

# Appendixes to "Life-Cycle Patterns of Interest Rate Markups in Small Firm Finance"

## Appendix1:

### The bankruptcy probability model SEBRA<sup>1</sup>

This appendix contains a brief description of the bankruptcy probability model SEBRA. More detailed presentations are given in Eklund, Larsen, and Bernhardsen (2001) and in Bernhardsen (2001).

The SEBRA model is estimated based on annual firm level accounting data covering almost all Norwegian limited liability firms. Estimating firm level bankruptcy or default probabilities from firms' financial statements has been common in the credit risk literature.<sup>2</sup> Moody's KMV RiskCalc<sup>TM</sup> (see Dwyer, Kocagil, and Stein (2006)) is also in the same tradition. The SEBRA model predicts the probability that a firm has its last year with a submitted account and within the next three years the firm is registered as bankrupt. All RHS variables, which are either firm or industry specific, are collected from the the Register of Public Accounts at the Brønnøysund Register Centre.<sup>3</sup> In the above-mentioned papers, the SEBRA model is estimated using data from 1990 to 1996. For this paper, however, the data used to estimate SEBRA cover the years 1990 – 2001. Nevertheless, we apply the same model specification as in Bernhardsen (2001) and in Eklund, Larsen, and Bernhardsen (2001). Firms with total assets less than NOK 500,000 ( $\approx$  € 65,000) are excluded. The total data set used consists of about 836,640 firm observations. The estimated model is a logit model in the predicted bankruptcy probability  $\hat{p}$  with the following RHS variables  $x_i$ :

- Earnings
  - earnings in per cent of total assets ( $tkr$ )
  
- Liquidity
  - liquid assets less short-term debt in per cent of operating revenues ( $lik$ )

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<sup>1</sup>We are grateful to Eivind Bernhardsen for providing us with the program used to estimate the SEBRA model.

<sup>2</sup>An early example is Ohlson (1980). A review of the literature can be found in Morris (1997).

<sup>3</sup>Electronic versions of these accounts have been supplied by Dun & Bradstreet.

- unpaid indirect taxes in per cent of total assets (*ube*)
- trade accounts payable in per cent of total assets (*lev*)
- Financial strength
  - equity in per cent of total assets (*eka*)
  - dummy for the event of book equity less than paid-in capital (*taptek*)
  - dummy for dividend payments the last accounting year (*div*)
- Industry variables
  - industry average for *eka* (*meaneka*)
  - industry average for *lev* (*meanlev*)
  - industry standard deviation for *tkr* (*stdtkr*)
- Age
  - dummy variable for each of the first 8 years of the firm's age ( $a_1$  to  $a_8$ )
- Size
  - total assets (*size*)

The structure of the model is as follow:

$$\hat{p} = \frac{1}{1 + e^{-\hat{y}}} \quad \text{where}$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 T_1(x_1) + \hat{\beta}_2 T_2(x_2) + \dots + \hat{\beta}_k T_k(x_k) \quad \text{and}$$

$$T_i(x_i) = \begin{cases} \frac{1}{1 + e^{-\left(\frac{x_i - \hat{\alpha}_i}{\hat{\delta}_i}\right)}} & \text{if } x_i \in \{eka, tkr, lik, lev, ube\} \\ x_i & \text{if } x_i \notin \{eka, tkr, lik, lev, ube\} \end{cases}$$

The values of the estimated coefficients are reported in the following table:

Estimated coefficients

Variable	$\beta$	$\alpha/\delta$	$1/\delta$
<i>eka</i>	-1.2283	-0.6970	0.0749
<i>tkr</i>	-1.0750	0.1092	0.2291
<i>lik</i>	-1.1847	3.7600	0.1894
<i>lev</i>	1.2627	0.2518	0.1929
<i>ube</i>	7.6555	1.3233	0.0256
$a_1$	0.5179	...	...
$a_2$	0.5595	...	...
$a_3$	0.5222	...	...
$a_4$	0.4326	...	...
$a_5$	0.3225	...	...
$a_6$	0.1914	...	...
$a_7$	0.1248	...	...
$a_8$	0.1116	...	...
<i>div</i>	-1.1595	...	...
<i>taptek</i>	0.5183	...	...
<i>size</i>	-0.0270	...	...
<i>meanlev</i>	2.5761	...	...
<i>meaneka</i>	-4.3491	...	...
<i>stdtkr</i>	4.2219	...	...
constant	-10.230	...	...

All coefficients are significantly different from 0 at significance level of 1 per cent, except the coefficient for  $a_8$  which has a  $p$ -value of 0.012 and  $\alpha/\delta$  for *tkr* with a  $p$ -value of 0.235.

As expected  $\hat{p}$  decreases in *tkr*, *eka*, and *lik*, and increases in *lev* and *ube*. For the first 8 years of a firm's life, the model predicts lower bankruptcy probability by each year, except from the first to the second year. After 8 years, age has by construction no effect on the bankruptcy probability. For the 5 non-linearly transformed variables, the marginal effect on  $\hat{p}$  is non-linear in the sense that the absolute value of the marginal effect has a peak around a certain value of  $x_i$ .

Syversten (2004) compares the predictive power of the SEBRA model, estimated on data from 1990 to 1996, with that of Moody's KMV Private Firm model for Norway.<sup>4</sup> Syversten applies "power curves" and their corresponding "accuracy ratios" to compare the bankruptcy predictions of SEBRA and the default probability pre-

<sup>4</sup>As KMV Private for Norway only covers about 3,500 firms and the SEBRA model covers more than 100,000 firms the comparison is based on a relatively small sample of the firms in the SEBRA model.

dictions of KMV Private to actual bankruptcies for the four years 1998 – 2001 and concludes that SEBRA’s accuracy is at least as good as the accuracy of KMV Private.

**Appendix 2:  
Results from Model 5**

Model 5 is the same as Model 2 but we exclude the variable  $coll_{i,t-1}$ , the ratio of firm’s collateralizable assets to its total debt lagged one year.

Results. dependent variable $m_{i,t}$	
Independent variable	Model 5
$\beta_0$	7.3981*** (12.66)
$d_{1;i,t}$	-.05579 (-0.12)
$d_{2;i,t}$	.3721 (-0.63)
$d_{3;i,t}$	.4873 (1.15)
$d_{4;i,t}$	1.3672** (2.52)
$d_{5;j,t}$	.5587 (0.64)
$d_{6;j,t}$	-.6127 (-0.55)
$d_{7;j,t}$	-4.5041*** (-5.85)
$VL_{c,k}$	26.669*** (4.02)
$HI_{c,t}$	-0.0004 (-1.12)
$dbtr_{t-1}$	-2.2073*** (-9.43)
$tass_{t-1}$	-0.0000 (0.31)
$F$ -test for $HI_{c,t}$ terms	0.0000
$F$ -test for $VL_{c,k}$ terms	0.0000
# clusters	36
# observations	30482
$R^2$	0.0189

For the sake of brevity we do not report the estimated coefficients of the interaction terms, by themselves these coefficients do not have any interesting economic interpretations.  $t$ -values are reported in the parentheses below the coefficients. The  $t$ -values are White-robust and adjusted for clustering of  $HI_{c,t}$ . \* represents a 10 per cent statistical significance, \*\* 5 per cent significance and \*\*\* 1 per cent significance. For the  $F$ -test we report the  $p$ -values.

Predicted Markups. Model 5

Volatility fractiles	Age groups, years														
	1-3		4-6		7-9		10-14		15-19		20-29		30-40		Above 40
5 pct.	5.21 (0.18)	↗**	5.51 (0.15)	→	5.82 (0.25)	→	6.08 (0.22)	→	6.07 (0.31)	→	5.94 (0.25)	↘***	4.91 (0.36)	↘**	3.93 (0.24)
25 pct.	5.34 (0.18)	↗**	5.67 (0.15)	↗*	6.06 (0.24)	→	6.31 (0.23)	→	6.38 (0.29)	→	6.29 (0.21)	↘***	5.56 (0.33)	↘**	4.57 (0.17)
50 pct.	5.48 (0.19)	↗***	5.85 (0.17)	↗**	6.33 (0.25)	→	6.57 (0.24)	↗*	6.73 (0.28)	→	6.66 (0.20)	→	6.28 (0.31)	↘***	5.28 (0.15)
75 pct.	5.67 (0.21)	↗***	6.08 (0.19)	↗***	6.68 (0.27)	→	6.91 (0.26)	↗**	7.19 (0.29)	→	7.16 (0.22)	→	7.22 (0.32)	↘***	6.21 (0.23)
95 pct.	6.11 (0.28)	↗**	6.62 (0.27)	↗***	7.49 (0.36)	→	7.69 (0.32)	↗*	8.25 (0.38)	→	8.30 (0.39)	↗**	9.39 (0.46)	→	8.34 (0.53)

Predicted interest rate markups reported as percentage points. Predicted standard errors in parantheses below. The Herfindahl index and the control variables are all set at their median values when the predictions are calculated. Increasing or decreasing arrows with one, two or three stars at the end indicate a 10, 5 or 1 per cent statistical significance in the difference between two neighbouring predictions. A horizontal arrow indicates no statistical significant difference between the predictions. The differences and their standard errors are calculated using the estimated model and covariance matrix.

When checking the effect of increased opaqueness within age groups we find, by looking at  $\hat{\gamma}_0$  and  $\hat{\gamma}_0 + \hat{\gamma}_j$  for  $j = 1 \dots 7$  and the corresponding covariance matrixes, that there is a significant increase in the markup for all age groups.

Rgearding the Herfindahl index ( $HI_{c,t}$ ) we find, by looking at  $\hat{\delta}_0$  and  $\hat{\delta}_0 + \hat{\delta}_j$  for  $j = 1 \dots 7$  and the corresponding covariance matrixes, that there is a positive and significant effect of higher bank market concentration on the markups for firms older than 40 years. We get a negative and significant effect in the age group 15–19 years, but no significant effect for any of the other age groups.

**Appendix 3:**  
**Descriptive statistics by the eight age groups**

Age group 1–3 years, 4,633 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	6,670	16,357	0	738,177
Total assets	5,161	16,836	501	575,734
Bank debt	2,053	9,520	0	333,961
Collateralizable assets to total debt	.4852	.4670	0	2
Total debt to total assets	.8165	.1341	.0722	.9998
Sales growth	96.99	1,384.9	−.9971	50,632
Interest rate	13.81	6.29	7.23	37.50
Interest rate markup	5.59	6.26	−11.71	30.15
Probability of bankruptcy	.0212	.0379	.0002	.3623
Volatility of bankruptcy probability	.0171	.0143	.0001	.1856
Firm age	2.6	.5	1	3
Herfindahl index banks	1,688	414	1,111	2,895
Sum market share of 3 largest banks	62.92	7.06	48.81	78.33

This table refers to the sample used for estimating Model 1 (see Section 4). Operating income, total assets, and bankdebt are measured in NOK thousands. Interest rate and interest rate markup are measured as percentage points. Market shares and the corresponding Herfindahl index are also measured as percenatge points. Probability of bankruptcy, measured as a ratio, is predicted from the SEBRA model. Firm age is measured in years. Collateralizable assets to total debt, total debt to total assets, and sales growth are also measured as ratios. For the precise definition of collateralizable assets to total debt, volatility of bankruptcy probability, and the Herfindahl index, see sections 3.3 and 3.4. table of descriptive statistics broken down on different age groups can be obtained from teh authors.

Age group 4–6 years, 6,201 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	9,798	73,280	0	4,120,349
Total assets	6,979	44,605	501	2,577,061
Bank debt	2,454	12,667	0	473,837
Collateralizable assets to total debt	.5153	.4761	0	2
Total debt to total assets	.7932	.1415	.0035	.9998
Sales growth	4.6571	113.03	−.09996	4,348
Interest rate	13.90	6.30	7.23	37.50
Interest rate markup	5.94	6.20	−12.62	30.27
Probability of bankruptcy	.0154	.0293	.0001	.2930
Volatility of bankruptcy probability	.0171	.0132	.0001	.14206
Firm age	4.9	.8	4	6
Herfindahl index banks	1,680	415	1,111	2,895
Sum market share of 3 largest banks	62.88	6.92	48.81	78.33

Age group 7–9 years, 4,623 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	9,362	28,299	0	1,036,267
Total assets	5,996	15,706	504	463,278
Bank debt	2,045	5,939	0	185,000
Collateralizable assets to total debt	.5527	.5048	0	2
Total debt to total assets	.7666	.1494	.0394	.9998
Sales growth	4.521	116.3	−.9992	5,982
Interest rate	14.21	6.51	7.23	37.50
Interest rate markup	6.49	6.44	−4.81	30.55
Probability of bankruptcy	.0121	.0244	.0001	.3333
Volatility of bankruptcy probability	.0166	.0123	.0006	.1118
Firm age	8.0	.8	7	9
Herfindahl index banks	1,666	401	1,111	2,895
Sum market share of 3 largest banks	62.81	6.85	48.81	78.33

Age group 10–14 years, 7,079 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	9,261	17,717	0	813,249
Total assets	5,914	29,238	500	2,217,586
Bank debt	1,785	5,925	0	323,227
Collateralizable assets to total debt	.5316	.5022	0	2
Total debt to total assets	.7521	.1547	.0023	.9999
Sales growth	5.0586	181.13	−.9999	11,652
Interest rate	14.41	6.56	7.23	37.50
Interest rate markup	6.75	6.51	−6.03	30.56
Probability of bankruptcy	.0105	.0209	.0001	.2661
Volatility of bankruptcy probability	.0159	.0109	.0007	.1102
Firm age	11.9	1.4	10	14
Herfindahl index banks	1,669	409	1,111	2,895
Sum market share of 3 largest banks	62.81	6.78	48.81	78.33

Age group 15–19 years, 3,117 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	10,732	20,851	0	576,553
Total assets	7,235	42,307	500	2,280,062
Bank debt	2,371	27,770	0	1,526,000
Collateralizable assets to total debt	.5564	.5228	0	2
Total debt to total assets	.7286	.1623	.0461	.9986
Sales growth	2.6002	101.42	−.9977	5488
Interest rate	14.48	6.65	7.23	37.50
Interest rate markup	6.85	6.61	−4.65	30.53
Probability of bankruptcy	.0094	.0197	.0001	.2579
Volatility of bankruptcy probability	.0151	.0101	.0008	.0741
Firm age	16.6	1.41	15	19
Herfindahl index banks	1636	388	1111	2895
Sum market share of 3 largest banks	62.50	6.62	48.81	78.33

Age group 20–29 years, 2,379 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	10,852	21,602	0	491,364
Total assets	7,794	26,755	502	1,024,570
Bank debt	2,038	5,453	0	117,350
Collateralizable assets to total debt	.6233	.5617	0	2
Total debt to total assets	.7101	.1732	.0905	.9981
Sales growth	.4492	10.971	−.9999	394.67
Interest rate	14.51	6.49	7.23	37.44
Interest rate markup	6.89	6.41	−3.00	29.89
Probability of bankruptcy	.0088	.0189	.0001	.2975
Volatility of bankruptcy probability	.0150	.0099	.0007	.0761
Firm age	23.6	2.8	20	29
Herfindahl index banks	1636	377	1111	2895
Sum market share of 3 largest banks	62.69	6.46	48.81	78.33

Age group 30–40 years, 960 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	20,846	213,290	0	6,517,853
Total assets	20,956	347,404	506	10.8·10 <sup>6</sup>
Bank debt	3,391	23,338	0	700,042
Collateralizable assets to total debt	.7200	.5982	0	2
Total debt to total assets	.6883	.1714	.0341	.9986
Sales growth	.4143	7.170	−.9953	157
Interest rate	13.98	6.47	7.23	37.50
Interest rate markup	6.43	6.43	.2.63	30.00
Probability of bankruptcy	.0082	.0186	.0001	.1896
Volatility of bankruptcy probability	.0134	.0095	.0013	.0640
Firm age	33.9	3.0	30	40
Herfindahl index banks	1619	369	1111	2895
Sum market share of 3 largest banks	62.70	6.26	48.81	78.33

Age group above 40 years, 1,181 observations

Variable	Mean	Std. Dev.	Min.	Max
Operating income	12,709	46,427	0	970,990
Total assets	111,075	24,048	509	306,379
Bank debt	3,092	7,209	0	75,477
Collateralizable assets to total debt	1.028	.6822	.0008	2
Total debt to total assets	.6313	.2154	.0124	.9987
Sales growth	1.715	55.185	−.9994	1,896
Interest rate	12.93	6.10	7.23	37.50
Interest rate markup	5.47	6.01	−6.10	30.49
Probability of bankruptcy	.0056	.0133	.0001	.1582
Volatility of bankruptcy probability	.0108	.0095	.0007	.0573
Firm age	64.6	18.8	41	148
Herfindahl index banks	1599	366	1111	2895
Sum market share of 3 largest banks	64.65	6.17	48.81	78.33

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