

What captures liquidity risk? Order based versus trade based liquidity measures

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Research question:

Does it matter what empirical measure we choose when examining the relationship between liquidity risk and expected returns?

What do we do?

- construct liquidity variables from detailed order and transactions data
- extract market-wide **common liquidity factors** that capture different "dimensions" of liquidity
- **estimate and compare models** that attempt to link liquidity risk to returns using different liquidity proxies

What do we find?

- **three common factors** → potential sources of liquidity risk
 - order based, trade based and “Amihud” (price impact) factor
- estimating a theoretical model → **only the “Amihud” factor** related to expected returns
- multifactor models → **the three liquidity variables** performs similarly to the Fama-French factors

A challenge to define and measure liquidity

- A single measure cannot capture the **different dimensions** of liquidity
 - Kyle (1985) defines a liquid market as: **tight**, **deep** and **resilient**
- Should liquidity be measured **pre-trade** or **post-trade**? [Aitken and Comerton-Forde, 2003]
 - **order based** measures (liquidity supply)
 - **trade based** measures (realized liquidity demand)
- **Data availability** problems
 - **tradeoff**: short duration of high-frequency data vs increased measurement noise using low-frequency data
 - detailed order book data only available for relatively short time periods

Is there a premium for liquidity risk?

- Theory
 - Yes, if liquidity is **time varying** [Acharya and Pedersen (2005), Jacoby et.al (2000)]
 - Yes, if liquidity shocks lead to **stochastic holding periods** [Huang (2003), Vayanos (2004)]
- Empirical evidence
 - **mixed**
 - Chordia et.al (2001), Hasbrouck and Seppi (2001), Eckbo and Norli (2002), Amihud (2002), Easley et.al (2002), Pastor and Stambaugh (2003), Acharya and Pederson (2005), Chen (2005), Bekaert et.al (2006), Korajczyk and Sadka (2006), ...
 - **mainly trade based liquidity variables**

- Aitken and Comerton-Forde (2003)
 - low correlations between trade based and order based measures
 - order based measures capture liquidity best during crisis
- Chen (2005), Korajczyk and Sadka (2006)
 - extract common liquidity factors from several aggregated liquidity measures
 - examine US data, long time series, trade based variables
 - both studies find significant risk premiums associated with their common liquidity factors

- Detailed **microstructure data** covering 5 years (1999-2004) from the Oslo Stock Exchange (OSE)
 - fully computerized **limit order book** system
 - detailed order and transactions data
 - can track every single order from its submission (through executions and amendments) to its fill or deletion
 - able to rebuild the full order book at any point in time

Use **factor analysis** to reduce the multidimensionality problem of liquidity

- construct a set of **14 liquidity variables** (5 trade based and 9 order based)
- use **common factor** method with an **orthogonal** factor rotation

Results - Factor analysis

Three common factors

	Rotated factor loadings		
	Factor 1	Factor 2	Factor 3
Trade based measures			
Trade frequency	0.00	0.89	0.09
Trading volume	0.53	0.67	0.41
Turnover	0.15	0.81	0.02
Sec. between trades	0.14	-0.67	0.41
Illiquidity ratio	-0.34	0.11	0.70
Order based measures			
Frequency of limit orders	0.39	0.71	-0.11
Depth inner quotes	0.75	0.09	0.12
Tick slope	0.85	0.33	-0.12
Price slope	0.74	0.14	0.34
Fraction of limit orders	0.54	-0.14	-0.19
Quoted spread	-0.84	-0.14	0.27
Relative quoted spread	-0.18	-0.27	0.84
Inter-quote time	-0.03	-0.59	0.45
Book symmetry	-0.14	0.01	-0.36
% of total shared variance	38 %	38 %	24 %

- factor 1: mainly order based variables
- factor 2: mainly trade based variables
- factor 3: combination (price impact)

What type of liquidity risk is priced?

Use the estimated factor structure to construct **3 marketwide factor series** and **3 company specific factor series** (for each company)

Examine what type of liquidity risk is priced in two ways:

- estimate the theoretical Acharya/Pedersen (JFE, 2005) model (**liquidity adjusted CAPM**) for each of our three common liquidity factors
 - Acharya/Pedersen uses the **illiquidity ratio** (similar to our factor 3)
- estimate a liquidity augmented **multifactor model**
 - construct liquidity risk factors based on the common liquidity factors

The Acharya/Pedersen (2005) model

- Liquidity adjusted CAPM
 - Rewrite the (frictionless economy) net returns CAPM into a gross return version with illiquidity costs.

$$E(r^i) = r^f + \overbrace{E(c^i)}^{\text{liquidity cost}} + \lambda \overbrace{\frac{\text{cov}(r^i, r^M)}{\text{var}(r^M - c^M)}}^{\beta_i^1 \text{ [market risk]}}$$
$$+ \underbrace{\lambda \frac{\text{cov}(c^i, c^M)}{\text{var}(r^M - c^M)} - \lambda \frac{\text{cov}(r^i, c^M)}{\text{var}(r^M - c^M)} - \lambda \frac{\text{cov}(c^i, r^M)}{\text{var}(r^M - c^M)}}_{\beta_i^2 \quad \beta_i^3 \quad \beta_i^4 \text{ [liquidity risks]}}$$

- c^i is the company specific liquidity cost measure, c^m is the marketwide liquidity cost measure
- we estimate the model for each of our three liquidity factors

AP-model: Beta loadings

$$E(r^i) = r^f + E(c^i) + \lambda \beta_i^{net} \quad \text{where} \quad \beta_i^{net} = \beta_i^1 + \beta_i^2 - \beta_i^3 - \beta_i^4$$

Estimates of λ based on each liquidity factor:

(a) Factor 1 (order based)

	Constant	E(c ^P)	β_1^P	β_{net}^P	R ² (%)
Model 1a	-0.29	0.01		0.75	0.15
Model 3a	-0.13		0.58		0.09

(b) Factor 2 (trade based)

	Constant	E(c ^P)	β_1^P	β_{net}^P	R ² (%)
Model 1b	1.21**	0.01		-1.00	0.23
Model 3b	0.86		-0.53		0.06

(c) Factor 3 ("Amihud factor")

	Constant	E(c ^P)	β_1^P	β_{net}^P	R ² (%)
Model 1c	-1.67**	0.01		2.30**	0.98
Model 3c	-1.26**		1.83**		0.54

A liquidity augmented multifactor model

Compare multifactor models (market model, Fama/French, liquidity augmented)

- construct portfolios based on firms' $cov(s_t^i, s_t^M)$ for each factor (β_2 in AP model)
- construct risk factors for each liquidity factor as difference in returns between the portfolio with the highest and lowest $cov(s_t^i, s_t^M)$
- examine whether any of the liquidity factors are priced

Multifactor estimation results

Risk Premia estimates							
<i>Model</i>	<i>Risk Factors</i>						delta-J (p-val.)
	Market	SMB	HML	F1	F2	F3	
CAPM	-0.023 [-11.02]						92.30 (0.00)
Fama-French	-0.024 [-13.40]	0.012 [3.05]	0.001 [0.94]				- -
CAPM and F1	-0.025 [-11.15]			0.005 [5.37]			94.38 (0.00)
CAPM and F2	-0.024 [-11.53]				0.004 [7.65]		18.66 (0.00)
CAPM and F3	-0.035 [-10.78]					0.009 [10.25]	137.63 (0.00)
CAPM and All Factors	-0.027 [-11.97]			0.006 [8.20]	0.005 [13.41]	0.004 [5.60]	4.63 (0.10)

- We find three potential liquidity risk sources → **three common factors**
 - separate order and trade based factors consistent with Aitken et.al. (2003)
- Our results indicate that the choice of empirical liquidity measure matters
 - in the Acharya/Pedersen framework the factor with loadings on variables related to **price impact** seem to contain most information about liquidity risk
 - however, in a multifactor model, the results are still inconclusive