamma$
- Variance specification as before:

$$\text{var}_{t-1}(F_t^y) = \exp(\alpha_y (S_t^e) + \theta_{1,y} y_{t-1} + \theta_{2,y} y_{t-1} d_{t-1})$$

$$\text{var}_{t-1}(F_t^\pi) = \exp(\alpha_\pi (S_t^e) + \theta_{1,\pi} \pi_{t-1} + \theta_{2,\pi} \pi_{t-1} d_{t-1})$$

$$\text{var}_{t-1}(F_t^i) = \exp(\alpha_i (S_t^{ir}) + \theta_i (S_t^{ir}) i_{t-1})$$

- Regime variables as before: $S_t = (S_t^m, S_t^e, S_t^{ir})$

- Rational expectations solution implies large number of highly nonlinear restrictions on parameters.
- Solving the model further complicated by the presence of regime switching and heteroskedasticity in the shocks.
- Bikbov (2005) shows using a simpler version that identification of the regimes is only possible when term structure data is added to the information set.
- Our solution: replace $E_t [X_{t+1}]$ by the median of the individual survey forecasts for the different state variables, denoted by X_t^f
- Follow-up paper:

Baele, Bekaert, Cho, Inghelbrecht, Moreno (??)

- Simple affine asset pricing models imply asset returns are constant beta functions of innovations in state variables.
- We allow betas to vary through time, but put sufficient structure on betas to avoid picking up non-fundamental sources:
 - ◆ **Duration Effects:** Interest rate sensitivity increases with duration
 - Bonds: duration decreases with interest rate level.
 - Equity: duration decreases with level of dividend yield
 - ◆ **Uncertainty:** dispersion in beliefs increases the effect of economic shocks on returns (David and Veronesi (04)).

$$\beta_{s,t-1}^y = \beta_{s,0}^y + \beta_{s,1}^y yd_{t-1} \quad \beta_{s,t-1}^\pi = \beta_{s,0}^\pi + \beta_{s,1}^\pi \pi d_{t-1} \quad \beta_{s,t-1}^i = \beta_{s,0}^i + \beta_{s,1}^i dy_{t-1}$$

$$\beta_{b,t-1}^y = \beta_{b,0}^y + \beta_{b,1}^y yd_{t-1} \quad \beta_{b,t-1}^\pi = \beta_{b,0}^\pi + \beta_{b,1}^\pi \pi d_{t-1} \quad \beta_{b,t-1}^i = \beta_{b,0}^i + \beta_{b,1}^i i_{t-1}$$

- Two-stage Procedure (FIRST state variable model, SECOND factor model)
- Model Selection
 - ◆ Test on Mean and Autocorrelation of $\hat{\varepsilon}_{s,t} \hat{\varepsilon}_{b,t}$
 - ◆ Distance (MAD) Model-Implied Conditional Correlation with:
 - Realized (ex-post) correlation
 - Conditional correlations from our statistical model.
 - ◆ Statistical Correlation Model evaluated on stock-bond residuals
 - ◆ R^2 of factor model

- Constant Beta Three Factor Model:
 - ◆ All indicators prefer structural over non-structural model.
 - Likely explanation : better identification of shocks.
 - ◆ Regime-Switching volatility specification.
 - Lagged values of level and uncertainty measures ‘redundant’

- Mean Equation:

$$y_t = \alpha_{IS} + 0.75 E_t [y_{t+1}] + (1 - 0.75) y_{t-1} + 0.025 (i_t - E_t [\pi_{t+1}]) + F_t^y$$

$$\pi_t = \alpha_{AS} + 0.40 E_t [\pi_{t+1}] + (1 - 0.40) \pi_{t-1} + 0.02 y_t + F_t^\pi$$

$$i_t = \alpha_{MP} + 0.78 i_{t-1} + (1 - 0.78) \begin{bmatrix} 1.88 \\ 0.92 \end{bmatrix} E_t [\pi_{t+1}] + \begin{bmatrix} 0.14 \\ 0.25 \end{bmatrix} y_t + F_t^i$$

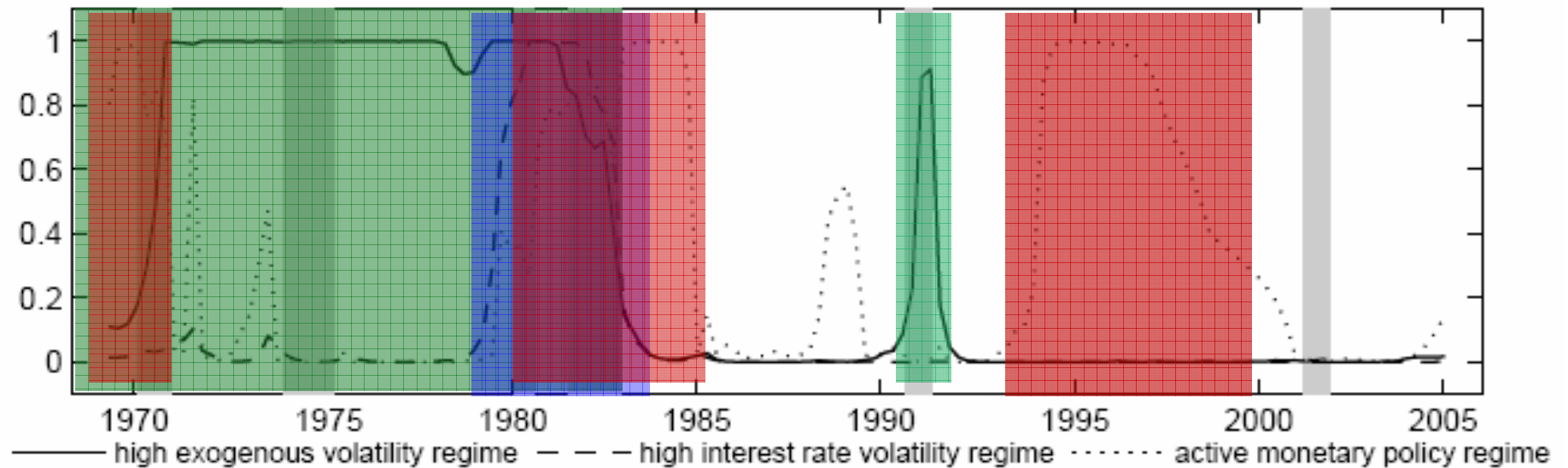
- Variance Equation:

$$\text{var}_{t-1}(F_t^y) = \exp(\begin{bmatrix} 0.29 & 0.44 \end{bmatrix})$$

$$\text{var}_{t-1}(F_t^\pi) = \exp(\begin{bmatrix} 0.20 & 0.37 \end{bmatrix})$$

$$\text{var}_{t-1}(F_t^i) = \exp(\begin{bmatrix} 0.07 & 0.27 \end{bmatrix})$$

Smoothed Probability of being in High Volatility Regime



- **Constant Beta Three Factor Model:**

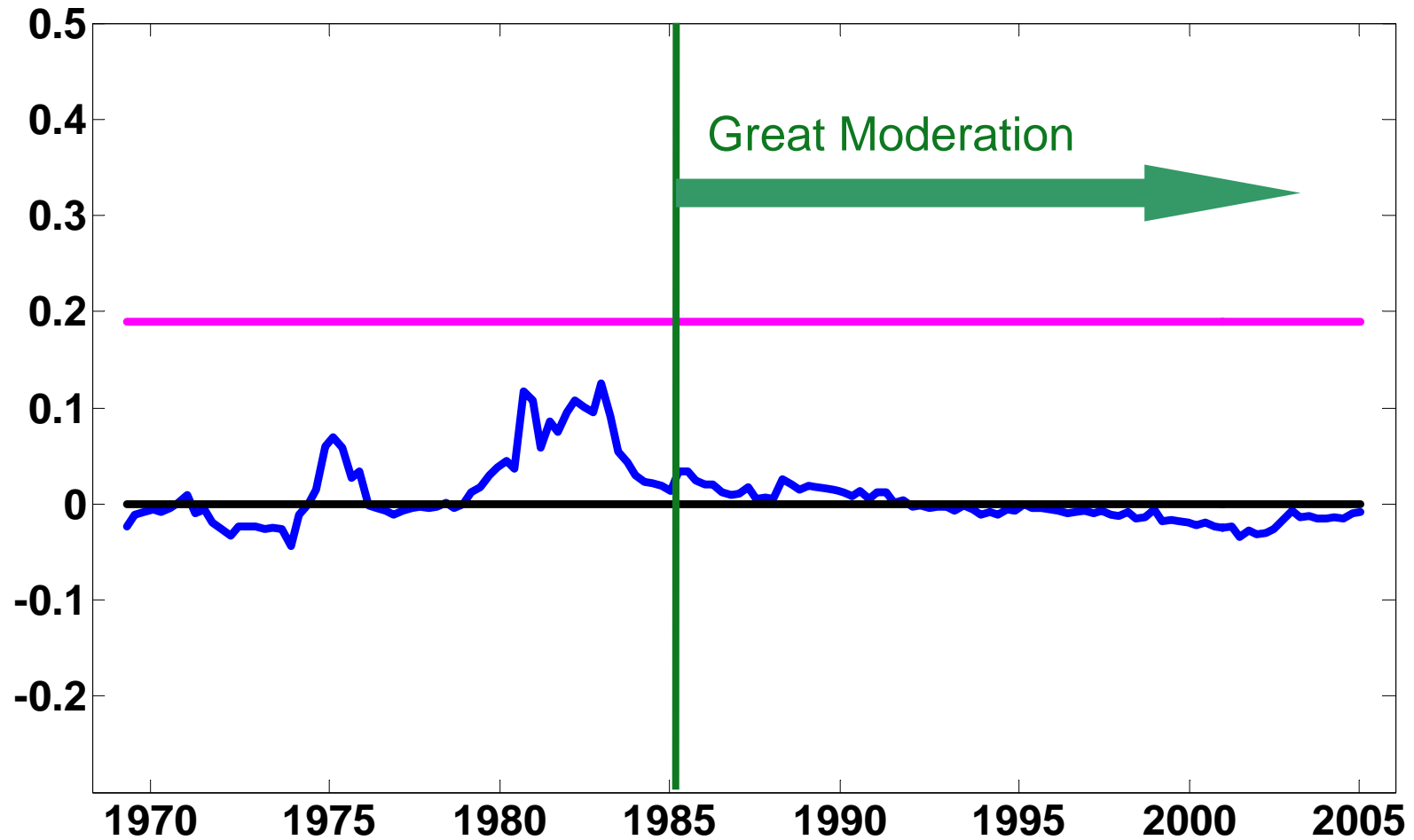
- ◆ All indicators prefer structural over non-structural model.
- ◆ Regime-Switching volatility specification.
- ◆ Beta estimates:

$$r_{s,t} - E_{t-1} [r_{s,t}] = 5.19F_t^y - 4.28F_t^\pi - 1.96F_t^i$$

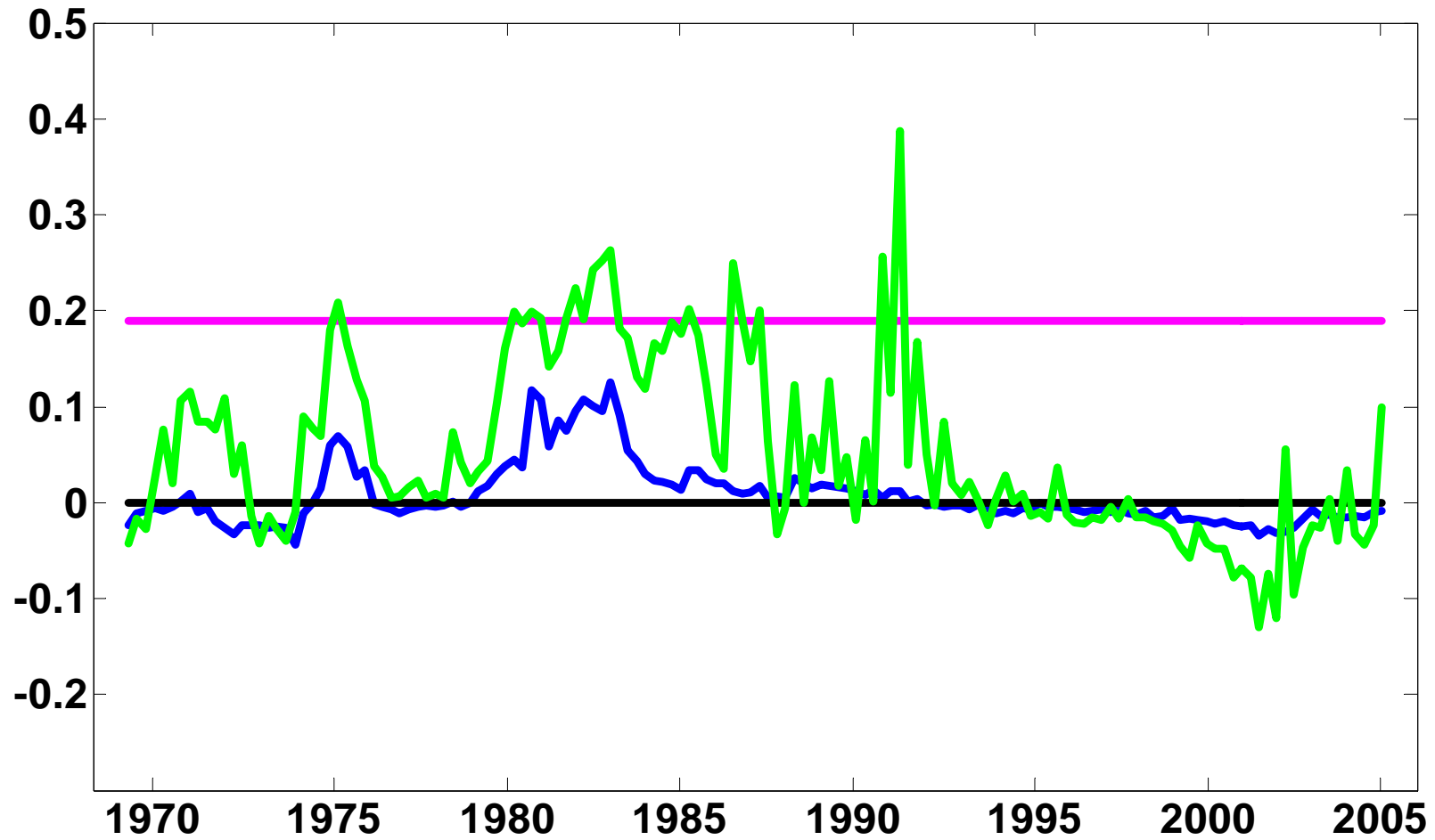
$$r_{b,t} - E_{t-1} [r_{b,t}] = -0.88F_t^y - 0.78F_t^\pi - 9.57F_t^i$$

- **Time-Varying Beta Three Factor Model:**

- ◆ At times betas get different signs, hence contribute to negative corr
- ◆ Poor statistical significance - Marginal increase in R^2 (for bonds)



- Output Gap, Inflation, Interest Rate as before.
- Expected inflation, Expected output gap.
 - ◆ Median of quarterly changes in Individual GDP Price Index Forecasts.
 - ◆ Median of quarterly changes in Individual Real GDP Forecasts.
- Inflation uncertainty, Output gap uncertainty.
 - ◆ Renewed interest in changes in fundamental uncertainty as source of asset price fluctuations (Bansal and Yaron (JF,04), Bekaert, Engstrom, Xing (05))
 - ◆ Possible correlation between uncertainty and risk premiums.
 - ◆ Average of standard deviations of the individual distributions of the forecasts for next quarter's change in GDP price index and real GDP.
- Cash Flow Growth.



- Our models may fail to capture time variation in risk premiums.
- Variation in equity premium essential in explaining the variability of the price-dividend ratio (e.g. [Cochrane \(RFS,92\)](#), [Ang and Bekaert \(RFS,06\)](#)).
- Sources of time-varying risk premiums:
 - ◆ Economic Uncertainty ([Bansal and Yaron \(JF,04\)](#))
 - ◆ Risk Aversion (Habit formation model of [Campbell and Cochrane \(JF,04\)](#))
 - ◆ Combination of the two ([Bekaert, Engstrom, and Xing \(06\)](#))
 - ◆ Our uncertainty and risk aversion measure are highly correlated.
- Economic (now) versus potentially behavioral (later) sources
- Campbell-Cochrane based measure from [Bekaert and Engstrom \(06\)](#)

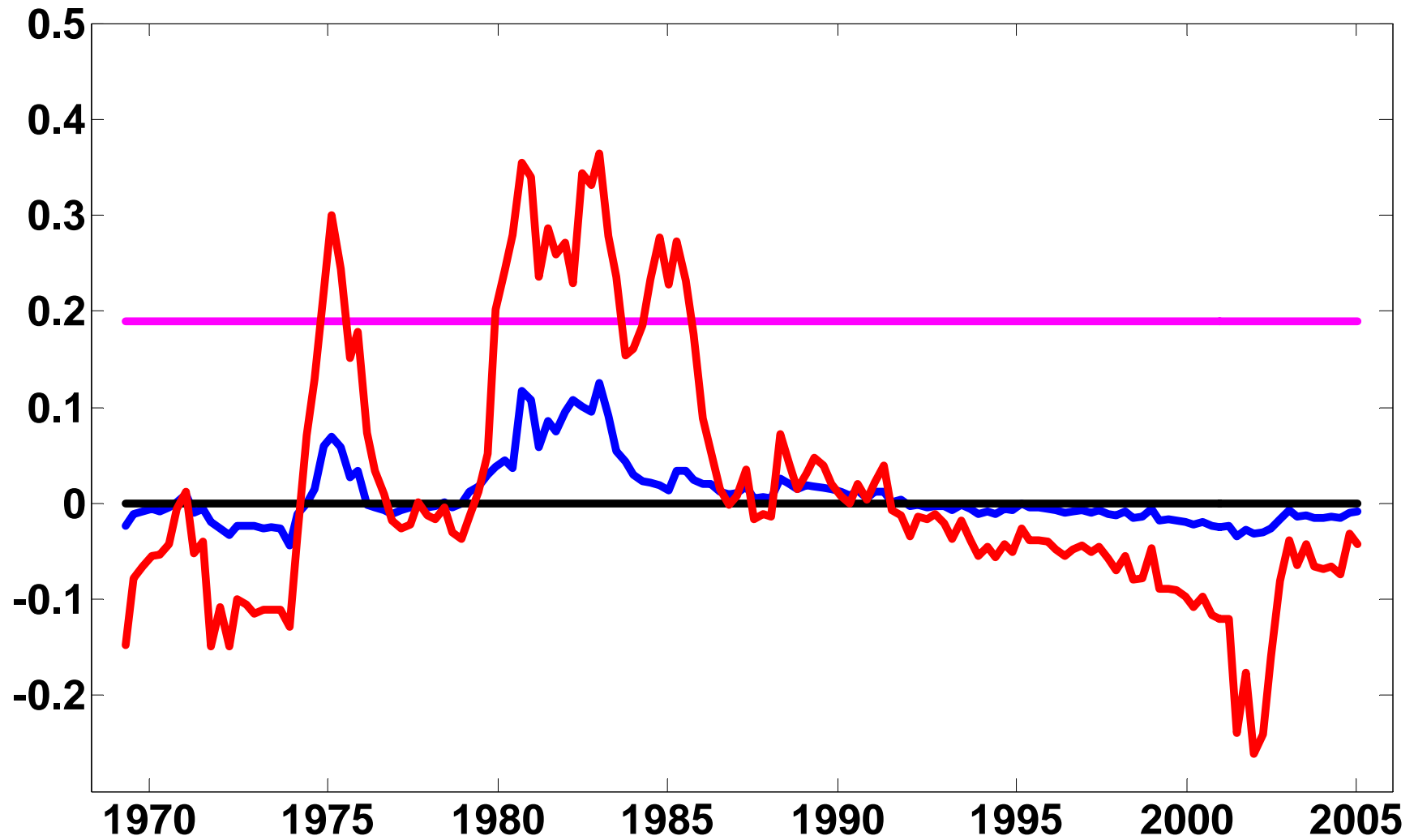
4-factor state variable model

- Results from constant beta model:

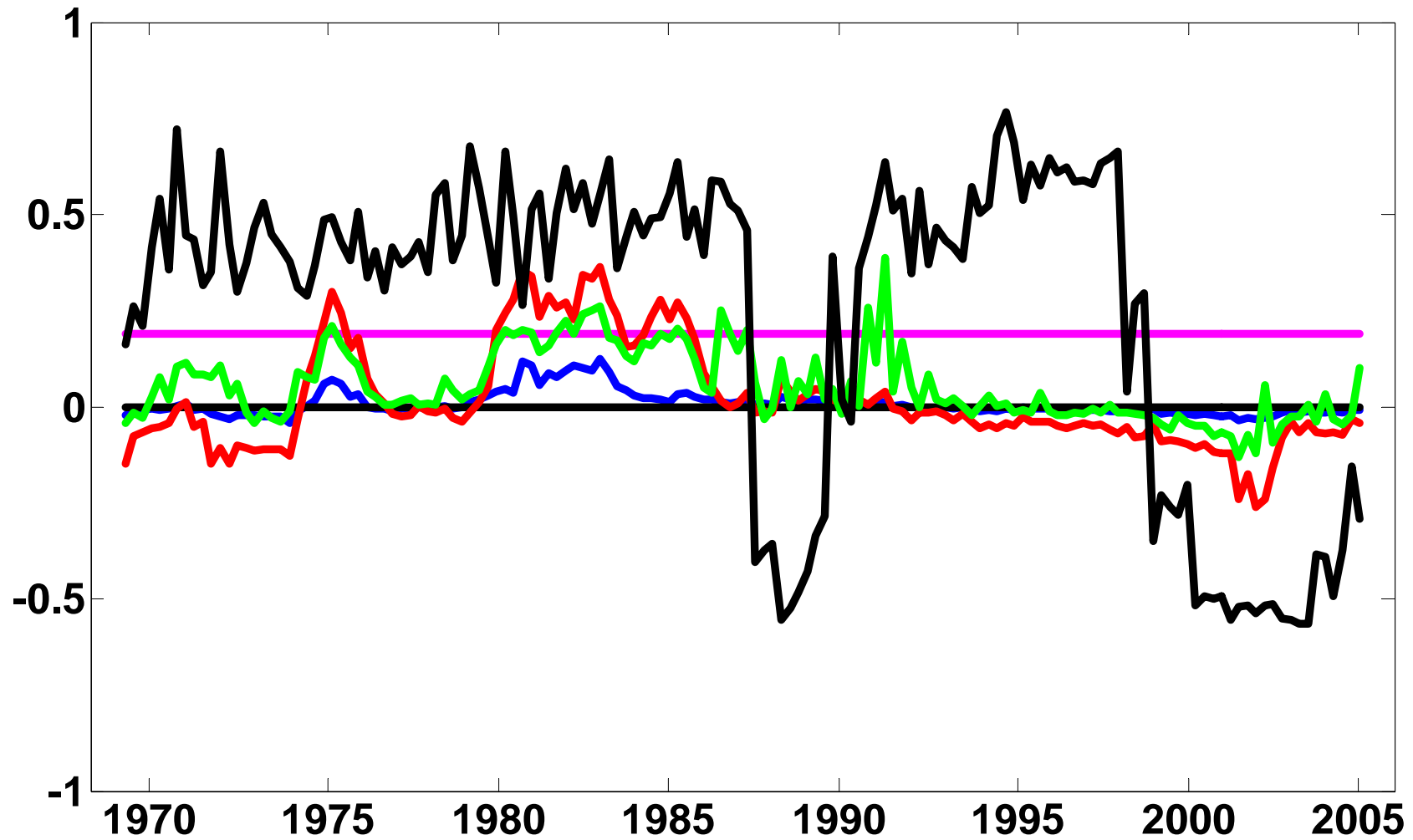
$$r_{s,t} - E_{t-1} [r_{s,t}] = 2.02F_t^y - 2.73F_t^\pi - 2.28F_t^i - 12.61F_t^{RA}$$

$$r_{b,t} - E_{t-1} [r_{b,t}] = -0.18F_t^y - 0.305F_t^\pi - 10.53F_t^i - 5.93F_t^{RA}$$

- State-dependent beta model:
 - ◆ Marginal improvement in model selection statistics.
 - ◆ Time-varying exposure to shocks in RA mostly negative for stocks and bonds, especially in times of high RA.



- Results do not meaningfully change when
 - ◆ We use real GDP growth, consumption growth instead of output gap (as required by NK models).
 - ◆ Alternative time-varying volatility or factor exposure specifications are used.
 - ◆ We allow for time-varying expected returns.
 - ◆ We allow for exogenous structural changes/breaks in the betas
 - NBER Recession Dummy.
 - Volcker Dummy (79-82), Post-Volcker Dummy.
 - Break Output Volatility (Great Moderation, 84-now).
 - Dummies for high exogenous, interest rate volatility, for active MP.



In search for alternative explanations

- Flight-to-safety

- ◆ Investors switch from the risky asset, stocks, to a safe haven, bonds, in times of increases stock market uncertainty (see e.g. [Connolly, Stivers, Sun \(JFQA,05\)](#))
- ◆ Price effects of these portfolio shifts are expected to generate negative stock-bond correlations.
- ◆ Measures of Stock Market Uncertainty:
 - VIX implied volatility index (only 85 – now).
 - Conditional Equity Market Volatility from our Statistical Model (full period)
- ◆ Implied/Statistical Volatility versus Economic Uncertainty.

■ Cross-Market Liquidity Effects

- ◆ Increasing evidence that liquidity is priced in both stock and bond markets.
- ◆ No consensus on commonality of stock and bond liquidity shocks
- ◆ Monetary policy stance can affect liquidity in both markets by altering the terms of margin borrowing and by alleviating the borrowing constraints of dealers.

=> If this effect dominates, we expect liquidity to contribute to higher stock-bond correlations.

- ◆ In crisis periods, flight-to-liquidity may mean that traders sell the less liquid stocks and buy liquid bonds. Resulting price-pressure may induce negative stock-bond correlation. Of course this effect is likely to be also related to episodes of flight-to-safety.

■ Liquidity Measures

◆ Bonds :

- Equal-weighted average of quoted bid-ask spreads of off-the-run Treasury Fixed income securities of 1 month, 3 months, 1, 2, 3, 5, 7, 10, 20, 30 years to maturity from [Goyenko \(05\)](#).
- Yield difference between off and on the run bonds (on/off-the-run spread)

◆ Equities :

- Proportion of zero daily firm returns and/or zero volume days, averaged over the quarter, from [Bekaert, Harvey, and Lundblad \(05\)](#).

- **Consumer Confidence**
 - ◆ Consumer confidence may contain an additional component that proxies for particular behavioral biases.
 - ◆ To the extent that individual investors are more prevalent in stock than in bond markets, stock prices may be bid up relative to bond prices in times of high consumer confidence.
 - ◆ We measure consumer confidence by the **University of Michigan's Consumer Sentiment Index**.

- Empirical Setup:

$$\hat{\varepsilon}_{s,t} \hat{\varepsilon}_{b,t} = \gamma_0 + \gamma_1' \varepsilon_{z,t}$$

- Stock/Bond residuals come from our best performing eight factor model. Results are robust to other specifications.
- We identify shocks in our information variables using a $VAR(n)$, where n is determined by the Schwartz criterion.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Constant	0,591 (0,028)	2,207 (0,025)	0,591 (0,028)	2,309 (0,078)	0,591 (0,028)	0,591 (0,026)	0,591 (0,024)	0,591 (0,021)	0,591 (0,022)	0,418 (0,078)	0,591 (0,020)
Equity Volatility	-157,569 (0,014)							-183,820 (0,005)	-130,573 (0,056)	-200,925 (0,003)	-127,653 (0,049)
VIX		-0,111 (0,036)									
Consumer Confidence			0,029 (0,403)					0,045 (0,227)	0,053 (0,148)	0,037 (0,318)	0,041 (0,259)
On/Off-the-Run Spread				-0,138 (0,037)							
Bond Illiquidity					18,869 (0,404)			22,844 (0,279)	14,161 (0,500)	14,189 (0,501)	33,792 (0,090)
Equity Illiquidity (zero return, volume)						-1454,492 (0,173)		-1764,899 (0,087)		-1876,766 (0,060)	
Equity Illiquidity (zero return)							-61,888 (0,079)		-63,413 (0,069)		-65,386 (0,055)
Interaction Bond-Equity Illiquidity										129,450 (0,029)	34,790 (0,027)
Adjusted R2	3,47%	6,73%	-0,17%	7,89%	0,00%	2,49%	4,49%	7,08%	5,74%	11,58%	8,84%

- We give **maximum flexibility** for economic **fundamentals** to explain the time variation in stock-bond correlations:
 - ◆ Wide range of fundamentals.
 - ◆ Flexible specifications.

- Despite flexibility, we **do not get close to explaining the average/conditional level of stock-bond correlations.**

- Alternative (**non-fundamental**) explanations look more **promising.**

- How much do fundamental – non-fundamental factors explain of stock – bond return **volatility**?
- Incorporate liquidity in the fundamental model.
- Stock-bond correlations at different frequencies (daily -> 5 year)